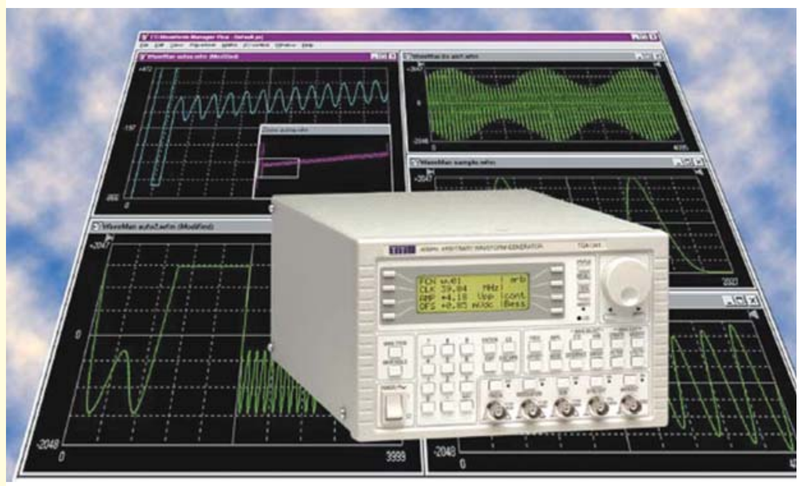
CHAPTER 20

HARMONICS

Learning Objectives

- Fundamental Wave and Harmonics
- Different Complex Waveforms
- General Equation of a Complex Wave
- R.M.S. Value of a Complex Wave
- Form Factor of a Copmplex Wave
- Power Supplied by a Complex Wave
- Harmonics in Single-phase A.C. Circuits
- Selective Resonance Due to Harmonics
- Effect of Harmonics on Measurement of Inductance and Capacitance
- Harmonics in Different Three-phase Systems
- Harmonics in Single and 3-Phase Transformers



Harmonics are the multiples of a sine wave (the fundamental frequency)

20.1. Fundamental Wave and Harmonics

Upto this stage, while dealing with alternating voltages and currents, it has been assumed that they have sinusoidal waveform or shape. Such a waveform is an ideal one and much sought after by the manufacturers and designers of alternators. But it is nearly impossible to realize such a waveform in practice. All the alternating waveforms deviate, to a greater or lesser degree, from this ideal sinusoidal shape. Such waveforms are referred to as ***non-sinusoidal*** or ***distorted*** or ***complex waveforms***.

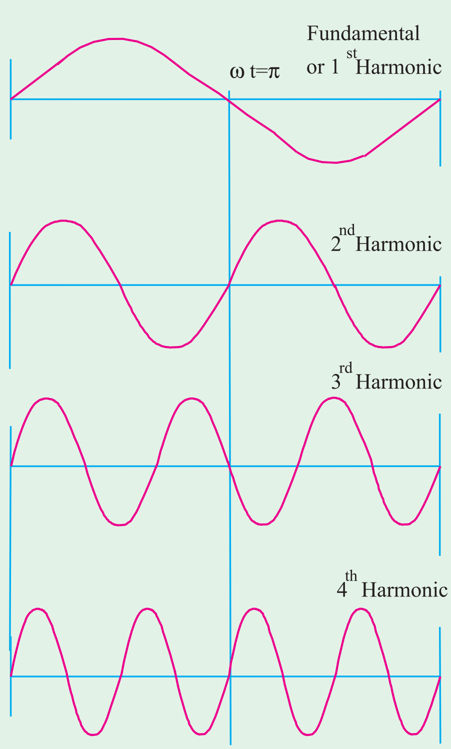


Fig. 20.1

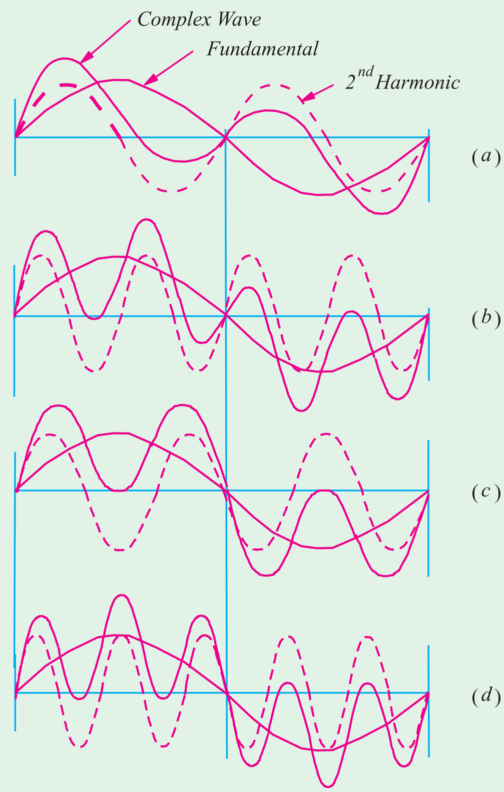


Fig. 20.2

Complex waveforms are produced due to the superposition of sinusoidal waves of different frequencies. Such waves occur in speech, music, TV, rectifier outputs and many other applications of electronics. On analysis, it is found that a complex wave essentially consists of

(a) a fundamental wave – it has the lowest frequency, say '*f*'

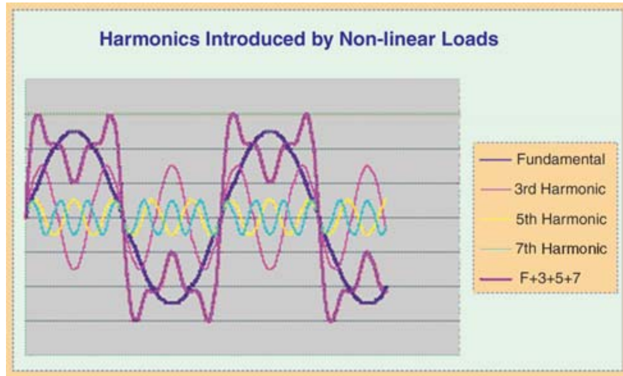
(b) a number of other sinusoidal waves whose frequencies are an integral multiple of the fundamental or basic frequency like $2f$, $3f$ and $4f$ etc.

The fundamental and its higher multiples form a ***harmonic*** series.

As shown in Fig. 20.1, fundamental wave itself is called the ***first*** harmonic. The ***second*** harmonic has frequency ***twice*** that of the fundamental, the ***third*** harmonic has frequency ***thrice*** that of the fundamental and so on.

Waves having frequencies of $2f$, $4f$ and $6f$ etc. are called ***even harmonics*** and those having frequencies of $3f$, $5f$ and $7f$ etc. are called *odd* harmonics. Expressing the above is angular frequencies, we may say that successive odd harmonics have frequencies of 3ω, 5ω and 7ω etc. and even harmonics have frequencies of 2ω, 4ω and 6ω etc.

As mentioned earlier, harmonics are introduced in the output voltage of an alternator due to many reason such as the irregularities of the flux distribution in it. Considerations of waveform and form factor are very important in the transmission of a. c. power but they are of much greater importance in radio work where the intelligibility of a signal is critically dependent on the faithful transmission of the harmonic structure of sound waves. In fact, it is only the rich harmonic content of the consonants and lesser at still plentiful harmonic content of vowels which helps the ear to distinguish a well regulated speech from a more rhythmical succession of musical sounds.



20.2. Different Complex Waveforms

Let us now find out graphically what the resultant shape of a complex wave is when we combine the fundamental with one of its harmonics. Two cases would be considered ***(i)*** when the fundamental and harmonic are in phase with each other and have equal or unequal amplitudes and ***(ii)*** when there is some phase difference between the two.

In Fig. 20.2 (*a*), the fundamental and second harmonic, both having the same amplitude, have been shown by the firm and broken line respectively. The resultant complex waveform is plotted out by algebraically adding the individual ordinates and is shown by thick line.

It may be noted that since the maximum amplitude of the harmonic is equal to the maximum amplitude of the fundamental, the complex wave is said to contain 100% of second harmonic.

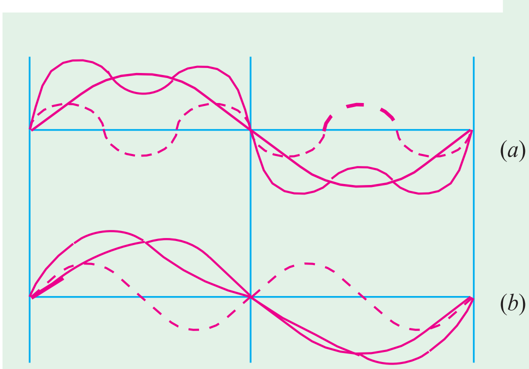


Fig. 20.3

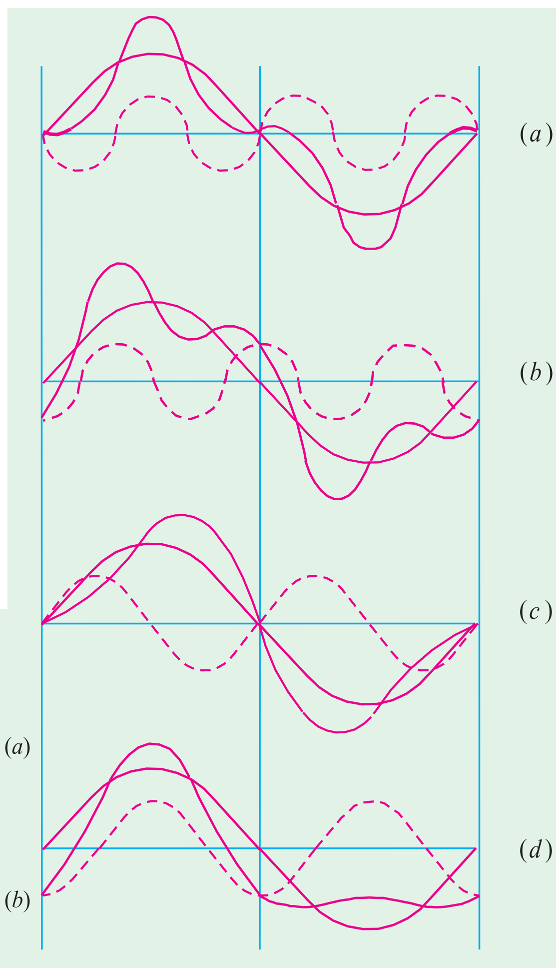


Fig. 20.4

The complex wave of Fig. 20.2 (*b*) is made up of the fundamental and 4th harmonic, that of Fig. 20.2 (*c*) consists of the fundamental and 3rd harmonic whereas that shown in Fig. 20.2 (*d*) is made up of the fundamental and 5th harmonic. Obviously, in all there cases, there is no phase difference between the fundamental and the harmonic.

Fig. 20.2 (*a*) and (*c*) have been reconstructed as Fig. 20.3 (*a*) and (*b*) respectively with the only difference that in this case, the amplitude of harmonic is half that of the fundamental *i.e.* the harmonic content is 50%.

The effect of phase difference between the fundamental and the harmonic on the shape of the resultant complex wave has been illustrated in Fig. 20.4.

Fig. 20.4 (*a*) shows the fundamental and second harmonic with phase difference of $\pi/2$ and Fig. 20.4 (*b*) shows the same with a phase difference of π. In Fig. 20.4 (*c*) and (*d*) are shown the fundamental and third harmonic with a phase difference of $\pi/2$ and π respectively. In all these figures, the amplitude of the harmonic has been taken equal to half that of the fundamental.

A careful examination of the above figures leads us to the following conclusions :

1. With ***odd*** harmonics, the positive and negative halves of the complex wave are symmetrical whatever the phase difference between the fundamental and the harmonic. In other words, the first and third quarters (*i.e.* ωt from 0 to $\pi/2$ and ωt from $3\pi/2$) and the second and fourth quarters (*i.e.* ωt from $\pi/2$ to π and ωt from $3\pi/2$ to 2π) are respectively similar.

2. (*i*) When ***even*** harmonics are present and their phase difference with the fundamental is 0 or π, then the first and fourth quarters of the complex wave are of the same phase but inverted and the same holds good for the second and third quarters.

(*ii*) When ***even*** harmonics are present and their phase difference with the fundamental is $\pi/2$ or $3\pi/2$, then there is no symmetry as shown in Fig. 20 (*a*).

3. It may also be noted that the resultant displacement of the complex wave (whether containing odd or even harmonics) is zero at $\omega t = 0$ only when the phase difference between the fundamental and the harmonics is either 0 to π.

The above conclusions are of great help in analysing a complex waveform into its harmonic constituents because a visual inspection of the complex wave enables us to rule out the presence of certain harmonics. For example, if the positive and negative half-cycles of a complex wave are symmetrical (*i.e.* the wave is symmetrical about $\omega t = 0$), then we need not look for even harmonics. In some cases, we may be able to forecast the types of harmonics to be expected from their mode of production. For example, in alternators which are symmetrically designed, we should expect *odd* harmonics only.

20.3. General Equation of a Complex Wave

Consider a complex wave which is built up of the fundamental and a few harmonics, each of which has its own peak value of phase angle. The fundamental may be represented by

$$e_1 = E_{1m} \sin(\omega t + \Psi_1)$$

the second harmonic by $e_2 = E_{2m} \sin(2\omega t + \Psi_2)$

the third harmonic by $e_3 = E_{3m} \sin(3\omega t + \Psi_3)$ and so on.

The equation for the instantaneous value of the complex wave is given by

$e = e_1 + e_2 + ... e_n = E_{1m} \sin(\omega t + \Psi_1) \times E_{2m} \sin(2\omega t + \Psi_2) + ... + I_{nm} \sin(n\omega t + \Psi_n)$
when E_{1m}, E_{2m} and E_{nm} etc. denote the maximum values or the amplitudes of the fundamental, second harmonic and *n*th harmonic etc. and Ψ_1, Ψ_2 and Ψ_n represent the phase differences with

respect to the complex wave* (*i.e.* angle between the zero value of complex wave and the corresponding zero value of the harmonic).

The number of terms in the series depends on the shape of the complex wave. In relatively simple waves, the number of terms in the series would be less, in others, more.

Similarly, the instantaneous value of the complex wave is given by

$$i = I_{1m}\sin(\omega t + \phi_1) + I_{2m}\sin(2\omega t + \phi_2) + ... + I_{nm}\sin(n\omega t + \phi_n)$$

Obviously $(\Psi_1 - \phi_1)$ is the phase difference between the harmonic voltage and current for the fundamental, $(\Psi_2 - \phi_2)$ for the second harmonic and $(\Psi_n - \Phi_2)$ for the *n*th harmonic.

20.4. R.M.S. Value of a Complex Wave

Let the equation of the given complex current wave be

$$i \quad I_{1m}\sin(\quad t \quad _1) \quad I_{2m}\sin(2 \quad t \quad _2) \quad ....I_{nm}\sin(n \quad t \quad _n)$$

Its r.m.s. value is given by $I = \sqrt{\text{average value of } i^2 \text{ over whole cycle}}$

Now $i^2 = [I_{1m}\sin(\omega t + \phi_1) + I_{2m}\sin(2\omega t + \phi_1) + ...I_{nm}\sin(n\omega t + \phi_n)]^2$

$$= I_{1m}{}^2\sin^2(\omega t + \phi_1) + I_{2m}{}^2\sin^2(2\omega t + \phi_2) + ...I_{nm}{}^2\sin^2(n\omega t + \phi_n)]^2$$

$$+ 2I_{1m}I_{2m}\sin(\omega t + \phi_1)\sin(2\omega t + \phi_2) + 2I_{1m}I_{3m}\sin(\omega t + \Phi_1)\sin(\omega t + \Phi_2) + ...$$

The right-hand side of the above equation consists of two types of terms

(i) harmonic self-products, the general expression for which is $\mathbf{I}_{pm}{}^2\sin^2(p\omega t + \phi_p)$ for the *p*th harmonic and

(ii) the products of different harmonics of the general form $2I_{pm}I_{qm}\sin(p\omega t + \phi_n)\sin(q\omega t + \phi_q)$

The average value of i^2 is the sum of the average values of these individual terms in the above equation. Let us now find the average value of the general term $I_{pm}{}^2\sin^2(p\omega t + \phi_p)$ over a whole cycle.

$$\text{Average value} = \frac{1}{\quad}\ {}_0^2\ I_{pm}{}^2\sin^2(p \quad t \quad _p)d(\quad t) \quad \frac{I_{pm}{}^2}{2}\ {}_0^2\ \sin^2(p \quad _p)d$$

$$= \frac{I_{pm}{}^2}{2}\ {}_0^2\ \frac{1 \quad \cos 2(p \quad _p)}{2}\ d \quad \frac{I_{pm}{}^2}{2}\Big| \quad \sin^2(p \quad _p)\Big|_0^2$$

$$= \frac{I_{pm}{}^2}{4\pi} \times 2\pi = \frac{I_{pm}{}^2}{2}$$

From this result, we can generalize that

$$\text{Average value of } I_{1m}{}^2\sin^2(\omega t + \phi_1) = \frac{I_{1m}{}^2}{2}$$

$$\text{Average value of } I_{2m}{}^2\sin^2(2\omega t + \phi_2) = \frac{I_{2m}{}^2}{2}$$

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* We could also express these phase angles with respect to the fundamentals wave instead of the complex wave.

Average value of $I_{nm}{}^2 \sin^2(n\omega t + \phi_n) = \frac{I_{nm}{}^2}{2}$ and so on.

Now, the average value of the product terms is

$$\frac{1}{2}\ {}_0^{2}\ I_{pm}I_{qm}\sin(p \quad t \quad {}_p)\sin(q \quad t \quad {}_q)\,d(\quad t)$$

$$\frac{I_{pm}I_{qm}}{5}\ {}_0^{2}\ \sin(p \quad {}_p)\sin(q \quad {}_q)\,d \quad 0$$

$\therefore$ Average value of $i^2 = \frac{I_{1m}{}^2}{2} + \frac{I_{2m}{}^2}{2} + ... + \frac{I_{nm}{}^2}{2}$

$\therefore$ r.m.s. value, $I = \sqrt{\text{average value of } i^2} \quad \sqrt{\frac{I_{1m}{}^2}{2} \quad \frac{I_{2m}{}^2}{2} \quad ..... \quad \frac{I_{nm}{}^2}{2}}$ ... (*i*)

$$= 0.707\sqrt{I_{1m}{}^2 + I_{2m}{}^2 + ... + I_{nm}{}^2}$$

Equation (*i*) above may also be put in the form

$$I = \sqrt{\frac{I_{1m}}{\sqrt{2}}^2 \quad \frac{I_{2m}}{\sqrt{2}}^2 \quad ..... \quad \frac{I_{nm}}{\sqrt{2}}^2} \quad \sqrt{I_1^2 \quad I_2^2 \quad .....I_n^2}$$

where $I_1 = I_{1m}/\sqrt{2}$ – r.m.s. value of fundamental

$I_2 = I_{2m}/\sqrt{2}$ – r.m.s. value of 2nd harmonic

$I_n = I_{nm}/\sqrt{2}$ – r.m.s. value of *n*th harmonic

Similarly, the r.m.s. value of a complex voltage wave is

$$E = 0.707\sqrt{E_{1m}{}^2 + E_{2m}{}^2 + ...E_{nm}{}^2} = \sqrt{E_1{}^2 + E_2{}^2 + ...E_n{}^2}$$

Hence, the **rule *is that the r.m.s. value of the complex current (or voltage) wave is given by the square-root of the sum of the squares of the r.m.s. values of its individual components.***

Note. If complex current wave contains a d.c. component of constant value I_D then its equation is given by

$$i = I_D + I_{1m}\sin(\omega t + \phi_1) + I_{2m}\sin(2\omega t + \phi_2) + ...I_{nm}\sin(n\omega t + \phi_n)$$

r.m.s. value, $I \quad \sqrt{I_D{}^2 \quad (I_{1m}/\sqrt{2})^2 \quad (I_{2m}/\sqrt{2})^2.....(I_{nm}/\sqrt{2})^2} \quad I_D{}^2 \quad I_1^2 \quad I_2{}^2 \quad ....I_n{}^2$

20.5. Form Factor of a Complex Wave

In general, it may be defined as $k_f = \frac{\text{R.M.S. value}}{\text{average value}}$

A general expression for form factor in some simple cases may be found as under :

(*i*) Sine Series. Suppose the equation of a complex voltage wave is

$$v = V_{1m}\sin\omega t \pm V_{3m}\sin 3\omega t \pm V_{5m}\sin 5\omega t$$

$$= V_{1m}\sin\theta \pm V_{3m}\sin 3\theta \pm V_{5m}\sin 5\theta \quad \text{where} \quad \omega = 2\pi/T.$$

Obviously, zeros occurs at $t = 0$ or at $\theta = 0°$ and $\theta = 180°$ or $t = T/2$.

Mean value over half-cycle is

$$V_{av} \quad \frac{1}{\quad}\ {}_0\ v d$$

$$\frac{1}{\pi}\left[V_{1m}\int_0^{\pi}\sin\theta .d\theta + V_{3m}\int_0^{\pi}\sin 3\theta .d\theta + V_{5m}\int_0^{\pi}\sin 5\theta .d\theta\right] = \frac{2}{\pi}\left(\frac{V_{1m}}{1}+\frac{V_{3m}}{3}+\frac{V_{5m}}{5}\right)$$

As found in Art. 20.4,

$$V = (V_1^2 + V_3^2 + V_5^2)^{1/2} = \frac{1}{\sqrt{2}}(V_{1m}^{\;2} + V_{3m}^{\;2} + V_{5m}^{\;2})^{1/2}$$

$$\therefore \quad k_f = \frac{(1/\sqrt{2})(V_{1m}^{\;2} + V_{3m}^{\;2} + V_{5m}^{\;2})^{1/2}}{\frac{2}{\pi}\left(V_{1m} + \frac{V_{3m}}{3} + \frac{V_{5m}}{5}\right)}$$

(*ii*) **Cosine Series.** Consider the following cosine series :

$$v = V_{1m}\cos\omega t \pm V_{3m}\cos 3\omega t \pm V_{5m}\cos 5\omega t$$
$$= V_{1m}\cos\theta \pm V_{3m}\cos 3\theta \pm V_{5m}\cos 5\theta$$

Obviously, in this case, zeros occur at $\theta = \pm\pi/2$ or 90°. Moreover, positive and negative half-cycles are symmetrical.

$$\therefore \quad V_{av} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}(V_{1m}\cos\theta - V_{3m}\cos 3\theta + V_{5m}\cos 5\theta)\,d\theta = \frac{2}{\pi}\left(V_{1m} - \frac{V_{3m}}{3} + \frac{V_{5m}}{5}\right)$$

$$\therefore \quad k_f = \frac{(1/\sqrt{2})(V_{1m}^{\;2} + V_{3m}^{\;2} + V_{5m}^{\;2})^{1/2}}{\frac{2}{\pi}\left(V_{1m} - \frac{V_{3m}}{3} + \frac{V_{5m}}{5}\right)}$$

Example 20.1. *A voltage given by v = 50 + 24 sin ωt–20 sin 2ωt is applied across the circuit shown in Fig. 20.5. What would be the readings of the instruments if ω = 10,000 rad/s. A_1 is thermoelectric ammeter, A_2 a moving-coil ammeter and V an electrostatic voltmeter.*

Solution. It may be noted that the thermoelectric ammeter and the electrostatic voltmeter record the r.m.s. values of the current and voltage respectively. But the moving coil ammeter records the average values. Since the average values of the sinusoidal waves are zero, hence the moving coil ammeter reads the d.c. component of the current only. The d.c. will pass only through the inductive branch and not through the capacitive branch.

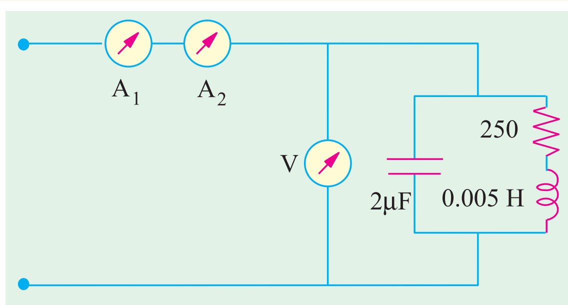

Fig. 20.5

$$\therefore \quad I_{DC} = \frac{V_{DC}}{R} = \frac{50}{250} = \mathbf{0.2\ A}$$

Equivalent impedance of the circuit at fundamental frequency is

$$\mathbf{Z_1} = \frac{(R + jX_{L1})(-jX_{C1})}{R + jX_{L1} - jX_{C1}} = \frac{(250 + j50)(-j50)}{250 + j(50 - 50)} = \frac{2{,}500 - j12{,}500}{250} = 10 - j50 = 51\angle -78°42'$$

$\therefore$ r.m.s. fundamental current $I_1 = I_{1m}/\sqrt{2} = 24/51\times\sqrt{2} = 0.33$ A

Equivalent impedance of the circuit at the second harmonic is

$$\mathbf{Z_2} = \frac{(R + jX_{L2})(-jX_{C2})}{R + jX_{L2} - jX_{C2}} = \frac{(250 + j100)(-j25)}{200 + j75} = 31.5\angle -88°43'$$

$\therefore$ r.m.s. value of second harmonic current

$$I_2 = I_{2m} / \sqrt{2} = 20 / 31.5 \times \sqrt{2} = 0.449 \text{ A}$$

r.m.s. current in the circuit is

$$I = \sqrt{I_{DC}{}^2 + I_1{}^2 + I_2{}^2} = \sqrt{0.2^2 + 0.33^2 + 0.449^2} = 0.593 \text{ A}$$

Hence, the reading of the thermoelectric ammeter is **0.593 A**

The voltmeter reading $V = \sqrt{50^2 + (24/\sqrt{2})^2 + (20/\sqrt{2})^2} =$ **54.66**

Example 20.2. *Draw one complete cycle of the following wave*

$$i = 100 \sin \omega t + 40 \sin 5 \omega t$$

Determine the average value, the r.m.s. value and form factor of the wave.

(Elect. Engineering, Osmania Univ.)

Solution. $I_{av} \quad \frac{2}{} \quad \frac{I_{2m}}{1} \quad \frac{I_{5m}}{5} \quad \frac{2}{} \quad \frac{100}{1} \quad \frac{40}{5}$ **68.7 A**

$$I = \sqrt{\frac{I_{1m}{}^2}{2} + \frac{I_{5m}{}^2}{2}} = \frac{1}{\sqrt{2}}(100^2 + 40^2)^{1/2} = \textbf{76.2 A}$$

Form factor $= \frac{I}{I_{av}} = \frac{76.2}{68.7} =$ **1.109**

20.6. Power Supplied by a Complex Wave

Let the complex voltage be represented by the equation

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + \ldots\ldots E_{nm} \sin n\omega t$$

be applied to a circuit. Let the equation of the resultant current wave be

$$i = I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \ldots\ldots I_{nm} \sin(n\omega t + \phi_n)$$

The instantaneous value of the power in the circuit is $p = ei$ watt

For obtaining the value of this product, we will have to multiply every term of the voltage wave, in turn, by every term in the current wave. The average power supplied during a cycle would be equal to the sum of the average values over one cycle of each individual product term. However, as proved in Art. 20.4 earlier, the average value of all product terms involving harmonics of different frequencies will be zero over one cycle, so that we need consider only the products of current and voltage harmonics of the same frequency.

Let us consider a general term of this nature *i.e.* $E_{nm} \sin n\omega t \times I_{nm} \sin(n\omega t - \phi_n)$ and find its average value over one cycle of the fundamental.

Average value of power $\frac{1}{2} \quad {}_0^{2} \quad E_{nm} I_{nm} \sin n \quad t \sin(n \quad t \quad {}_n) d(\quad t)$

$$\frac{E_{nm} I_{nm}}{2} \quad {}_0^{2} \quad \sin n \quad \sin(n \quad {}_n) d$$

$$\frac{E_{nm} I_{nm}}{2} \quad {}_0^{2} \quad \frac{\cos \quad {}_n \quad \cos(2n \quad {}_n)}{2} d$$

$$= \frac{E_{nm} I_{nm} \cos \phi}{2} = \frac{E_{nm}}{\sqrt{2}} \cdot \frac{I_{nm}}{\sqrt{2}} \cdot \cos \phi_n = E_n I_n \cos \phi_n$$

where E_n and I_n are the r.m.s. values of the voltage and current respectively. Hence, total average power supplied by a complex wave is ***the sum of the average power supplied by each harmonic component acting independently***.

$\therefore$ Total power is $P = E_1 I_1 \cos\phi_1 + E_2 I_2 \cos\phi_2 + ... E_n I_n \cos\phi_n$

The overall power factor is given by

$$\text{pf.*} = \frac{\text{total watts}}{\text{total voltamperes}} = \frac{E_1 I_1 \cos\phi_1 + E_2 I_2 \cos\phi_2 + ...}{E \times I}$$

when E = r.m.s. value of the complex voltage wave

I = r.m.s. value of the complex current wave

Example 20.3. *A single-phase voltage source 'e' is given by*

$$e = 141\ \sin\omega t + 42.3 \sin 3\omega t + 28.8 \sin 5\,\omega t$$

The corresponding current in the load circuit is given by

$$i = 16.5 \sin(\omega t + 54.5^\circ) + 8.43 \sin(3\omega t - 38^\circ) + 4.65 \sin(5\omega t - 34.3^\circ)$$

Find the power supplied by the source.

(Electrical Circuits, Nagpur Univ. 1991)

Solution. In problems of such type, it is best to deal with each harmonic separately

Power at fundamental

$$= E_1 I_1 \cos\phi_1 = \frac{E_{1m}}{\sqrt{2}} \cdot \frac{I_{1m}}{\sqrt{2}} \cos\phi_1 = \frac{E_{1m} I_{1m}}{2} \cos\phi_1 = \frac{141 \times 16.5}{2} \cos 54.5^\circ = 675.5\ \text{W}$$

Power at 3rd harmonic $= \dfrac{E_{3m} I_{3m}}{2} \cos\phi_3 = \dfrac{42.3 \times 8.43}{2} \cos 38^\circ = 140.5\ \text{W}$

Power at 5th harmonic $= \dfrac{28.8 \times 4.65}{2} \cos 34.3^\circ = 55.5\ \text{W}$

Total power supplied = 675.5 + 140.5 + 55.5 = **871.5 W**

Example 20.4. *A complex voltage is given by e = 60 sin ωt + 24 sin (3 ωt + ω/6) + 12 sin (5 ω t + π/3) is applied across a certain circuit and the resulting current is given by*

$$i = 0.6\ \sin(\omega t - 2\pi/10) + 0.12 \sin(\omega t - 2\pi/24) + 0.1 \sin(5\pi - 3\pi/4)$$

*Find **(i)** r.m.s value of current and voltage **(ii)** total power supplied and **(iii)** the overall power factor.*

Solution. In such problems where harmonics are involved, it is best to deal with each harmonic separately.

Power at fundamental $= E_1 I_1 \cos\phi_1 = \dfrac{E_{1m} I_{1m}}{2} \cos\phi_1 = \dfrac{60 \times 0.6}{2} \times \cos 36^\circ = 14.56\ \text{W}$

Power at 3rd harmonic $= \dfrac{E_{3m} I_{3m}}{2} \cos 45^\circ = \dfrac{24 \times 0.12}{2} \times 0.707 = 1.02\ \text{W}$

Power at 5th harmonic $= \dfrac{E_{5m} I_{5m}}{2} \cos 75^\circ = \dfrac{12 \times 0.1}{2} \times 0.2588 = 0.16\ \text{W}$

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* When harmonics are present, it is obvious that the overall p.f. of the circuit cannot be stated lagging or leading. It is simply the ratio of power in watts of voltamperes.

(*i*) R.M.S. current $I = \sqrt{I_1^2 \quad I_3^2 \quad I_5^2} \qquad \sqrt{\left(\frac{I_{1m}}{\sqrt{2}}\right)^2 \quad \left(\frac{I_{3m}}{\sqrt{2}}\right)^2 \quad \left(\frac{I_{5m}}{\sqrt{2}}\right)^2}$

$$= \sqrt{\frac{0.6^2}{2} + \frac{0.12^2}{2} + \frac{0.1^2}{2}} = \mathbf{0.438\ A}$$

R.M.S. volts, $E = \sqrt{\frac{60^2}{2} + \frac{24^2}{2} + \frac{12^2}{2}} = \mathbf{46.5\ V}$

(*ii*) Total power = 14.56 + 1.02 + 0.16 = **15.74 W**

(*iii*) Overall p.f. $= \frac{\text{watts}}{\text{voltamperes}} = \frac{15.74}{46.5 \times 0.438} = \mathbf{0.773}$

20.7. Harmonics in Single-phase A.C. Circuits

If an alternating voltage, containing various harmonics, is applied to a single-phase circuit containing linear circuit elements, then the current so produced also contains harmonics. Each harmonic voltage will produce its own current independent of others. By the principle of superposition, the combined current can be found. We will now consider some of the well-known elements like pure resistance, pure inductance and pure capacitance and then various combinations of these. In each case, we will assume that the applied complex voltage is represented by

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + \ldots + E_{nm} \sin n\omega t$$

(*a*) Pure Resistance

Let the circuit have a resistance of R which is independent of frequency.

The instantaneous current i_1 due to fundamental voltage is

$$i_1 = \frac{E_{1m} \sin \omega t}{R}$$

Similarly, $i_2 = \frac{E_{2m} \sin 2\omega t}{R}$ for 2nd harmonic

and $i_n \quad \frac{E_{nm} \sin n \quad t}{R}$ ... for nth harmonic

total current $i = i_1 + i_2 + \ldots + i_n$

$$= \frac{E_{1m} \sin \omega t}{R} + \frac{E_{2m} \sin 2\omega t}{R} + \ldots + \frac{E_{nm} \sin n\omega t}{R}$$

$$= I_{1m} \sin \omega t + I_{2m} \sin 2\omega t + \ldots + I_{nm} \sin n\omega t$$

It shows that

(*i*) the waveform of the resulting current is similar to that of the applied voltage *i.e.* the two waves are identical.

(*ii*) the percentage of harmonic content in the current wave is the same as in the applied voltage.

(*b*) Pure Inductance

Let the inductance of the circuit be L henry whose reactance varies directly as the frequency of the applied voltage. Its reactance for the fundamental would be $X_1 = \omega L$; for the second harmonic, $X_2 = 2\omega L$, for the third harmonic, $X_3 = 2\omega L$ and for the nth harmonic $X_n = n\omega L$.

However, for every harmonic term, the current will lag behind the voltage by 90°.

Current due to fundamental, $i_1 = \frac{E_{1m}}{\omega L}\sin(\omega t - \pi/2)$

Current due to 2nd harmonic, $i_2 = \frac{E_{2m}}{2\omega L}\sin(2\omega t - \pi/2)$

Current due to 3rd harmonic, $i_3 = \frac{E_{3m}}{3\omega L}\sin(3\omega t - \pi/2)$

Current due to nth harmonic, $i_n = \frac{E_{nm}}{n\omega L}\sin(n\omega t - \pi/2)$

$\therefore$ Total current $i = i_1 + i_2 + ...i_n$

$$\frac{E_{1m}}{\;L}\sin(\;t \quad /2) \quad \frac{E_{2m}}{2\;L}\sin(2\;t \quad /2) \quad .... \quad \frac{E_{nm}}{n\;L}\sin(n\;t \quad /2)$$

It can be seen from the above equation that

(*i*) the waveform of the current differs from that of the applied voltage.

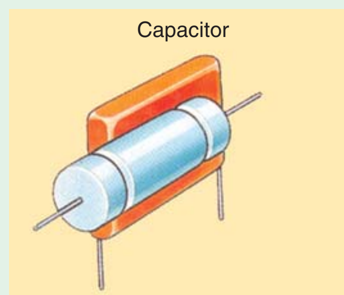
(*ii*) for the nth harmonic, the percentage harmonic content in the current-wave is $1/n$ of the corresponding harmonic content in the voltage wave. It means that in an inductive circuit, the current waveform shows less distortion that the voltage waveform. In this case, current more nearly approaches a sine wave than it does in a circuit containing resistance.

(*c*) Pure Capacitance

In this case,

$X_1 = \frac{1}{\omega C}$ – for fundamental ; $X_2 = \frac{1}{2\omega C}$ – for 2nd harmonic

$X_3 = \frac{1}{3\omega C}$ – for 3rd harmonic ; $X_n = \frac{1}{n\omega C}$ – for nth harmonic



The amount of charge the device can store for a given voltage difference is called the capacitance

$$i_1 = \frac{E_{1m}}{1/\omega C}\sin(\omega t + \pi/2) = \omega C E_{1n}\sin(\omega t + \pi/2)$$

$$i_2 = \frac{E_{2m}}{1/2\omega C}\sin(2\omega t + \pi/2) = 2\omega C E_{2m}\sin(2\omega t + \pi/2)$$

$$i_n = \frac{E_{nm}}{1/n\omega C}\sin(n\omega t + \pi/2) = n\omega C E_{nm}\sin(n\omega t + \pi/2)$$

For every harmonic term, the current will lead the voltage by 90°.

Now $i = i_1 + i_2 + ... + i_n$

$$= \omega C E_{1m}\sin(\omega t + \pi/2) + 2\omega C E_{2m}\sin(2\omega t + \pi/2) + ... + n\omega C E_{nm}\sin(n\omega t + \pi/2)$$

This equation shows that

(*i*) the current and voltage waveforms are dissimilar.

(*ii*) percentage harmonic content of the current is larger than that of the applied voltage wave. For example, for nth harmonic, it would be n time larger.

(*iii*) as a result, the current wave is more distorted than the voltage wave.

(*iv*) effect of capacitor on distortion is just the reverse of that of inductance.

Example 20.5. *A complex wave of r.m.s. value 240 V has 20% 3rd harmonic content, 5% 5th harmonic content and 2% 7th harmonic content. Find the r.m.s. value of the fundamental and of each harmonic.* **(Elect. Circuits, Gujarat Univ.)**

Solution. Let V_1, V_3, V_5 and V_7 be the r.m.s. values of the fundamental and harmonic voltages. Then

$$V_3 = 0.2\ V_1\ ;\ \ V_5 = 0.05\ V_1 \text{ and } V_7 = 0.02\ V_1$$

$$240 = (V_1^2 + V_3^2 + V_5^2 + V_7^2)^{1/2}$$

$$\therefore\ 240 = [V_1^2 + (0.2\ V_1)^2 + (0.05\ V_1)^2 + (0.02\ V_1)^2]^{1/2}$$

$$\therefore\ V_1 = \mathbf{235\ V}\ ;\ V_3 = 0.2 \times 235 = \mathbf{47\ V}$$

$$V_5 = 0.05 \times 235 = \mathbf{11.75\ V}\ ;\ V_7 = 0.02 \times 235 = \mathbf{4.7\ V}$$

Example 20.6. *Derive an expression for the power, power factor and r.m.s. value for a complex wave.*

A voltage e = 250 sin $\omega t + 50$ *sin* $(3\omega t + \pi/3)$ *+ 2 sin* $(\omega t + 5\pi/6)$ *is applied to a series circuit of resistance* $20\ \Omega$ *and inductance 0.05 H. Derive* **(a)** *an expression for the current* **(b)** *the r.m.s. value of the current and for the voltage* **(c)** *the total power supplied and (d) the power factor. Take* ω *= 314 rad/s.* **(Electrical Circuits, Nagpur Univ. 1991)**

Solution. For Fundamental

$$X_1 = \omega L = 314 \times 0.05 = 15.7\ \Omega;\ \mathbf{Z}_1 = 20 + j15.7 = 25.4\angle 38.1°\ \Omega$$

For Third Harmonic

$$X_3 = 3\omega L = 3 \times 15.7 = 47.1\Omega;\ \mathbf{Z}_3 = 20 + j47.1 = 51.2\angle 67°\ \Omega$$

For Fifth Harmonic

$$X_5 = 5\,\omega\,L = 5 \times 15.7 = 78.5\Omega;\ \mathbf{Z}_5 = 20 + j78.5 = 81\angle 75.7°\ \Omega$$

(a) Expression for the current is

$$i = \frac{250}{25.4}\sin(\omega t - 38.1°) + \frac{50}{51.2}\sin(3\omega t + 60° - 67°) + \frac{20}{81}\sin(5\omega t + 150° - 75.7°)$$

$$\therefore\ \ i = 9.84\sin(\omega t - 38.1°) + 0.9\sin(3\omega t - 7°) + 0.25\sin(5\omega t + 74.3°)$$

(b) *R.M.S. current* $I = \sqrt{\dfrac{I_{1m}^2}{2} + \dfrac{I_{3m}^2}{2} + \dfrac{I_{5m}^2}{2}}$

$$I^2 = \frac{9.84^2}{2} + \frac{0.9^2}{2} + \frac{0.25^2}{2} = 48.92$$

$$\therefore\qquad I = \sqrt{48.92} = \mathbf{6.99\ A}$$

R.M.S. voltage $V = \sqrt{\dfrac{250^2}{2} + \dfrac{50^2}{2} + \dfrac{20^2}{2}} = \mathbf{180.8\ V}$

(c) Total power $= I^2R = 48.92 \times 20 = \mathbf{978\ W}$

(d) Power factor $\dfrac{\text{Watts}}{\text{VI}}\ \ \dfrac{978}{180.8\ \ 6.99}$ **0.773**

Example 20.7. *An r.m.s. current of 5 A, which has a third harmonic content, is passed through a coil having a resistance of 1 Ω and an inductance of 10 mH. The r.m.s. voltage across the coil is 20 V. Calculate the magnitude of the fundamental and harmonic components of current if the fundamental frequency is 300/2π Hz. Also, find the power dissipated.*

Solution. (*i*) Fundamental Frequency

$\omega = 300$ rad/s ; $X_L = 300 \times 10^{-2} = 3\ \Omega$ ∴ $\mathbf{Z}_1 = 1 + j\,3 = 3.16\ \angle\ 71.6°$ ohm

If V_1 is the r.m.s. value of the fundamental voltage across the coil, then

$$V_1 = I_1 Z_1 = 3.16\ I_1$$

(*ii*) Third Harmonic

$X_3 = 3 \times 3 = 9\ \Omega$; $\mathbf{Z}_3 = 1 + j\,9 = 9.05\ \angle 83.7°$ ohm ; $V_3 = I_3 Z_3 = 9.05\ I_3$

Since r.m.s. current of the complex wave is 5 A and r.m.s. voltage drop 20 V

$$5 = \sqrt{I_1^2 + I_3^2} \quad \text{and} \quad 20 = \sqrt{V_1^2 + V_3^2}$$

Substituting the values of V_1 and V_3, we get, $20 = [(3.16\ I_1)^2 + (9.05\ I_3)^2]^{1/2}$

Solving for I_1 and I_3, we have I_1 = **4.8 A** and I_3 = **1.44 A**

Power dissipated = $I^2R = 5^2 \times 1$ = **25 W**

Example 20.8. *An e.m.f. represented by the equation e = 150 sin 314 t + 50 sin 942 t is applied to a capacitor having a capacitance 20 μF. What is the r.m.s. value of the charging current ?*

Solution. For Fundamental

X_{C1} 1/ C $10^6/20$ 314 159 ; I_{1m} E_{1m}/X_{C1} 150/159 0.943 A

For Third Harmonic

$X_{C1} = 1/3\ \omega C = 159/3 = 53\ \Omega$ ∴ $I_{3m} = E_{3m}/X_{C1} = 50/53 = 0.943$ A

r.m.s. value of charging current,

$$I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2}} = \sqrt{\frac{0.943^2}{2} + \frac{0.943^2}{2}}$$

or ***I* = 0.943 A**

Example 20.9. *The voltage given by v = 100 cos 314 t + 50 sin (1570t – 30º) is applied to a circuit consisting of a 10 Ω resistance, a 0.02 H inductance and a 50 μF capacitor. Determine the instantaneous current through the circuit. Also find the r.m.s. value of the voltage and current.*

Solution. For Fundamental

$\omega = 314$ rad/s; $X_L = 314 \times 0.02 = 6.28\ \Omega$

$X_C = 10^6/314 \times 50 = 63.8\ \Omega$; $X = X_L - X_C = 6.28 - 63.8 = -57.32\ \Omega$

$Z = \sqrt{10^2 + (-57.32)^2} = 58.3\ \Omega$; $I_{1m} = 100/58.3 = 1.71$ A

$\phi_1 = \tan^{-1}(-57.32/10) = -80.2°$ (lead) ; $i_1 = 1.71\cos(314t + 80.2°)$

For Fifth Harmonic

Inductive reactance = $5\ X_L = 5 \times 6.28 = 31.4\ \Omega$

Capacitive reactance = $X_C/5 = 63.8/5 = 12.76\ \Omega$

Net reactance = 31.4 – 12.76 = 18.64 Ω

$$Z = \sqrt{10^2 + 18.64^2} = 21.2\Omega$$

I_{5m} = 50/21.2 = 2.36 A; ϕ_5 = $\tan^{-1}$ (18.64/10) = 61.8º (lag)

i_5 = 2.36 sin (1570 t – 30º – 61.8º) = 2.36 sin (1570t – 91.8º)

Hence, total instantaneous current is

$i = i_1 + i_5$ = 1.71 cos (314 t + 80.2º) + 2.36 sin (1570t – 91.8º)

$$\text{R.M.S. voltage} = \sqrt{\frac{100^2}{2} + \frac{50^2}{5}} = \mathbf{79.2\ V}$$

$$\text{R.M.S. current} = \sqrt{\frac{1.71^2}{2} + \frac{2.36^2}{2}} = \mathbf{2.06\ A}$$

Example 20.10. *A 6.36 μF capacitor is connected in parallel with a resistance of 500 Ω and the combination is connected in series with a 500-Ω resistor. The whole circuit is connected across an a.c. voltage given by e = 300 sin ωt + 100 sin* $(3\omega t + \pi/6)$.

If ω = 314 rad/s, find

(i) *power dissipated in the circuit*

(ii) *an expression for the voltage across the series resistor*

(iii) *the percentage harmonic content in the resultant current.*

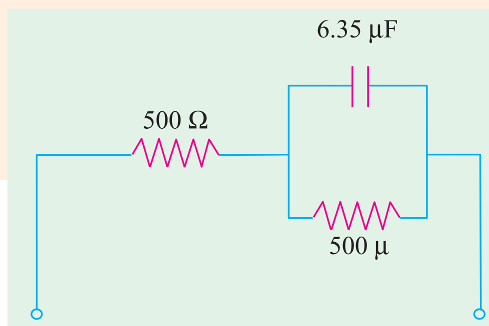


Fig. 20.6

Solution. For Fundamental

$$X_{C1} = \frac{1}{\omega C} = \frac{10^6}{314 \times 6.36} = 500\ \Omega$$

The impedance of the whole series-parallel circuit is given by

$$\mathbf{Z}_1 \quad 500 \quad \frac{500(\quad j500)}{500 \quad j500} \quad 750 \quad j250 \quad 791 \qquad 18.4^\circ$$

For Third Harmonic

$$X_{C3} = 1/3\omega\, C = 500/3 = 167\ \Omega$$

$$\therefore \quad \mathbf{Z}_3 \quad 500 \quad \frac{500(\quad j167)}{500 \quad j167} \quad 550 \quad j150 \quad 570 \qquad 15.3^\circ$$

$$\therefore \quad i = \frac{300}{791}\sin(\omega t + 18.4^\circ) + \frac{100}{570}\sin(3\omega t + 45.3^\circ)$$

$$= 0.397\ \sin(\omega t + 18.4^\circ) + 0.175 \sin(3\omega t + 45.3)$$

(i) Power dissipated = $\dfrac{E_{1m} I_{1m}}{2}\cos\phi_1 + \dfrac{E_{3m} I_{3m}}{2}\cos\phi_3$

$$= \frac{300 \times 0.379}{2} \times \cos 18.4° + \frac{100 \times 0.175}{2} \cos 15.3 = \textbf{62.4 W}$$

(ii) The voltage drop across the series resistor would be

$$E_R = iR = 500[0.379 \sin(\omega t + 18.4°) + 0.175 \sin(3\omega t + 45.3°)]$$

$$e_R = \textbf{189.5 sin } (\omega t + 18.4°) + 87.5 \sin(3\omega t + 45.3°)$$

(iii) The percentage harmonic content of the current is = 87.5/189.5 × 100 = **46.2%**

Example 20.11. *An alternating voltage of v = 1.0 sin 500t + 0.5 sin 1500t is applied across a capacitor which can be represented by a capacitance of 0.5 μF shunted by a resistance of 4,000 Ω. Determine*

(i) *the r.m.s. value of the current* **(ii)** *the r.m.s. value of the applied voltage*

(iii) *the p.f. of the circuit.* **(Circuit Theory and Components, Madras Univ.)**

Solution. For Fundamental [Fig. 20.7 (*a*)]

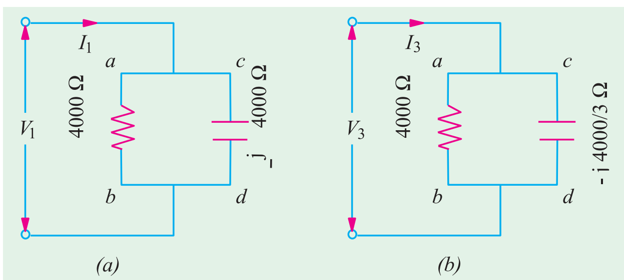


Fig. 20.7

$V_1 = 1.0/\sqrt{2} = 0.707$ V

Let, $V_1 = (0.707 + j0)$

Capacitive reactance

$= jX_{C1} = -j\ 10^6/500 \times 0.5$

$= j\ 4000\ \Omega$; $R = 4{,}000\ \Omega$

$\therefore\ I_{ab1} = 0.707/4{,}000 = 0.177$ mA

$I_{cd1} = 0.707/-j4{,}000 = j0.177$ m/A

$\therefore\ I_1 = 0.177 + j0.177$

$= 0.25\ \angle 45°$ mA

Hence, I_1 lead the fundamental voltage by 45º.

$P_{ab1} = 0.707 \times 0.177 = 0.125$ mW ; $P_{cd1} = 0$

For Third Harmonic [Fig. 20.7 (*b*)]

$\mathbf{V_3} = 0.5/\sqrt{2} = 0.3535\ \angle 0°$; $R = 4{,}000\ \Omega$: $\mathbf{X_{C3}} = -j4{,}000/3\ \Omega$

$\mathbf{I}_{ab3} = 0.3535/4000 = 0.0884\ \angle 0°$ mA ; $\mathbf{I}_{cd3} = 0.3535/-j(4{,}000/3) = j\ 0.265$ mA

$\mathbf{I_3} = 0.0884 + j0.265 = 0.28\ \angle 71.6°$ mA

$P_{ab3} = 0.3535 \times 0.0884 = 0.0313$ mW ; $P_{cd3} = 0$

(i) *R.M.S.* current $= \sqrt{I_1^2 + I_3^2} = \sqrt{0.25^2 + 0.28^2} =$ **0.374 mA**

(ii) *R.M.S.* voltage $= \sqrt{(1/\sqrt{2})^2 + (0.5/\sqrt{2})^2} =$ **0.79 V**

(iii) Power factor = watts/voltampere

Wattage = (0.125 + 0.0313) × 10^{-3} = 0.1563 × 10^{-3} W

Volt-amperes = 0.79 × 0.374 = 0.295 ; p.f. = 0.1563 × 10^{-3}/0.295 = **0.0005**

20.8. Selective Resonance due to Harmonics

When a complex voltage is applied across a circuit containing both inductance and capacitance, it may happen that the circuit resonates at one of the harmonic frequencies of the applied voltage. This phenomenon is known as ***selective resonance.***

If it is a series circuit, then large currents would be produced at resonance, even though the applied voltage due to this harmonic may be small. Consequently, it would result in large harmonic voltage appearing across both the capacitor and the inductance.

If it is a parallel circuit, then at resonant frequency, the resultant current drawn from the supply would be minimum.

It is because of the possibility of such selective resonance happening that every effort is made to eliminate harmonics in supply voltage.

However, the phenomenon of selective resonance has been usefully employed in some wave analyses for determining the harmonic content of alternating waveforms. For this purpose, a variable inductance, a variable capacitor, a variable non-inductive resistor and a fixed non-inductive resistance or shunt for an oscillograph are connected in series and connected to show the wave-form of the voltage across the fixed non-inductive resistance. The values of inductance and capacitance are adjusted successively to give resonance for the first, third, first and seventh harmonics and a record of the waveform is obtained by the oscillograph. A quick inspection of the shape of the waveform helps to detect the presence or absence of a particular harmonic.

Example 20.12. *An e.m.f. e = 200 sin ωt + 40 sin 3 ωt + 10 sin 5 ωt is impressed on a circuit comprising of a resistance of 10 Ω, a variable inductor and a capacitance of 30 μF, all connected in series. Find the value of the inductance which will give resonance with triple frequency component of the pressure and estimate the effective p.f. of the circuit,* $\omega = 300$ *radian/second.* **(Elect. Engg. I, Bombay Univ.)**

Solution. For resonance at third harmonic

$3\omega L = 1/3\omega C \quad \therefore \quad L = 1/9\omega^2 C = 10^6/9 \times 300^2 \times 30 =$ **0.041 H**

$$\mathbf{Z}_1 = 10 \quad j \quad 300 \quad 0.041 \quad \frac{10^6}{300 \quad 30} = 10 + j\,(12.3 - 111.1) = 10 - j98.8 = 99.3 \angle -84.2^\circ$$

$$Z_3 \quad 10 \quad j \quad 3 \quad L \quad \frac{1}{3 \quad C} \quad 10 \quad j(36.9 \quad 37.0) \quad 10 \quad 0^\circ$$

$$Z_5 \quad 10 \quad j \quad 5 \quad L \quad \frac{1}{5 \quad L} \quad 10 \quad j(61.5 \quad 22.2) \quad 10 \quad j39.3 \quad 40.56 \quad 75.7^\circ$$

$I_{1m} = 200/99.3 = 2.015$ A ; $I_{3m} = 40/10 = 4$A ; $I_{5m} = 10/40.56 = 0.246$ A

$$I = \sqrt{\frac{2.015^2}{2} + \frac{4^2}{2} + \frac{0.246^2}{2}} = \sqrt{10.06} = 3.172 \text{ A}$$

$$V = \sqrt{\frac{200^2}{2} + \frac{40^2}{2} + \frac{10^2}{2}} = 144.5 \text{ V} \text{ ; Power} = I^2R = 10.06 \times 10 = 100.6 \text{ W}$$

Volt-amperes $VI = 144.5 \times 3.172 = 458$ VA ; Power factor = 100.6/458 = **0.22**

Example 20.13. *A coil having R = 100 Ω and L = 0.1 H is connected in series with a capacitor across a supply, the voltage of which is given by e = 200 sin 314t + 5 sin 3454t. What capacitance would be required to produce resonance with the 11th harmonic. Find* ***(a)*** *the equation of the current and* ***(b)*** *the r.m.s. value of the current, if this capacitance is in circuit.*

Solution. For series resonance, $X_L = X_C$

Since resonance is required for 11th harmonic whose frequency is 3454 rad/s, hence

$$3454\,L = \frac{1}{3454\,C} ; C = \frac{1}{3454^2 \times 0.1} \text{ farad} = \mathbf{0.838} \ \mu\mathbf{F}$$

(*a*) For Fundamental

Inductive reactance = $\omega L = 314 \times 0.1 = 31.4\ \Omega$

Capacitive reactance = $1/\omega C = 10^6 / 0.836 \times 314 = 3796\,\Omega$

$\therefore$ Net reactance = 3796 – 31.4 = 3765 Ω ; Resistance = 100 Ω

$\therefore$ $$Z_1 = \sqrt{100^2 + 3765^2} = 3767\ \Omega\ ;\ \tan\ \phi_1 = 3765/100 = 37.65$$

$\therefore$ $\phi_1 = 88°28'$ (leading) = 1.546 radian

Now $E_{1m} = 200\ V$; $Z_1 = 3767\ \Omega$ $\therefore$ $I_{1m} = 200/3767 = 0.0531$ A

Eleventh Harmonic

New reactance = 0; Impedance $Z_{11} = 100\ \Omega$

$\therefore$ Current $I_{11m} = 5/100 = 0.05$ A ; $\phi_{11} = 0$ ... at resonance

Hence, the equation of the current is

$$i = \frac{200}{3767}(\sin 314t + 1.546) + \frac{5}{100}\sin(3454t + 0)$$

$$\boldsymbol{i = 0.0531 \sin (314 t + 1.546) + 0.05 \sin 3454 t}$$

(*b*) $I = \sqrt{(0.0531)^2/2 + (0.05)^2/2} =$ **0.052 A**

20.9. Effect of Harmonics on Measurement of Inductance and Capacitance

Generally, with the help of ammeter and voltmeter readings, the value of impedance, inductance and capacitance of a circuit can be calculated. But while dealing with complex voltages, the use of instrument readings does not, in general, give correct values of inductance and capacitance except in the case of a circuit containing only pure resistance. It is so because, in the case of resistance, the voltage and current waveforms are similar and hence the values of r.m.s. volts and r.m.s amperes (as read by the voltmeter and ammeter respectively) would be the same whether they ware sinusoidal or non-sinusoidal (*i.e.* complex).

(*i*) Effect on Inductances

Let L be the inductance of a circuit and E and I the r.m.s. values of the applied voltage and current as read by the instruments connected in the circuit. For a complex voltage

$$E = 0.707\sqrt{(E_{1m}{}^2 + E_{3n}{}^2 + E_{5m}{}^2 + ...)}$$

Hence $$I = 0.707\sqrt{\left(\frac{E_{1m}}{L}\right)^2 \quad \left(\frac{E_{3m}}{3\ \ L}\right)^2 \quad \frac{E_{5m}}{5\ \ L} \quad .....}$$

$$\frac{0.707}{L}\sqrt{E_{1m}{}^2 \quad \frac{1}{9}E_{3m}{}^2 \quad \frac{1}{25}E_{5m}{}^2 \quad ...}$$

$\therefore$ $$L \quad \frac{0.707}{I}\sqrt{E_{1m}{}^2 \quad \frac{1}{9}E_{3m}{}^2 \quad \frac{1}{25}E_{5m}{}^2 \quad .....}$$

For calculating the value of L from the above expression, it is necessary to known the absolute value of the amplitudes of several harmonic voltages. But, in practices, it is more convenient to deal with relative values than with absolute values. For this purpose, let us multiply and divide the right-hand side of the above expression by E but write the E ***in the denominator*** in its form 0.707 $\sqrt{(E_{1m}{}^2 + E_{3m}{}^2 + E_{5m}{}^2 + ...)}$

$\therefore$ $$L \quad \frac{0.707}{I}\sqrt{E_{1m}{}^2 \quad 1/9.E_{3m}{}^2 \quad 1/25.E_{5m}{}^2 \quad .....} \qquad \frac{E}{0.707\sqrt{(E_{1m}{}^2 \quad E_{3m}{}^2 \quad E_{5m}{}^2 \quad ......)}}$$

or $$L = \frac{E}{\omega I}\sqrt{\frac{E_{1m}^2 + 1/9\,E_{3m}^2 + 1/25\,E_{5m}^2 + ...)}{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}} = \frac{E}{\omega I}\sqrt{\frac{1 + 1/9\,(E_{3m}/E_{1m})^2 + 1/25\,(E_{5m}/E_{1m})^2 + ...}{1 + (E_{3m}/E_{1m})^2 + (E_{5m}/E_{1})^2 + ...}}$$

If the effect of harmonics were to be neglected, then the value of the inductance would appear to be $E/\omega I$ but the true or actual value is less than this. The apparent value has to be multiplied by the quantity under the radical to get the true value of inductance when harmonics are present.

The quantity under the radical is called the correction factor *i.e.*

True inductance (L) = Apparent inductance (L') × correction factor

(*ii*) Effect on Capacitance

Let the capacitance of the circuit be C farads and E and I the instrument readings for voltage and current. Since the instruments read r.m.s. values, hence, as before,

$$E = 0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}$$

Hence $$I = 0.707\sqrt{\left(\frac{E_{1m}}{1/\omega C}\right)^2 + \left(\frac{E_{3m}}{1/3\omega C}\right)^2 + \left(\frac{E_{5m}}{1/5\omega C}\right)^2 + ...}$$

$$= 0.707\sqrt{(\omega CE_{1m})^2 + (3\omega CE_{3m})^2 + (5\omega C\,E_{5m})^2 + ...}$$

$$= 0.707\ \omega C\sqrt{E_{1m}^2 + 9E_{3m}0^2 + 25E_{sm}^2 + ...}$$

$$\therefore\quad C = \frac{I}{0.707\omega\sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + ...)}}$$

Again, we will multiply and divide the right-hand side E but in this case, we will write E in the *numerator* in its form $[0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}]$

$$\therefore\quad C = \frac{1}{0.707\,\omega E\sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25\,E_{5m}^2 + ...)}} \times 0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}$$

$$= \frac{1}{\omega E}\sqrt{\frac{E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...}{E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + ...}} = \frac{1}{\omega E}\sqrt{\frac{1 + (E_{3m}/E_{1m})^2 + (E_{5m}/E_{1m})^2 + ...}{1 + 9(E_{3m}/E_{1m})^2 + 25(E_{5m}/E_{1m})^2 + ...}}$$

Again, if the effects of harmonics were neglected, the value of capacitance would appear to be $I/\omega E$ but its true value is less than this. For getting the true value, this apparent value will have to be multiplied by the quantity under the radical (which, therefore, is referred to as correction factor).*

∴ True capacitance (C) = Apparent capacitance (C') × correction factor

Example 20.14. *A current of 50-Hz containing first, third and fifth harmonics of maximum values 100, 15 and 12 A respectively, is sent through an ammeter and an inductive coil of negligibly small resistance. A voltmeter connected to the terminals shows 75 V. What would be the current indicated by the ammeter and what is the exact value of the inductance of the coil in henrys ?*

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* It may be noted that this correction factor is different from that in the case of pure inductance.

Solution. The r.m.s. current is

$$I = 0.707\sqrt{I_{1m}{}^2 + I_{3m}{}^2 + I_{5m}{}^2} = 0.707\sqrt{(100^2 + 15^2 + 12^2)} = 72 \text{ A}$$

Hence, current indicated by the ammeter is **72 A**

Now $$E = 0.707\sqrt{(E_{1m}{}^2 + E_{3m}{}^2 + E_{5m}{}^2)}$$

Also $$I_{1m} = \frac{E_{1m}}{\omega L}; I_{3m} = \frac{E_{3m}}{3\omega L}; I_{5m} = \frac{E_{5m}}{5\omega L}$$

$$\therefore \quad E_{1m} = I_{1m} \cdot \omega L; E_{3m} = I_{3m} 3\omega L; E_{5m} = I_{5m} \cdot 5\omega L$$

$$\therefore \quad E = 0.707\sqrt{(I_{1m}\omega L)^2 + (I_{3m} 3\omega L)^2 + (I_{5m} 5\omega L)^2} = 0.707\omega L\sqrt{I_{1m}{}^2 + 9I_{3m}{}^2 + 25I_{5m}{}^2}$$

$$\therefore \quad 75 = 0.707\, L \times 2\pi \times 50\sqrt{100^2 + 9 \times 15^2 + 25 \times 12^2}$$

$\therefore$ L = **0.0027 H**

Note. Apparent inductance $L' = \dfrac{E}{\omega I} = \dfrac{75}{2\pi \times 50 \times 72} = 0.00331$ H

Example 20.15. *The capacitance of a 20 μF capacitor is checked by direct connection to an alternating voltage which is supposed to be sinusoidal, an electrostatic voltmeter and a dynamometer ammeter being used for measurement. If the voltage actually follows the law,*

$$e = 100 \sin 250\, t + 20 \sin (500 - \phi) + 10 \sin (750\, t - \phi)$$

Calculate the value of capacitance as obtained from the direct ratio of the instrument readings.

Solution. True value, $C = 20$ μF

Apparent value C' = value read by the instruments

Now, $C = C'$ × correction factor.

Let us find the value of correction factor.

Here $E_{1m} = 100$; $E_{2m} = 20$ and $E_{3m} = 10$

$$\therefore \text{ Correction factor } \sqrt{\frac{E_{1m}{}^2 \quad E_{2m}{}^2 \quad E_{3m}{}^2}{E_{1m} \quad 4E_{2m}{}^2 \quad 9E_{5m}{}^2}} \quad \sqrt{\frac{100^2 \quad 20^2 \quad 10^2}{100^2 \quad 4 \quad 20^2 \quad 9 \quad 10^2}} \quad 0.9166$$

$20 = C' \times 0.9166$ $\therefore$ $C' = 20 / 0.9166$ = **21.82 μF.**

21.10. Harmonics in Different Three-phase Systems

In tree-phase systems, harmonics may be produced in the same way as in single-phase systems.

Hence, for all calculation they are treated in the same manner *i.e.* each harmonic is treated separately. Usually, even harmonics are absent in such systems. But care must be exercised when dealing with odd, especially, third harmonics and all multiples of 3rd harmonic (also called the triple-*n* harmonics).

(*a*) Expressions for Phase E.M.Fs.

Let us consider a 3-phase alternator having identical phase windings (R, Y and B) in which harmonics are produced. The three phase e.m.fs. would be represented in their proper phase sequence by the equation.

$$e_R = E_{1m}(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5) + ...$$

$$e_Y = E_{1m}\sin\left(\omega t - \frac{2\pi}{3} + \Psi_1\right) + E_{3m}\sin 3\left(\omega t - \frac{2\pi}{3} + \Psi_3\right) + E_{5m}\sin 5\left(\omega t - \frac{2\pi}{3} + \Psi_5\right)$$

$$e_B = E_{1m}\sin\left(\omega t - \frac{4\pi}{3} + \Psi_1\right) + E_{3m}\sin 3\left(\omega t - \frac{4\pi}{3} + \Psi_3\right) + E_{5m}\sin 5\left(\omega t - \frac{4\pi}{3} + \Psi_5\right)$$

On simplification, these become

$$e_R = E_{1m}\sin(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5) + ... \quad \text{– as before}$$

$$e_Y = E_{1m}\sin\left(\omega t - \frac{2\pi}{3} + \Psi_2\right) + E_{3m}\sin(3\omega t - 2\pi + \Psi_3) + E_{5m}\sin\left(5\omega t - \frac{10\pi}{3} + \Psi_5\right) + ...$$

$$= E_{1m}\sin\left(\omega t - \frac{2\pi}{3} + \Psi_1\right) + E_{3m}\sin(3\omega t + \Psi_3) + E_{5m}\sin\left(5\omega t - \frac{4\pi}{3} + \Psi_5\right) + ...$$

$$e_B = E_{1m}\sin\left(\omega t - \frac{4\pi}{3} + \Psi_1\right) + E_{3m}\sin(3\omega t + \Psi_3) + E_{5m}\sin\left(5\omega t - \frac{2\pi}{3} + \Psi_5\right) + ...$$

From these expressions, it is clear that

(*i*) All third harmonics are equal in all phases of the circuit *i.e.* they are in time phase.

(*ii*) Fifth harmonics in the three phases have a negative phase sequence of R, B, Y because the fifth harmonic of blue phase reaches its maximum value before that in the yellow phase.

(*iii*) All harmonics which are not multiples of three, have a phase displacement of 120º so that they can be dealt with in the usual manner.

(*iv*) At any instant, all the e.m.fs. have the same direction which means that in the case of a Y-connected system they are directed either away from or towards the neutral point and in the case of Δ-connected system, they flow in the same direction.

Main points can be summarized as below :

(*i*) all triple-n harmonics *i.e.* 3rd, 9th, 15th etc. are in phase,

(*ii*) the 7th, 13th and 19th harmonics have positive phase rotation of R, Y, B.

(*iii*) the 5th, 11th and 17th harmonics have a negative phase sequence of R, B, Y.

(*b*) Line Voltage for a Star-connected System

In this system, the line voltages will be the ***difference*** between successive phase voltages and hence will contain no third harmonic terms because they, being identical in each phase, will cancel out. The fundamental will have a line voltage $\sqrt{3}$ times the phase voltage. Also, fifth harmonic has line voltage $\sqrt{3}$ its phase voltage.

But it should be noted that in this case the r.m.s. value of the line voltage will be less than $\sqrt{3}$ times the r.m.s. value of the phase voltage due to the absence of third harmonic term from the line voltage. It can be proved that for any line voltage.

$$\text{Line value} = \sqrt{3}\sqrt{\frac{E_1{}^2 + E_5{}^2 + E_7{}^2}{E_1{}^2 + E_3{}^2 + E_5{}^2 + E_7{}^2}}$$

where E_1, E_3 etc. are r.m.s. values of the phase e.m.fs.

(*c*) Line Voltage for a Δ-connected System

If the winding of the alternator are delta-connected, then the resultant e.m.f. acting round the closed mesh would be the sum of the phase e.m.fs. The sum of these e.m.fs. is zero for fundamental, 5th, 7th, 11th etc. harmonics. Since the third harmonics are in phase, there will be a resultant third harmonic e.m.f. of three times the phase value acting round the closed mesh. It will produce a circulating current whose value will depend on the impedance of the windings at the third harmonic frequency. It means that the third harmonic e.m.f. would be short-circuited by the windings with the result that there will be no third harmonic voltage across the lines. The same is applicable to all triple-*n* harmonic voltages. Obviously, the line voltage will be the phase voltage but without the triple-*n* terms.

Example 20.16. *A 3-ϕ generator has a generated e.m.f. of 230 V with 15 per cent third harmonic and 10 per cent fifth harmonic content. Calculate*

(i) *the r.m.s. value of line voltage for Y-connection.*

(ii) *the r.m.s. value of line voltage for Δ-connection.*

Solution. Let E_1, E_3, E_5 be the r.m.s. values of the phase e.m.fs. Then

$E_3 = 0.15\ E_1$ and $E_5 = 0.1\ E_1$

$$\therefore \qquad 230 = \sqrt{E_1{}^2 + (0.15E_1)^2 + (0.1E_1)^2}$$

$E_1 = 226$ V $\quad \therefore \quad E_3 = 0.15 \times 226 = 34$ V and $E_5 = 0.1 \times 226 = 22.6$ V

(*i*) r.m.s. value of the fundamental line voltage = $\sqrt{3} \times 226 = 392$ V

r.m.s. value of third harmonic line voltage = 0

r.m.s. value of 5th harmonic line voltage $\sqrt{3} \times 22.6 =$ **39.2 V**

$\therefore$ r.m.s. value of line voltage $V_L = \sqrt{392^2 + 39.2^2} =$ **394 V**

(*ii*) In Δ-connection, again the third harmonic would be absent from the line voltage

$\therefore$ r.m.s. value of line voltage $V_L = \sqrt{226^2 + 22.6^2} =$ **227.5 V**

(*d*) Circulating Current in Δ-connected Alternator

Let the three symmetrical phase e.m.fs. of the alternator be represented by the equations,

$$e_R = E_{1m}\sin(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5) + \ldots\ldots$$

$$e_Y = E_{1m}\sin(\omega t + \Psi_1 - 2\pi/3) + E_{3m}\sin(3\omega t + \Psi_3) + E_{5m} + \sin(5\omega t + \Psi_5 - 4\pi/3)\ldots\ldots$$

$$e_B = E_{1m}\sin(\omega t + \Psi_1 - 4\pi/3) + E_{3m}\sin(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5 - 2\pi/3) + \ldots.$$

The resultant e.m.f. acting round the Δ-connected windings of the armature is the sum of these e.m.fs. Hence it is given by $e = e_R + e_Y + e_B$

$$\therefore \quad e = 3E_{3m}\sin(3\omega t + \phi_3) + 3E_{9m}\sin(9\omega t + \Psi_9) + 3E_{15}\sin(15\omega t + \Psi_{15}) + \ldots\ldots$$

If R and L represent respectively the resistance and inductance per phase of the armature winding, then the circulating current due to the resultant e.m.f. is given by

$$i_i \quad \frac{3E_{3m}\sin(3\ \ t\ \ \ \ _3)}{3\sqrt{(R^2\ \ 9L^2\ \ ^2)}}\ \ \frac{3E_{9m}\sin(9\ \ t\ \ \ \ _9)}{3\sqrt{(R^2\ \ 81L^2\ \ ^2)}}\ \ \frac{3E_{15m}\sin(15\ \ t\ \ \ \ _{15})}{3\sqrt{(R^2\ \ 225L^2\ \ ^2)}}\ \ \ldots.$$

$$= \frac{E_{3m}\sin(3\omega t+\Psi_3)}{\sqrt{(R^2+9L^2\omega^2)}} + \frac{E_{9m}\sin(9\omega t+\Psi_9)}{\sqrt{(R^2+81L^2\omega^2)}} + \frac{E_{15m}\sin(15\omega t+\Psi_{15})}{\sqrt{(R^2+225L^2\omega^2)}}$$

The r.m.s. value of the current is given by

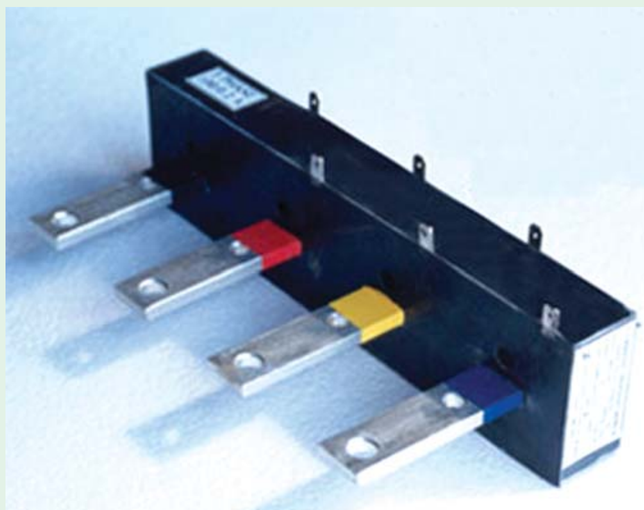
$$I_C = 0.707\sqrt{[E_{3m}{}^2/(R^2+9L^2\omega^2) + E_{9m}{}^2/(R^2+81L^2\omega^2) + (E_{15m}{}^2/(R^2+225L^2\omega^2) + ...]}$$

(e) Three-phase four-wire System

In this case, there will be no third harmonic component in line voltage. For the 4-wire system, each phase voltage (*i.e.* line to neutral) may contain a third harmonic component. If it is actually present, then current will flow in the *Y*-connected load. In case load is balanced, the resulting third harmonic line currents will all be in phase so that neutral wire will have to carry three times the third harmonic line current. There will be no current in the neutral wire either at fundamental frequency, or any harmonic frequency other than the triple-*n* frequency.

20.11. Harmonics in Single and 3-phase Transformers

The flux density in transformer course is usually maintained at a fairly high value in order to keep the required volume of iron to the minimum. However, due to the non-linearity of magnetisation curve, some third harmonic distortion is always produced. Also, there is usually a small percentage of fifth harmonic. The magnetisation current drawn by the primary contains mainly third harmonic whose proportion depends on the size of the primary applied voltage. Hence, the flux is sinusoidal.



3-phase current transformer

In the case of three-phase transformers, the production of harmonics will be affected by the method of connection and the type of construction employed.

(a) Primary Windings Δ-CONNECTED

Each primary phase can be considered as separately connected across the sinusoidal supply.

(i) The core flux will be sinusoidal which means that magnetizing current will contain 3rd harmonic component in addition to relatively small amounts of other harmonics of higher order.

(ii) In each phase, these third harmonic currents will be in phase and so produce a circulating current round the mesh with the result that there will be no third harmonic component in the line current.

(b) Primary Winding Connected in 4-wire Star

Each phase of the primary can again be considered as separately connected across a sinusoidal supply.

(*i*) The flux in the transformer core would be sinusoidal and so would be the output voltage.

(*ii*) The magnetizing current will contain 3rd harmonic component. This component being in phase in each winding will, therefore, return through the neutral wire.

(*c*) Primary Windings Connected 3-wire Star

Since there is no neutral wire, there will be no return path for the 3rd harmonic component of the magnetizing current. Hence, there will exist a condition of forced magnetization so that core flux must contain third harmonic component which is in phase in each limb of the transformer core. Although there will be a magnetic path for these fluxes in the case of shell type 3-phase transformer, yet in the case of three-limb core type transformer, the third harmonic component of the flux must return *via* the air. Because of the high reluctance magnetic path in such transformers, the third harmonic flux is reduced to a very small value. However, if the secondary of the transformer is delta-connected, then a third harmonic circulating current would be produced. This current would be in accordance with Lenz's law tend to oppose the very cause producing it *i.e.* it would tend to minimize the third harmonic component of the flux.

Should the third and fifth secondary be *Y*-connected, then provision of an additional Δ-connected winding, in which this current can flow, becomes necessary. This tertiary winding additionally served the purpose of preserving magnetic equilibrium of the transformer in the case of unbalanced loads. In this way, the output voltage from the secondary can be kept reasonably sinusoidal.

Example 20.17. *Determine whether the following two waves are of the same shape*

$e = 10 \sin(\omega t + 30^\circ) - 50 \sin(3\omega t - 60^\circ) + 25 \sin(5\omega t + 40^\circ)$

$i = 1.0 \sin(\omega t - 60^\circ) + 5 \sin(3\omega t - 150^\circ) + 2.5 \cos(5\omega t - 140^\circ)$

(Principles of Elect. Engg-II Jadavpur Univ.)

Solution. Two waves possess the same waveshape

(*i*) if they contain the same harmonics

(*ii*) if the ratio of the corresponding harmonics to their respective fundamentals is the same

(*iii*) if the harmonics are similarly spaced with respect to their fundamentals.

In other words,

(*a*) the ratio of the magnitudes of corresponding harmonics must be constant and

(*b*) with fundamentals in phase, the corresponding harmonics of the two waves must be in phase.

The test is applied first by checking the ratio of the corresponding harmonics and then coinciding the fundamentals by shifting one wave. If the phase angles of the corresponding harmonics are the same, then the two waves have the same shape.

In the present case, condition (*i*) is fulfilled because the voltage and current waves contain the same harmonics, *i.e.* third and fifth.

Secondly, the ratio of the magnitude of corresponding current and voltage harmonics is the same *i.e.* 1/10.

Now, let the fundamental of the current wave be shifted ahead by 90º so that it is brought in phase with the fundamental of the voltage wave. It may be noted that the third and fifth harmonics of the current wave will be shifted by 3 × 90º = 270° and 5 × 90º = 450º respectively. Hence, the current wave becomes

$$i' = 1.0 \sin(\omega t - 60^\circ + 90^\circ) + 5 \sin(3\omega t - 150^\circ + 270^\circ) + 2.5 \cos(5\omega t - 140^\circ + 450^\circ)$$

$$= 1.0 \sin(\omega t + 30^\circ) + 5 \sin(3\omega t + 120^\circ) + 2.5 \cos(5\omega t + 310^\circ)$$

$$= 1.0\sin(\omega t + 30°) - 5\sin(3\omega t - 60°) + 2.5\sin(5\omega t + 40°)$$

It is seen that now the corresponding harmonics of the voltage and current waves are in phase. Since all conditions are fulfilled, the two waves are of the same waveshape.

Tutorial Problem No. 20.1

1. A series circuit consist of a coil of inductance 0.1 H and resistance 25 Ω and a variable capacitor. Across this circuit is applied a voltage whose instantaneous value is given by

$$v = 100\sin\omega t + 20\sin(3\omega t - 45) + 5\sin(5\omega t - 30) \text{ where } \omega = 314 \text{ rad/s}$$

Determine the value of C which will produce response at third harmonic frequency and with this value of C, find

(*a*) an expression for the current in the circuit (*b*) the r.m.s. value of this current (*c*) the total power absorbed.

[11.25μF, (*a*) i = 0.398 sin (ω*t* + 84.3) + 0.8 sin (3 ω*t* + 45) + 0.485 sin (5 ω*t* + 106) (*b*) 0.633 A (*c*) 10 W]

2. A voltage given by v = 200 sin 314 t + 520 sin (942 t + 45°) is applied to a circuit consisting of a resistance of 20 Ω, and inductance of 20 mH and a capacitance of 56.3 μF all connected in series.

Calculate the r.m.s. values of the applied voltage and current. Find also the total power absorbed by the circuit. **[146 V ; 3.16 A 200 W]**

3. A voltage given by v = 100 sin ω*t* + 8 sin 3 ω*t* is applied to a circuit which has a resistance of 1 Ω, an inductance of 0.02 H and a capacitance of 60μF. A hot-wire ammeter is connected in series with the circuit and a hot-wire voltmeter is connected to the terminals. Calculate the ammeter and voltameter readings and the power supplied to the circuit. **[71 V ; 5.18 A ; 26.8 W]**

4. A certain coil has a resistance of 20 Ω and an inductance of 0.04 H. If the instantaneous current flowing in it is represented by i = 5 sin 300 t + 0.8 sin 900 t amperes, derive an expression for the instantaneous value of the voltage applied across the ends of the coil and calculate the r.m.s. value of that voltage. **[V = 117 sin (300 t + 0.541) + 33 sin (900 t + 1.06) ; 0.86 V]**

5. A voltage given by the equation $v = \sqrt{2}\,100\sin 2\pi \times 50t + \sqrt{2.20}\sin 2\pi.$ 150t is applied to the terminals of a circuit made up of a resistance of 5 Ω, an inductance of 0.0318 H and a capacitor of 12.5 μF all in series. Calculate the effective current and the power supplied to the circuit.

[0.547 A ; 1.5 W]

6. An alternating voltage given by the expression v = 1,000 sin 314t + 100 sin 942t is applied to a circuit having a resistance of 100 Ω and an inductance of 0.5 H. Calculate r.m.s. value of the current and p.f. **[3.81 A ; 0.535]**

7. The current in a series circuit consisting of a 159 μF capacitor, a reactor with a resistance of 10 Ω and an inductances of 0.0254 H is given by $i = \sqrt{2}(8\sin\omega t + 2\sin 3\omega t)$ amperes. Calculate the power input and the power factor. Given $\omega = 100\pi$ radian/second. **[680 W ; 0.63]**

8. If the terminal voltage of a circuit is 100 sin $\omega t + 50\sin(3\omega t + \pi/4)$ and the current is 10 $\sin(\omega t + \pi 3)$ $+5\sin 3\omega t$, calculate the power consumption. **[522.6 W]**

9. A single-phase load takes a current of 4 sin $(\omega t + \pi/6) + 1.5\sin(3\omega t + \pi/3)$ A from source represented by 360 sin ω*t* volts. Calculate the power dissipated by the circuit and the circuit power factor.

[623.5 W ; 0.837]

10. An e.m.f. given by e = 100 sin $\quad t \quad 40\sin(\quad t \quad /6) \quad 10\sin(5 \quad t \quad /3)$ volts is applied to a series circuit having a resistance of 100 Ω, an inductance of 40.6 mH and a capacitor of 10 μF. Derive an expression for the current in the circuit. Also, find the r.m.s. value of the current and the power dissipated in the circuit. Take ω= 314 rad/second.

[0.329 A, 10.8 W]

11. A p.d. of the form $v = 400\sin\omega t + 30\sin 3\omega t$ is applied to a rectifier having a resistance of 50Ω in one direction and 200 in the reverse direction. Find the average and effective values of the current and the p.f. of the circuit.

[1.96 A, 4.1 A, 0.51]

12. A coil having R = 2 Ω and L = 0.01 H carries a current given by i = 50 + 20 sin 300 t

A moving-iron ammeter, a moving-coil voltmeter and a dynameter wattmeter, are used to indicate current, voltage and power respectively. Determine the readings of the instruments and equation for the p.d.

[121.1 V ; 52 A ; 5.4 kW, V = 100 + 72 sin (300 t + 0.982)]

13. Two circuits having impedances at 50 Hz of (10 + j6) Ω and (10 – j6) respectively are connected in parallel across the terminals of an a.c. system, the waveform of which is represented by v = 100 sin $\omega t + 35$ sin $3\omega t + 10\sin 5\omega t$, the fundamental frequency being 50 Hz. Determine the ratio of the readings of two ammeters, of negligible resistance, connected one in each circuit.

[6.35 ; 6.72]

14. Explain what is meant by harmonic resonance in a.c. circuits.

A current having an instantaneous value of 2 $(\sin\omega t + \sin 3\omega t)$ amperes is passed through a circuit which consists of a coil of resistance R and inductance L in series with a capacitor C. Derive an expression for the value of ω at which the r.m.s. circuit voltage is a minimum. Determine the voltage if the coil has inductance 0.1 H and resistance 150 Ω and the capacitance is 10 μ F. Determine also the circuit voltage at the fundamental resonant frequency.

[$\omega = 1/\sqrt{(LC)}$; 378 V ; 482 V]

15. An r.m.s. current of 5 A which has a third-harmonic content, is passed through a coil having a resistance of 1Ω and an inductance of 10 mH. The r.m.s. voltage across the coil is 20 V. Calculate the magnitudes of the fundamental and harmonic components of current if the fundamental frequency is $300/2\pi$ Hz Also, find the power dissipated.

[4.8 A ; 1.44 A ; 25 W]

16. Derive a general expression for the form factor of a complex wave containing only odd-order harmonics. Hence, calculate the form factor of the alternating current represented by

i = 2.5 sin 157 t + 0.7 sin 471 t + 0.4 sin 785 t

[1.038]

OBJECTIVE TESTS – 20

1. Non-sinusoidal waveforms are made up of
(*a*) different sinusoidal waveforms
(*b*) fundamental and even harmonics
(*c*) fundamental and odd harmonics
(*d*) even and odd harmonics only.

2. The positive and negative halves of a complex wave are symmetrical when
(*a*) it contains even harmonics
(*b*) phase difference between even harmonics and fundamental is 0 or π
(*c*) it contains odd harmonics
(*d*) phase difference between even harmonics and fundamental is either $\pi/2$ or $3\pi/2$.

3. The r.m.s. value of the complex voltage given by $v = 16\sqrt{2}\sin\omega t + 12\sqrt{2}\sin 3\omega t$ is (*a*) $20\sqrt{2}$ (*b*) 20

(*c*) $28\sqrt{2}$ (*d*) 192

4. In a 3-phase system, ___th harmonic has negative phase sequence of RBY.

(a) 9 (*b*) 13

(*c*) 5 (*d*) 15

5. A complex current wave is given by the equation i = 14 sin $\omega t + 2\sin 5\omega t$. The r.m.s. value of the current is ___ ampere.

(*a*) 16 (*b*) 12

(*c*) 10 (*d*) 8

6. When a pure inductive coil is fed by a complex voltage wave, its current wave

(*a*) has larger harmonic content

(*b*) is more distorted

(*c*) is identical with voltage wave

(*d*) shows less distortion.

7. A complex voltage wave is applied across a pure capacitor. As compared to the fundamental voltage, the reactance offered by the capacitor to the third harmonic voltage would be

(*a*) nine time (*b*) three times

(*b*) one-third (*d*) one-ninth

8. Which of the following harmonic voltage components in a 3-phase system would be in phase with each other ?

(*a*) 3rd, 9th, 15th etc.

(*b*) 7th, 13th, 19th etc.

(*c*) 5th, 11th, 17th etc.

(*d*) 2nd, 4th, 6th etc.

ANSWERS

1. *(a)* **2.** *(c)* **3.** *(b)* **4.** *(c)* **5.** *(c)* **6.** *(d)* **7.** *(c)* **8.** *(d)*

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