Contents

CHAPTER 19

POLYPHASE CIRCUITS

Learning Objectives

- ➢ Generation of Polyphase Voltages
- ➢ Phase Sequence
- ➢ Interconnection of Three Phases
- ➢ Star or Wye (Y) Connection
- ➢ Voltages and Currents in Y-Connection
- ➢ Delta (D) or Mesh Connection
- ➢ Balanced Y/D and D/Y Conversions
- ➢ Star and Delta Connected Lighting Loads
- ➢ Power Factor Improvement
- ➢ Parallel Loads
- ➢ Power Measurement in 3-phase Circuits
- ➢ Three Wattmeter Method
- ➢ Two Wattmeter Method
- ➢ *Balanced* or *Unbalanced load*
- ➢ Variations in Wattmeter Readings
- ➢ Leading Power Factor
- ➢ Power Factor-*Balanced Load*
- ➢ Reactive Voltamperes with One Wattmeter
- ➢ One Wattmeter Method
- ➢ Double Subscript Notation
- ➢ Unbalanced Loads
- ➢ Four-wire Star-connected Unbalanced Load
- ➢ Unbalanced Y-connected Load Without Neutral
- ➢ Millman's Thereom
- ➢ Application of Kirchhoff's Laws
- ➢ Delta/Star and Star/Delta Conversions
- ➢ Unbalanced Star-connected Non-inductive Load
- ➢ Phase Sequence Indicators

This is a mercury arc rectifier 6-phase device, 150 A rating with grid control

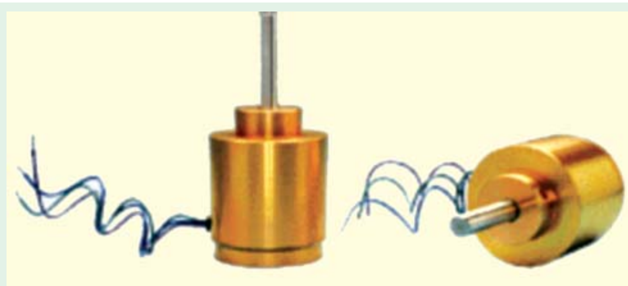
19.1. Generation of Polyphase Voltage

The kind of alternating currents and voltages discussed in chapter 12 to 15 are known as single-phase voltage and current, because they consist of a single alternating current and voltage wave. A single-phase alternator was diagrammatically depicted in Fig. 11.1 (*b*) and it was shown to have one armature winding only. But if the number of armature windings is increased, then it becomes polyphase alternator and it produces as many independent voltage waves as the number of windings or phases. These windings are displaced from one another by equal angles, the values of these angles being determined by the number of phases or windings. In fact, the word 'poly-phase' means poly (*i.e.* many or numerous) and phases (*i.e.* winding or circuit).

In a two-phase alternator, the armature windings are displaced 90 electrical degrees apart. A 3-phase alternator, as the name shows, has three independent armature windings which are 120 electrical degrees apart. Hence, the voltages induced in the three windings are 120° apart in time-phase. With the exception of two-phase windings, it can be stated that, in general, the electrical displacement between different phases is $360/n$ where n is the number of phases or windings.

Three-phase systems are the most common, although, for certain special jobs, greater number of phases is also used. For example, almost all mercury-arc rectifiers for power purposes are either six-phase or twelve-phase and most of the rotary converters in use are six-phase. All modern generators are practically three-phase. For transmitting large amounts of power, three-phase is invariably used. The reasons for the immense popularity of three-phase apparatus are that **(*i*)** it is more efficient **(*ii*)** it uses less material for a given capacity and **(*iii*)** it costs less than single-phase apparatus etc.

In Fig. 19.1 is shown a two-pole, stationary-armature, rotating-field type three-phase alternator. It has three armature coils aa', bb' and cc' displaced 120° apart from one another. With the position and clockwise rotation of the poles as indicated in Fig. 19.1, it is found that the e.m.f. induced in conductor '*a*' for coil aa' is maximum and its direction* is away from the reader. The e.m.f. in conductor '*b*' of coil bb' would be maximum and away from the reader when the N-pole has turned through 120° *i.e.* when *N-S* axis lies along bb'. It is clear that the induced e.m.f. in conductor '*b*' reaches its maximum value 120° later than the maximum value in conductor '*a*'. In the like manner, the maximum e.m.f. induced (in the direction away from the reader) in conductor '*c*' would occur 120° later than that in '*b*' or 240° later than that in '*a*'.



The rotary phase converter

Thus the three coils have three e.m.fs. induced in them which are similar in all respects except that they are 120° out of time phase with one another as pictured in Fig. 19.3. Each voltage wave is assumed to be sinusoidal and having maximum value of E_m.

In practice, the space on the armature is completely covered and there are many slots per phase per pole.

* The direction is found with the help of Fleming's Right-hand rule. But while applying this rule, it should be remembered that the relative motion of the conductor with respect to the field is anticlockwise although the motion of the field with respect to the conductor is clockwise as shown. Hence, thumb should point to the left.

Fig. 19.2 illustrates the relative positions of the windings of a 3-phase, 4-pole alternator and Fig. 19.4 shows the developed diagram of its armature windings. Assuming full-pitched winding and the direction of rotation as shown, phase '*a*' occupies the position under the centres of *N* and *S*-poles. It starts at S_a and ends or finishes at F_a.

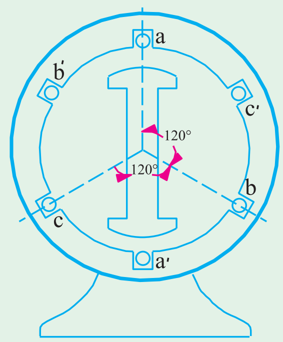


Fig. 19.1

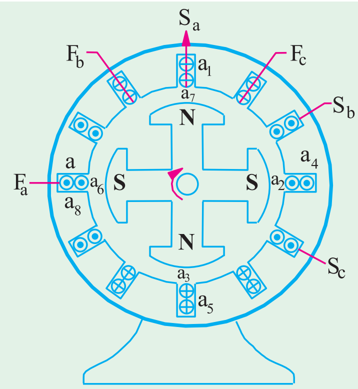


Fig. 19.2

The second phase '*b*' start at S_b which is 120 electrical degrees apart from the start of phase '*a*', progresses round the armature clockwise (as does '*a*') and finishes at F_b. Similarly, phase '*c*' starts at S_c, which is 120 electrical degrees away from S_b, progresses round the armature and finishes at F_c. As the three circuits are exactly similar but are 120 electrical degrees apart, the e.m.f. waves generated in them (when the field rotates) are displaced from each other by 120°. Assuming these waves to be sinusoidal and counting the time from the instant when the e.m.f. in phase '*a*' is zero, the instantaneous values of the three e.m.fs. will be given by curves of Fig. 193.

Their equations are :

$$e_a = E_m \sin \omega t \quad \text{... (i)}$$

$$e_b = E_m \sin(\omega t - 120°) \quad \text{... (ii)}$$

$$e_c = E_m \sin(\omega t - 240°) \quad \text{... (iii)}$$

As shown in Art. 11.23. alternating voltages may be represented by revolving vectors which indicate their maximum values (or r.m.s. values if desired). The actual values of these voltages vary from peak positive to zero and to peak negative values in one revolution of the vectors. In Fig. 19.5 are shown the three vectors representing the r.m.s. voltages of the three phases E_a, E_b and E_c (in the present case $E_a = E_b = E_c = E$, say).

It can be shown that the sum of the three phase e.m.fs. is zero in the following three ways :

(*i*) The sum of the above three equations **(*i*), (*ii*)** and **(*iii*)** is zero as shown below :

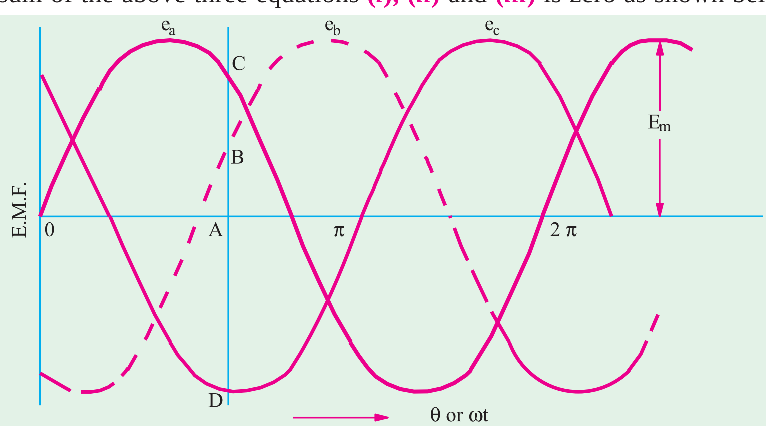


Fig. 19.3

Resultant instantaneous e.m.f. = $e_a + e_b + e_c$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120^\circ) + E_m (\omega t - 240)$$

$$= E_m [\sin \omega t + 2 \sin(\omega t - 180^\circ) \cos 60^\circ]$$

$$= E_m [\sin \omega t - 2 \sin \omega t \cos 60^\circ] = 0$$

(ii) The sum of ordinates of three e.m.f. curves of Fig. 19.3 is zero. For example, taking ordinates *AB* and *AC* as positive and *AD* as negative, it can be shown by actual measurement that

$$AB + AC + (-AD) = 0$$

(iii) If we add the three vectors of Fig. 19.5 either vectorially or by calculation, the result is zero.

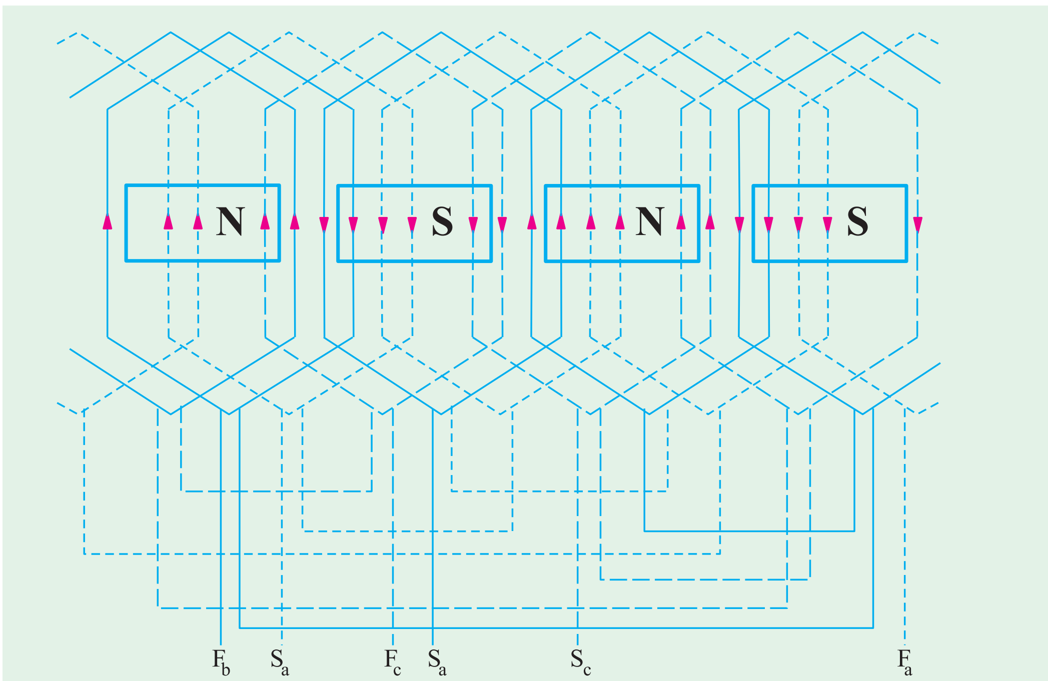


Fig. 19.4

Vector Addition

As shown in Fig. 19.6, the resultant of E_a and E_b is E_r and its magnitude is $2E \cos 60° = E$ where $E_a = E_b = E_c = E$.

This resultant E_r is equal and opposite to E_c. Hence, their resultant is zero.

By Calculation

Let us take E_a as reference voltage and assuming clockwise phase sequence

$$\mathbf{E_a} = E\angle 0^\circ = E + j0$$

$$\mathbf{E_b} = E\angle -240^\circ = E\angle 120^\circ = E(-0.05 + j0.866)$$

$$\mathbf{E_c} = E\angle 240^\circ = E\angle -120^\circ = E(-0.05 - j0.866)$$

$$\therefore \mathbf{E_a} + \mathbf{E_b} + \mathbf{E_c} = (E + j0) + E(-0.5 + 0.866) + E(-0.05 - j0.866) = 0$$

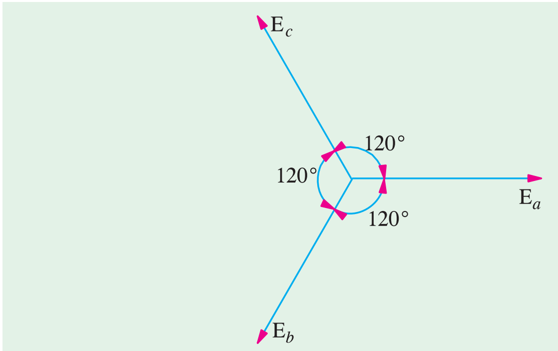


Fig. 19.5

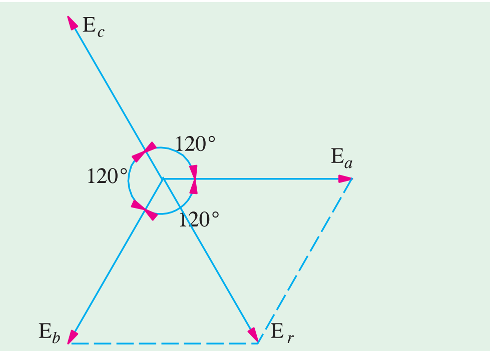


Fig. 19.6

19.2. Phase Sequence

By phase sequence is meant the order in which the three phases attain their peak or maximum values. In the development of the three-phase e.m.fs. in Fig. 19.7, clockwise rotation of the field system in Fig. 19.1 was assumed. This assumption made the e.m.fs. of phase '*b*' lag behind that

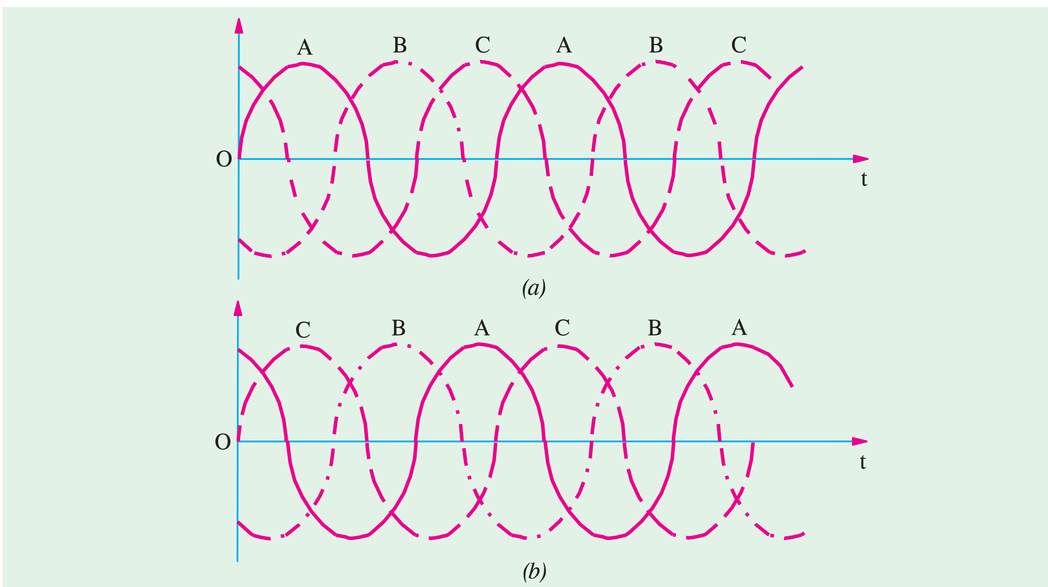


Fig. 19.7

of '*a*' by 120° and in a similar way, made that of '*c*' lag behind that of '*b*' by 120° (or that of '*a*' by 240°). Hence, the order in which the e.m.fs. of phases *a*, *b* and *c* attain their maximum values is *a b c*. ***It is called the phase order* or *phase sequence*** $a \rightarrow b \rightarrow c$ as illustrated in Fig. 19.7 (*a*).

If, now, the rotation of the field structure of Fig. 19.1 is reversed *i.e.* made anticlockwise, then the order in which the three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become $a \rightarrow b \rightarrow c$. This means that e.m.f. of phase '*c*' would now lag behind that of phase '*a*' by 120° instead of 240° as in the previous case as shown in Fig. 19.7 (*b*). By repeating the letters, this phase sequence can be written as ***acbacba*** which is the same thing as ***cba***. Obviously, a three-phase system has only two possible sequences : ***abc*** and ***cba*** (*i.e.* ***abc*** read in the reverse direction).

19.3. Phase Sequence At Load

In general, the phase sequence of the voltages applied to load is determined by the order in which the 3-phase lines are connected. The phase sequence can be reversed by interchanging any pair of lines. In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation. In the case of 3-phase unbalanced loads, the effect of sequence reversal is, in general, to cause a completely different set of values of the currents. Hence, when working on such systems, it is essential that phase sequence be clearly specified otherwise unnecessary confusion will arise. Incidentally, reversing the phase sequence of a 3-phase generator which is to be paralleled with a similar generator can cause extensive damage to both the machines.

Fig. 19.8 illustrates the fact that by interchanging any two of the three cables the phase sequence ***at the load*** can be reversed though sequence of 3-phase supply remains the same *i.e.* ***abc***. It is customary to define phase sequence at the load by reading repetitively from top to bottom. For example, load phase sequence in Fig. 19.8 (*a*) would be read as ***abcabcabc***– or simply ***abc***. The changes are as tabulated below :

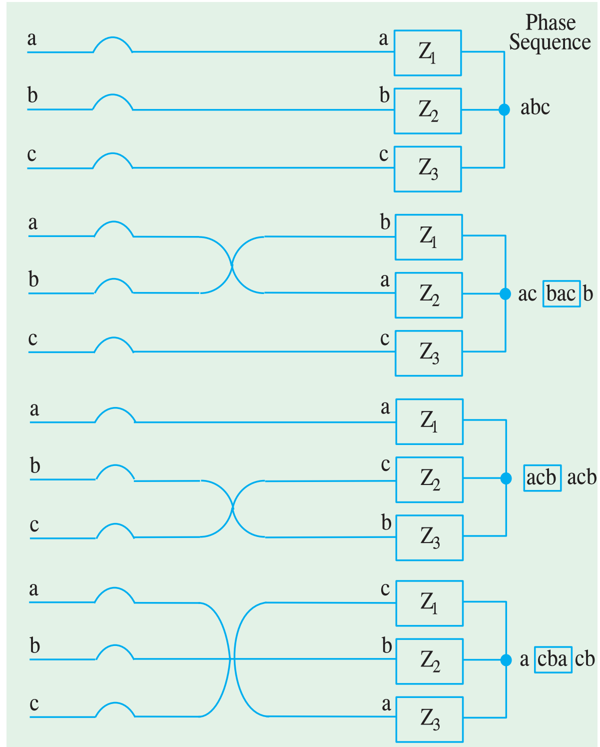


Fig. 19.8

Induction motor for drilling applications

Cables Interchanged	*Phase Sequence*
a and *b*	*b a* *c b a c b a* *c* – or *c b a*
b and *c*	*a* *c b a c b a* *c b* – or *c b a*
c and *a*	*c b a c b a* *c b a* – or *c b a*

19.4. Numbering of Phases

The three phases may be numbered 1, 2, 3 or *a*, *b*, *c* or as is customary, they may be given three colours. The colours used commercially are red, yellow (or sometimes white) and blue. In this case, the sequence is *RYB*.

Obviously, in any three-phase system, there are two possible sequences in which the three coil or phase voltages may pass through their maximum values *i.e.* red → yellow → blue (*RYB*) or red → blue → yellow (*RBY*). By convention, sequence *RYB* is taken as positive and *RBY* as negative.

19.5. Interconnection of Three Phases

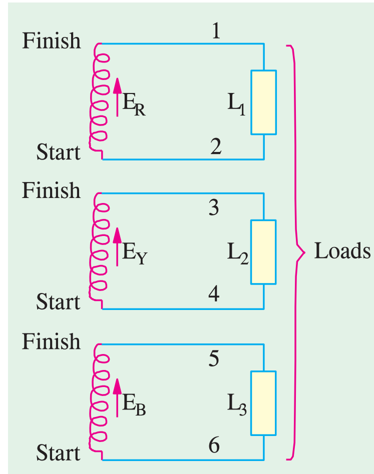


Fig. 19.9

If the three armature coils of the 3-phase alternator (Fig. 19.8) are not interconnected but are kept separate, as shown in Fig. 19.9, then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence, the three phases are generally inter-connected which results in substantial saving of copper. The general methods of interconnection are

3-phase alternator

(*a*) Star or Wye (Y) connection and

(*b*) Mesh or Delta (Δ) connection.

19.6. Star or Wye (Y) Connection

In this method of interconnection, the ***similar**** ends say, 'star' ends of three coils (it could be 'finishing' ends also) are joined together at point *N* as shown in Fig. 19.10 (*a*).

The point *N* is known as ***star point* or *neutral point***. The three conductors meeting at point *N* are replaced by a single conductor known as ***neutral conductor*** as shown in Fig. 19.10 (*b*). Such an interconnected system is known as four-wire, 3-phase system and is diagrammatically shown in Fig. 19.10 (*b*). If this three-phase voltage system is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120° out of phase with each other. Hence, their vector sum is zero.

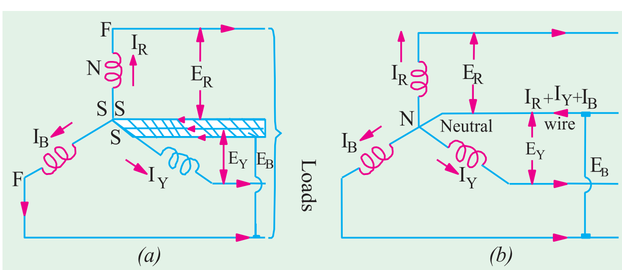


Fig. 19.10

i.e. $\mathbf{I_R + I_Y + I_B = 0}$ ... vectorially

The neutral wire, in that case, may be omitted although its retention is useful for supplying lighting loads at low voltages (Ex. 19.22). The p.d. between any terminal (or line) and neutral (or star) point gives the ***phase*** or ***star*** voltage. But the p.d. between any two lines gives the line-to-line voltage or simply line voltage.

19.7. Values of Phase Currents

When considering the distribution of current in a 3-phase system, it is extremely important to bear in mind that :

* As an aid to memory, remember that first letter *S* of Similar is the same as that of Star.

(i) the arrow placed alongside the currents I_R I_Y and I_B flowing in the three phases [Fig. 19.10 (*b*)] indicate the directions of currents when they are assumed to be ***positive*** and not the directions at a particular instant. It should be clearly understood ***that at no instant will all the three currents flow in the same direction either outwards or inwards.*** The three arrows indicate that first the current flows outwards in phase *R*, then after a phase-time of 120°, it will flow outwards from phase *Y* and after a further 120°, outwards from phase *B*.

(ii) the current flowing outwards in one or two conductors is always equal to that flowing inwards in the remaining conductor or conductors. In other words, ***each conductor in turn, provides a return path for the currents of the other conductors.***

In Fig. 19.11 are shown the three phase currents, having the same peak value of 20 A but displaced from each other by 120°. At instant '*a*', the currents in phases *R* and *B* are each + 10 A (*i.e.* flowing outwards) whereas the current in phase *Y* is – 20A (*i.e.* flowing inwards). In other words, at the instant '*a*', phase *Y* is acting as return path for the currents in phases *R* and *B*. At instant *b*, I_R = +15 A and I_Y = +5 A but I_B = –20A which means that now phase *B* is providing the return path.

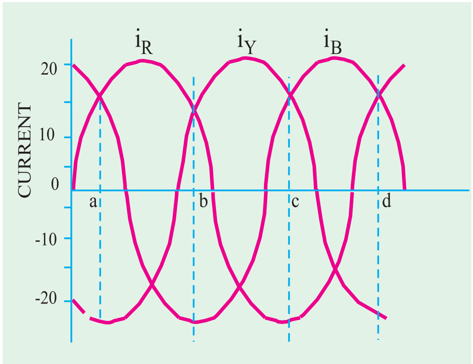


Fig. 19.11

At instant *c*, I_Y = +15 *A* and I_B = +5A and I_R = – 20A.

Hence, now phase *R* carries current inwards whereas *Y* and *B* carry current outwards. Similarly at point *d*, I_R = 0, I_B = 17.3 A and I_Y = – 17.3 A. In other words, current is flowing outwards from phase *B* and returning *via* phase *Y*.

In addition, it may be noted that although the distribution of currents between the three lines is continuously changing, yet at any instant the algebraic sum of the ***instantaneous*** values of the three currents is zero *i.e.*

$$i_R + i_Y + i_B = 0 \quad \text{– algebraically.}$$

19.8. Voltages and Currents in Y-Connection

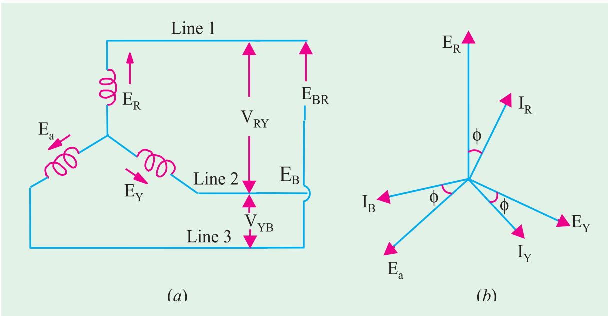


Fig. 19.12

The voltage induced in each winding is called the ***phase*** voltage and current in each winding is likewise known as ***phase*** current. However, the voltage available between any pair of terminals (or outers) is called ***line*** voltage (V_L) and the current flowing in each ***line*** is called ***line*** current (I_L).

As seen from Fig. 19.12 (*a*), in this form of interconnection, there are two phase windings between each pair of terminals but since their ***similar*** ends have been joined together, they are in opposition. Obviously, the ***instantaneous*** value of p.d. between any two terminals is the ***arithmetic difference*** of the two phase e.m.fs. concerned. However, the r.m.s. value of this p.d. is given by the ***vector difference*** of the two phase e.m.fs.

The vector diagram for phase voltages and currents in a star connection is shown in Fig. 19.12.

(*b*) where a balanced system has been assumed.* It means that $E_R = E_Y = E_{ph}$ (phase e.m.f.).

Line voltage V_{RY} between line 1 and line 2 is the vector difference of E_R and E_Y.

Line voltage V_{YB} between line 2 and line 3 is the vector difference of E_Y and E_B.

Line voltage V_{BR} between line 3 and line 1 is the vector difference of E_B and E_R.

(*a*) Line Voltages and Phase Voltages

The p.d. between line 1 and 2 is $V_{RY} = E_R - E_Y$ ... vector difference.

Hence, V_{RY} is found by compounding E_R and E_Y reversed and its value is given by the diagonal of the parallelogram of Fig. 19.13. Obviously, the angle between E_R and E_Y reversed is 60°. Hence if $E_R = E_Y = E_B$ = say, E_{ph} – the phase e.m.f., then

$$V_{RY} = 2 \times E_{ph} \times \cos(60°/2)$$

$$= 2 \times E_{ph} \times \cos 30° = 2 \times E_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3}\, E_{ph}$$

Similarly, $V_{YB} = E_Y - E_B = \sqrt{3} \cdot E_{ph}$ ...vector difference

and $V_{BR} = E_B - E_R = \sqrt{3} \cdot E_{ph}$

Now $V_{RY} = V_{YB} = Y_{BR} =$ line voltage, say V_L. Hence, in star connection $V_L = \sqrt{3} \cdot E_{ph}$

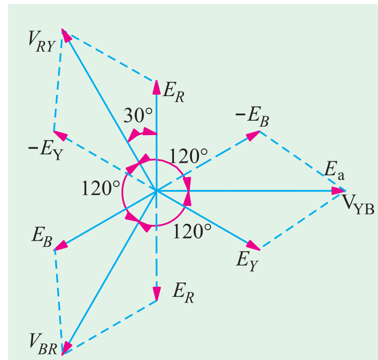


Fig. 19.13

It will be noted from Fig. 19.13 that

1. Line voltages are 120° apart.

2. Line voltages are 30° ahead of their respective ***phase*** voltages.

3. The angle between the line currents and the corresponding line voltages is $(30 + \phi)$ with current lagging.

(*b*) Line Currents and Phase Currents

It is seen from Fig. 19.12 (*a*) that each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 = I_R ; Current in line 2 = I_Y; Current in line 3 = I_B

Since $I_R = I_Y = I_B$ = say, I_{ph} – the phase current

∴ line current $I_L = I_{ph}$

(*c*) Power

The total active or true power in the circuit is the sum of the three phase powers. Hence,

total active power = 3 × phase power or $P = 3 \times V_{ph} I_{ph} \cos\phi$

Now $V_{ph} = V_L / \sqrt{3}$ and $I_{ph} = I_L$

Hence, in terms of line values, the above expression becomes

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi \quad \text{or} \quad P = \sqrt{3}\, V_L\, I_L \cos\phi$$

* A balanced system is one in which (*i*) the voltages in all phases are equal in magnitude and offer in phase from one another by equal angles, in this case, the angle = 360/3 = 120°, (*ii*) the currents in the three phases are equal in magnitude and also differ in phase from one another by equal angles.

A 3-phase balanced load is that in which the loads connected across three phases are identical.

It should be particularly noted that ϕ is the angle between ***phase*** voltage and ***phase*** current and not between the line voltage and line current.

Similarly, total reactive power is given by **Q** = $\sqrt{3} V_L I_L \sin\phi$

By convention, reactive power of a coil is taken as positive and that of a capacitor as negative.

The total apparent power of the three phases is

$S = \sqrt{3} V_L I_L$ Obviously, $S = \sqrt{P^2 + Q^2}$ – Art. 13.4

Example 19.1. *A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.* **(Elect. Engg., Bhagalpur Univ.)**

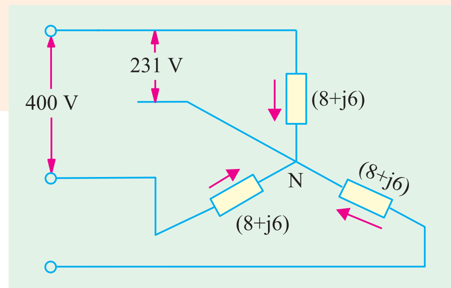


Fig. 19.14

Solution. $Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Omega$

$V_{ph} = 400/\sqrt{3} = 231$ V

$I_{ph} = V_{ph}/Z_{ph} = 231/10 = 23.1$ A

(i) $I_L = I_{ph}$ = **23.1 A**

(ii) p.f. = cos ϕ = R_{ph}/Z_{ph} = 8/10 = **0.8 (lag)**

(iii) Power P $\sqrt{3} V_L I_L \cos$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$ = **12,800 W** [Also, $P = 3I_{ph}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800$ W]

(iv) Total volt-amperes, $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 23.1$ = **16,000 VA**

Example 19.2. *Phase voltages of a star connected alternator are $E_R = 231 \angle 0°$ V; $E_Y = 231 \angle -120°$ V ; and $E_B = 231 \angle +120°$V. What is the phase sequence of the system ? Compute the line voltages E_{RY} and E_{YB}.* **(Elect. Mechines AMIE Sec. B Winter 1990)**

Solution . The phase voltage $E_B = 231\angle -120°$ can be written as $E_B = 231\angle -240°$. Hence, the three voltages are: $E_R = 231\angle -0°$, $E_Y = 231\angle -120°$ and $E_B = 231\angle -240°$. It is seen that E_R is the reference voltage, E_Y lags behind it by 120° whereas E_B lags behind it by 240°. Hence, phase sequence is *RYB*. Moreover, it is a symmetrical 3-phase voltage system.

$\therefore$ $E_{RY} = E_{YB} = \sqrt{3} \times 231 =$ **400 V**

Example 19.3 *Three equal star-connected inductors take* 8 *kW at a power factor* 0.8 *when connected across a* 460 *V*, 3-*phase*, 3-*phase*, 3-*wire supply. Find the circuit constants of the load per phase.* **(Elect. Machines AMIE Sec. B 1992)**

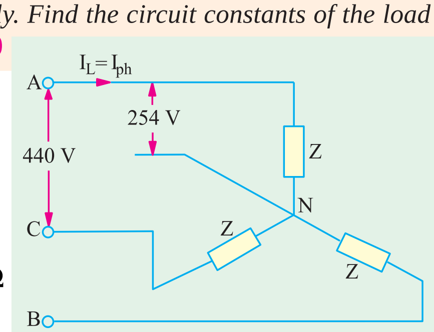


Fig. 19.15

Solution. $P = \sqrt{3} V_L I_L \cos\phi$ or

$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$

$\therefore$ I_L = 12.55 A $\therefore$ I_{ph} = 12.55 *A*;

$V_{ph} = V_L/\sqrt{3}$ $460/\sqrt{3}$ $265V$

$I_{ph} = V_{ph}/Z_{ph}; \therefore Z_{ph} = V_{ph}/I_{ph} = 265/12.55 = 21.1\Omega$

$R_{ph} = Z_{ph} \cos\phi = 21.1 \times 0.8 = 16.9\ \Omega$

$X_{ph} = Z_{ph} \sin\phi \; = 21.1 \times 0.6 = 12.66\,\Omega$;

The circuit is shown in Fig. 19.15.

Example 19.4. *Given a balanced* $3-\phi$, *3-wire system with Y-connected load for which line voltage is 230 V and impedance of each phase is (6 + J8) ohm. Find the line current and power absorbed by each phase.* **(Elect. Engg - II Pune Univ. 1991)**

Solution. $Z_{ph} = \sqrt{6^2 + 8^2} = 10\,\Omega;\; V_{ph} = V_L / \sqrt{3} = 230/\sqrt{3} = 133\,\text{V}$

$\cos\phi = R/Z = 6/10 = 0.6; I_{ph} = V_{ph}/Z_{ph} = 133/10 = 13.3\,\text{A}$

$\therefore \qquad I_L = I_{ph} =$ **13.3 A**

Power absorbed by each phase = $I_{ph}{}^2 R_{ph} = 13.3^2 \times 6 =$ **1061 W**

Solution by Symbolic Notation

In Fig. 19.16 (*b*), V_R, V_Y and V_B are the phase voltage whereas I_R, I_Y and I_B are phase currents. Taking V_R as the reference vector, we get

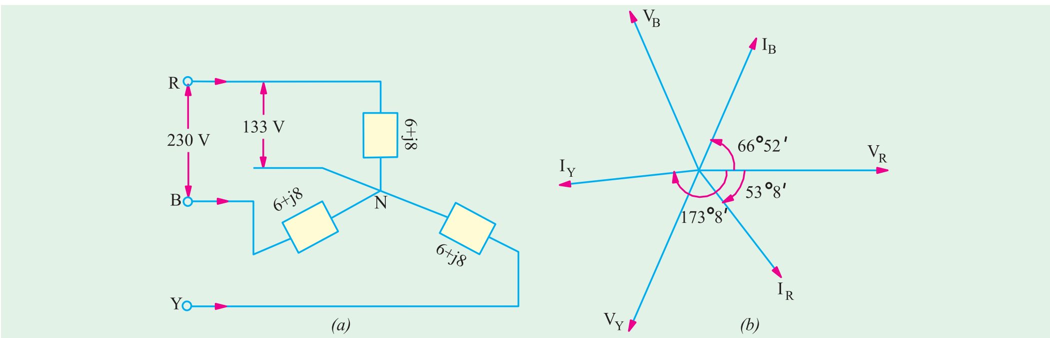


Fig 19.16

$\mathbf{V}_R \quad 133 \quad 0 \quad 133 \quad j0$ volt

$\mathbf{V}_Y \quad 133 \quad 120 \quad 133(\; 0.5 \quad j0.866) \quad (\; 66.5 \quad j115)$ volt

$\mathbf{V}_B = 133 \angle 120° = 133\,(-0.5 + j\,0.866) = (-66.5 + j\,115)$ volt

$\mathbf{Z} = 6 + j8 = 10 \angle 53°\,8'$; $\mathbf{I}_R = \dfrac{\mathbf{V}_R}{\mathbf{Z}} \quad \dfrac{133 \quad 0}{10 \quad 53 \; 8} \quad 13.3 \quad 53°8$

This current lags behind the reference voltage by $53°8'$ [Fig. 19.16 (*b*)]

$\mathbf{I}_Y = \dfrac{\mathbf{V}_Y}{\mathbf{Z}} \quad \dfrac{133 \quad 120}{10 \quad 53 \; 8} \quad 13.3 \quad 173 \; 8$

It lags behind the reference vector *i.e.* V_R by $173°8'$ which amounts to lagging behind its phase voltage V_Y by $53°8'$.

$\mathbf{I}_B \quad \dfrac{\mathbf{V}_B}{\mathbf{Z}} \quad \dfrac{133 \quad 120}{10 \quad 53 \; 8} \quad 13.3 \quad 66°52$

This current leads V_R By $66°52'$ which is the same thing as ***lagging*** behind its phase voltage by $53°8'$. For calculation of power, consider *R*-phase

$\mathbf{V}_R = (133 - j0)$; $I_R = 13.3\,(0.6 - j0.8) = (7.98 - j10.64)$

Using method of conjugates, we get

$\mathbf{P}_{VA} = (133 - j0)\,(7.98 - j10.64) = 1067 - j1415$

$\therefore$ Real power absorbed/phase = 1067 W – as before

Example 19.5. *When the three identical star-connected coils are supplied with 440 V, 50 Hz, 3-ϕ supply, the 1-ϕ wattmeter whose current coil is connected in line R and pressure coil across the phase R and neutral reads 6 kW and the ammeter connected in R-phase reads 30 Amp. Assuming RYB phase sequence find:*

(i) resistance and reactance of the coil, *(ii) the power factor, of the load*

(iii) reactive power of 3-ϕ load. **(Elect. Engg.-I, Nagpur Univ. 1993)**

Solution. $V_{ph} = 440/\sqrt{3} = 254$ V; $I_{ph} = 30$A (Fig. 19.17.)

Now, $V_{ph}\,I_{ph}\cos\phi = 6000$; $254 \times 30 \times \cos\phi = 6000$

$\therefore$ $\cos\phi = 0.787$; $\phi = 38.06°$ and $\sin\phi = 0.616°$; $Z_{ph} = V_{ph}/I_{ph} = 254/30 = 8.47\,\Omega$

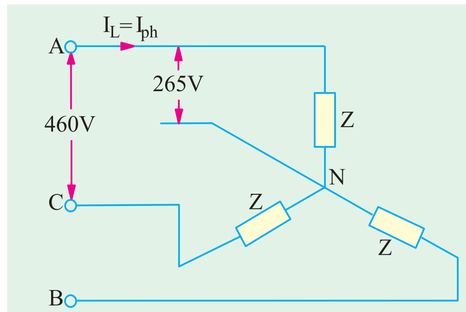


Fig. 19.17

(*i*) Coil resistance $R = Z_{ph}\cos\phi = 8.47 \times 0.787 = 6.66\,\Omega$

$X_L = Z_{ph}\sin\phi = 8.47 \times 0.616 =$ **5.22** Ω

(*ii*) p.f. = cos ϕ = **0.787 (lag)**

(*iii*) Reactive power = $\sqrt{3}\,V_L I_L \sin$ $\sqrt{3}$ 440 30 0.616 14,083VA **14.083kVA**

Example 19.6 *Calculate the active and reactive components in each phase of Y-connected 10,000 V, 3-phase alternator supplying 5,000 kW at 0.8 p.f. If the total current remains the same when the load p.f is raised to 0.9, find the new output.*

(Elements of Elect. Engg.-I, Bangalore Univ.)

Solution. $5000 \times 10^3 = \sqrt{3}$ 10,000 I_L 0.8; I_L I_{ph} 361A

active component= $I_L\cos\phi = 361 \times 0.8 =$ **288.8 A**

reactive component= $I_L\sin\phi = 361 \times 0.6 =$ **216.6 A**

New power P = $\sqrt{3}V_L I_L\cos\phi = \sqrt{3}\times 10^4 \times 361 \times 0.9$ = **5,625 kW**

[or new power = 5000 × 0.9/0.8 = 5625 kW]

Example 19.7. *Deduce the relationship between the phase and line voltages of a three-phase star-connected alternator. If the phase voltage of a 3-phase star-connected alternator be 200 V, what will be the line voltages (a) when the phases are correctly connected and (b) when the connections to one of the phases are reversed.*

Solution. (*a*) When phases are correctly connected, the vector diagram is as shown in Fig. 19.12. (*b*). As proved in Art. 19.7

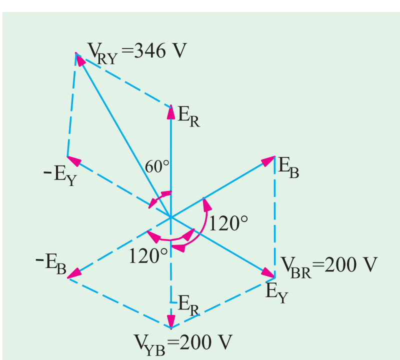


Fig. 19.18

$V_{RY} = V_{YB} V_{BR} = \sqrt{3}.E_{Ph}$

Each line voltage = $\sqrt{3}\times 200$ = **346 V**

(*b*) Suppose connections to *B*-phase have been reversed. Then voltage vector diagram for such a case is shown in Fig. 19.18. It should be noted that E_B has been drawn in the reversed direction, so that angles between the three-phase voltages are 60° (instead of the usual 120°)

$V_{RY} = E_R - E_Y$ ... vector difference

$= 2 \times E_{ph} \times \cos 30° = \sqrt{3}\times 200$ = **346 V**

$V_{YB} = E_Y - E_B$ ... vector difference

$$= 2 \times E_{ph} \times \cos 60° = 2 \times 200 \times \frac{1}{2} = \mathbf{200V}$$

$$V_{BR} = E_B - E_R \quad \text{... vector difference} = 2 \times E_{ph} \times \cos 60° = 2 \times 200 \times \frac{1}{2} = \mathbf{200\ V}$$

Example 19.8 *In a 4-wire, 3-phase system, two phases have currents of 10A and 6A at lagging power factors of 0.8 and 0.6 respectively while the third phase is open-circuited, Calculate the current in the neutral and sketch the vector diagram.*

Solution. The circuit is shown in Fig. 19.19 (*a*).

$$\phi_1 = \cos^{-1}(0.8) = 36°54'; \ \phi_2 = \cos^{-1}(0.6) = 53°6'$$

Let V_R be taken as the reference vector. Then

$$\mathbf{I}_R = 10\angle -36°54' = (8 - j6) \quad \mathbf{I}_y = 6\angle -173°6' = (-6 - j0.72)$$

The neutral current $\mathbf{I}_N$, as shown in Fig. 19.16 (*b*), is the sum of these two currents.

$$\therefore \quad \mathbf{I}_N = (8 - j6) + (-6 - j0.72) = 2 - j6.72 = 7\angle -73°26'$$

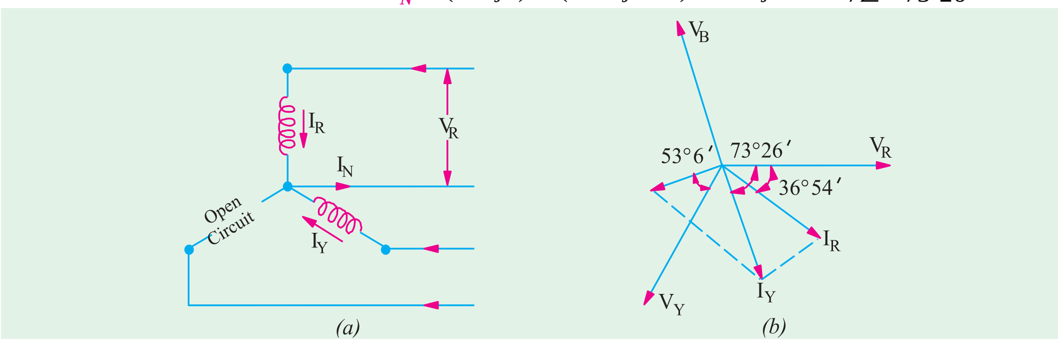


Fig. 19.19

Example 19.9 (*a*). *Three equal star-connected inductors take 8 kW at power factor 0.8 when connected a 460-V, 3-phase, 3-wire supply. Find the line currents if one inductor is short-circuited.*

Solution. Since the circuit is balanced, the three line voltages are represented by

$$V_{ab} = 460\angle 0°; V_{bc} = 460\angle -120° \text{ and } V_{ca} = 460\angle 120°$$

The phase impedance can be found from the given data :

$$8000 = \sqrt{3} \times 460 \times I_L \times 0.8 \quad \therefore \ I_L = I_{ph} = 12.55 \text{ A}$$

$$Z_{ph} = V_{ph} / I_{ph} = 460 / \sqrt{3} \times 12.55 = 21.2\ \Omega;$$

$$\therefore \quad Z_{ph} = 21.2\angle 36.9° \text{ because } \phi = \cos^{-1}(0.8) = 36.9°$$

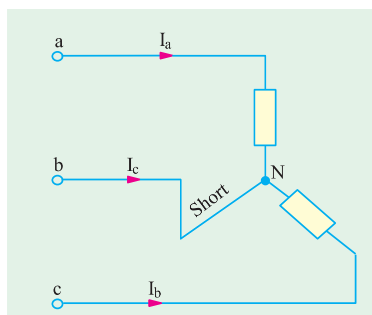


Fig. 19.20

As shown in the Fig. 19.20, the phase *c* has been short-circuited. The line current $I_a = V_{ac}/Z_{ph} = -V_{ca}/Z_{ph}$ because the current enters at point *a* and leaves from point *c*.

$$\therefore \quad I_a = -460\angle 1200°/21.2\angle 36.9° = 21.7\angle 83.1°$$

Similarly, $I_b = V_{bc}/Z_{ph} = 460 \angle 120°/21.2 \angle 36.9° = 21.7 \angle -156.9°$. The current I_c can be found by applying KVL to the neutral point N.

$$\therefore \quad I_a + I_b + I_c = 0 \quad \text{or} \quad I_c = -I_a - I_b$$

$\therefore \quad I_C = 21.7\angle 83.1° - 21.7\angle -156.9° = 37.3\angle 53.6°$

Hence, the magnitudes of the three currents are : 21.7 A; 21.7 Al 37.3 A.

Example 19.9 (*b*). *Each phase of a star-connected load consists of a non-reactive resistance of 100 Ω in parallel with a capacitance of 31.8 μF.*

Calculate the line current, the power absorbed, the total kVA and the power factor when connected to a 416-V, 3-phase, 50-Hz supply.

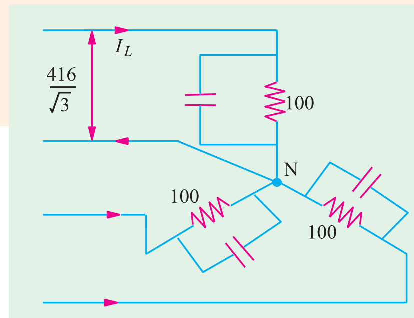


Fig. 19.21

Solution. The circuit is shown in Fig. 14.20.

$$V_{ph} = (416/\sqrt{3})\angle 0° = 240\angle 0° = (240 + j0)$$

Admittance of each phase is

$$\mathbf{Y}_{ph} \quad \frac{1}{R} \quad j \;\; C \quad \frac{1}{100} \quad j314 \quad 31.8 \quad 10^{6}$$

$$= 0.01 + j0.01$$

$\therefore \quad \mathbf{I}_{ph} = \mathbf{V}_{ph} \,.\, \mathbf{Y}_{ph} = 240(0.01 + j0.01)$

$= 2.4 + j2.4 = 3.39 \angle 45°$

Since $\quad I_{ph} = I_L$ – for a star connection $\therefore$ I_L **= 3.39 A**

Power factor = cos 45° = **0.707 (leading)**

Now $\quad \mathbf{V}_{ph} = (240 + j0)$; $I_{ph} = 2.4 + j2.4$

$\therefore \quad \mathbf{P}_{VA} = (240 + j0)\,(2.4 + j\,2.4)$

$= 240 \times 2.4 - j2.4 \times 240 = 576 - j576 = 814.4 \angle -45°$ ... per phase

Hence, total power = 3 × 576 = 1728 W = **1.728 kW**

Total voltampers = 814.4 × 3 = 2,443 VA ; kilovolt amperes = **2.433 kVA**

Example 19.10. *A three pahse 400-V, 50 Hz, a.c. supply is feeding a three phase delta-connected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp.* **[Nagpur University, November 1999]**

Solution. Impedance per phase $r + jX_L - jX_C$

$$X_L = 2\pi \times 50 \times 0.15 = 47.1\ \Omega$$

$$X_C = \frac{10^6}{32{\cdot}37} = 26.54\ \Omega$$

$$\cos\phi = \frac{25}{32.37}$$ Lagging, since inductive reactance is dominating.

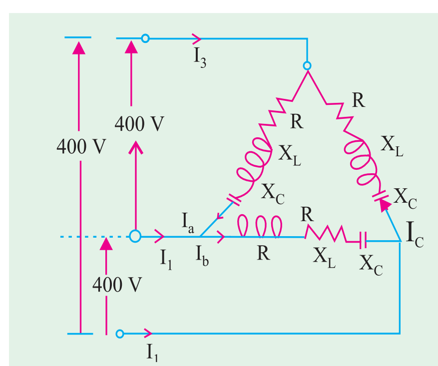


Fig. 19.22

$$\text{Phase Current} = \frac{400}{25 + j20.56} = 12.357$$

$$\text{Line Current} = \sqrt{3} \times 12.357 = 21.4 \text{ amp}$$

Since the power factor is 0.772 lagging, P = total three phase power = $\sqrt{3} V_L \; I_L \; \cos\phi \times 10^{-3}$ kW

= $\sqrt{3}$ × 400 × 21.4 × 0.772 × 10^{-3} = 11.446 kW

S = total 3 ph kVA $\frac{11.446}{0.772}$ = 14.83 kVA 14.83 kVA

Q = total 3 ph "reactive kilo-volt-amp" $\sqrt{3}$ = $(S^2 - P^2)^{0.50}$ = 9.43 kVAR lagging

Example 19.11. *Three phase star-connected load when supplied from a 400 V, 50 Hz source takes a line current of 10 A at 66.86º w.r.t. to its line voltage. Calculate : **(i)** Impedance-Parameters, **(ii)** P.f. and active-power consumed. Draw the phasor diagram.*

[Nagpur University, April 1998]

Solution. Draw three phasors for phase-voltages.

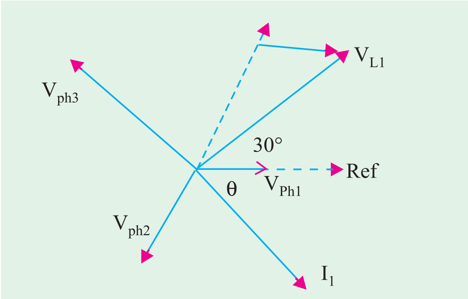


Fig. 19.23

These are V_{ph1}, V_{ph2}, V_{ph3} in Fig 19.23. As far as phase number 1 is concerned, its current is I_1 and the associated line voltage is V_{L1} . V_{L1} and V_{ph1} differ in phase by 30°. A current differing in phase with respect to line voltage by 66.86º and associated with V_{ph1} can only be lagging, as shown in Fig. 19.23. This means ϕ = 36.86º, and the corresponding load power factor is 0.80 lagging.

$Z = V_{ph}/I_{ph}$ = 231/10 = 23.1 ohms

R = Z cos ϕ = 23.1 × 0.8 = 18.48 ohms

X_L = Z sin ϕ = 23.1 × ?0.6 = 13.86 ohms

Total active power consumed = 3 V_{ph} I_{ph} cos ϕ

= 3 × 231 × 10 × 0.8 × 10^{-3} kW = 5.544 kW

or total active power = 3 × I^2R = 3 × 10^2 × 18.48 = 5544 watts

For complete phasor diagram for three phases, the part of the diagram for Phase 1 in Fig 19.23 has to be suitably repeated for phase-numbers 2 and 3.

19.9. Delta (Δ)* or Mesh Connection

In this form, of interconnection the ***dissimilar*** ends of the three phase winding are joined together *i.e.* the 'starting' end of one phase is joined to the 'finishing' end of the other phase and so on as showing in Fig. 19.24 (*a*). In other words, the three windings are joined in series to form a closed mesh as shown in Fig. 19.24 (*b*).

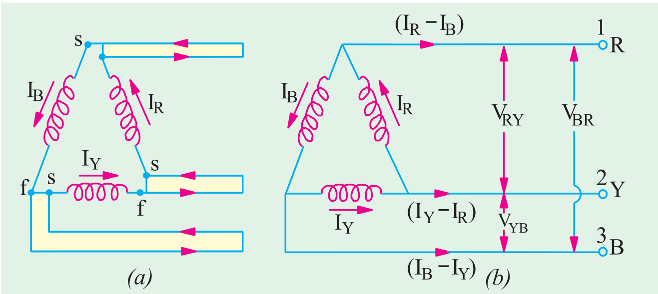


Fig. 19.24

Three leads are taken out from the three junctions as shown as outward directions are taken as positive.

It might look as if this sort of interconnection results in

* As an aid to memory, remember that first letter *D* of Dissimilar is the same as that of Delta.

shortcircuiting the three windings. However, if the system is balanced then sum of the three voltages round the closed mesh is zero, hence no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.

This type of connection is also referred to as 3-phase, 3-wire system.

(*i*) Line Voltages and Phase Voltages

It is seen from Fig. 19.24 (*b*) that there is only one phase winding completely included between any pair of terminals. Hence, in Δ-connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is *R Y B*, the voltage having its positive direction from *R* to *Y* leads by 120° on that having its positive direction from *Y* to *B*. Calling the voltage between lines 1 and 2 as V_{RY} and that between lines 2 and 3 as V_{YB}, we find that V_{RY} lead V_{YB} by 120°. Similarly, V_{YB} leads V_{BR} by 120º as shown in Fig. 19.23. Let $V_{RY} = V_{YB} = V_{BR}$ = line voltage V_L. Then, it is seen that $V_L = V_{ph}$.

(*ii*) Line Currents and Phase Currents

It will be seen from Fig. 19.24 (*b*) that current in each line is the ***vector difference*** of the two phase currents flowing through that line. For example

Current in line 1 is I_1 $\quad I_R \quad I_B$

Current in line 2 is I_2 $\quad I_Y \quad I_R$ vector difference

Current in line 3 is I_3 $\quad I_B \quad I_Y$

Current in line No. 1 is found by compounding I_R and I_B reversed and its value is given by the diagonal of the parallelogram of Fig. 19.25. The angle between I_R and I_B reversed (*i.e.* $-I_B$) is 60°. If $I_R = I_Y$ = phase current I_{ph} (say), then

Current in line No. 1 is

$$I_1 = 2 \times I_{ph} \times \cos(60º/2) = 2 \times I_{ph}\sqrt{3}/2 = \sqrt{3}\,I_{ph}$$

Current in line No. 2 is

$I_2 = I_B - I_Y$ ... vector difference $= \sqrt{3}I_{ph}$ and current in line No. 3 is $I_3 = I_B - I_Y$ $\therefore$ Vector difference $= \sqrt{3}\cdot I_{ph}$

Since all the line currents are equal in magnitude *i.e.*

$I_1 = I_2 - I_3 = I_L$

$$\therefore \quad I_L = \sqrt{3}\,I_{ph}$$

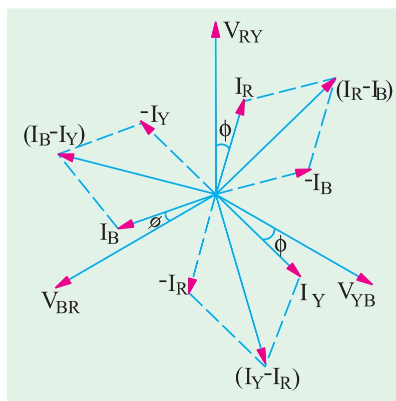


Fig. 19.25

With reference to Fig. 19.25, it should be noted that

1. line currents are 120º apart ;

2. line currents are 30º behind the respective phase currents ;

3. the angle between the line currents and the corresponding line voltages is (30 + ϕ) with the current lagging.

(*iii*) Power

Power/phase $= V_{ph}I_{ph}\cos\phi$; Total power $= 3 \times V_{ph}I_{ph}\cos\phi$. However, $V_{ph} = V_L$ and $I_{ph} = I_L/\sqrt{3}$

Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3}\,V_L I_L \cos\phi$$

where ϕ is the phase power factor angle.

19.10. Balanced Y/Δ and Δ/Y Conversion

In view of the above relationship between line and phase currents and voltages, any balanced *Y*-connected system may be completely replaced by an equivalent Δ-connected system. For example, a 3-phase, *Y*-connected system having the voltage of V_L and line current I_L may be replaced by a Δ-connected system in which phase voltage is V_L and phase current is $I_L/\sqrt{3}$.

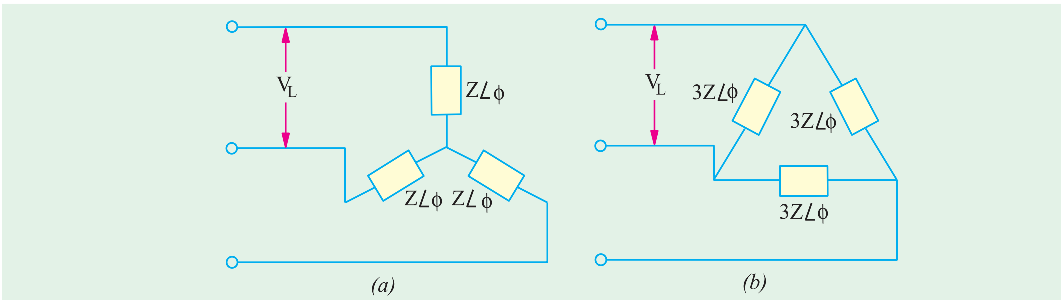


Fig. 19.26

Similarly, a balanced *Y*-connected load having equal branch impedances each of $Z\angle\phi$ may be replaced by an equivalent Δ-connected load whose each phase impedance is $3Z\angle\phi$. This equivalence is shown in Fig. 19.26.

For a balanced star-connected load, let

V_L = line voltage; I_L = line current ; Z_Y = impedance/phase

$\therefore \quad V_{ph} \quad V_L/\sqrt{3}. \;\; I_{ph} \quad I_L; \;\; Z_Y \quad V_L/(\sqrt{3}\,I_L)$

Now, in the equivalent Δ-connected system, the line voltages and currents must have the same values as in the *Y*-connected system, hence we must have

$$V_{ph} = V_L, \quad I_{ph} = I_L/\sqrt{3} \quad \therefore \quad Z_\Delta = V_L/(I_L/\sqrt{3}) = \sqrt{3}\,V_L/I_L = 3Z_Y$$

$$\therefore \qquad Z_\Delta\angle\phi = 3Z_Y\angle\phi \qquad\qquad (\quad V_L/I_L = \sqrt{3}Z_Y)$$

or $\quad \mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad$ or $\quad \mathbf{Z}_Y = \mathbf{Z}_\Delta/3$

The case of unbalanced load conversion is considered later. (Art. 19.34)

Example 19.12. *A star-connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm/phase. The line voltage is 230 V. Determine* **(a)** *current in the load branch,* **(b)** *power consumed by the load,* **(c)** *power factor of load, (d) reactive power of the load.*
(Elect. Engg. A.M.Ae. S.I. June 1991)

Solution. Considering the Δ-connected load, we have $Z_{ph} = \sqrt{8^2+6^2} = 10\,\Omega$; $V_{ph} = V_L =$ **230 V**

(a) $I_{ph} = V_{ph}/Z_{ph} = 230/10 = 23$ A

(b) $I_L = \sqrt{3}\,I_{ph} = \sqrt{3}\times 23 = 39.8\ A$; $\;P \quad \sqrt{3}\,V_L I_L \cos \quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.8$ **12,684 W**

(c) p.f. $\cos\phi = R\,|\,Z = 8/10 =$ **0.8 (lag)**

(d) Reactive power $Q = \sqrt{3}\;V_L\,I_L \sin \quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.6$ **9513 W**

Example 19.13. *A 220-V, 3-φ voltage is applied to a balanced delta-connected 3-φ load of phase impedance (15 + j20)* Ω

(a) *Find the phasor current in each line.* **(b)** *What is the power consumed per phase ?*
(c) *What is the phasor sum of the three line currents ? Why does it have this value ?*
(Elect. Circuits and Instruments, B.H.U.)

Solution. The circuit is shown in Fig. 19.27 (*a*).

$$V_{ph} = V_L = 220\ V;\ Z_{ph} = \sqrt{15^2 + 20^2} = 25\Omega,\ I_{ph} = V_{ph}/Z_{ph} = 220/25 = 8.8\,\text{A}$$

(*a*) $I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 8.8 =$ **15.24 A** (*b*) $P = I_{ph}{}^2 R_{ph} = 8.8^2 \times 15 =$ **462 W**

(*c*) Phasor sum would be zero because the three currents are equal in magnitude and have a mutual phase difference of 120°.

Solution by Symbolic Notation

Taking V_{RY} as the reference vector, we have [Fig. 19.27 (*b*)]

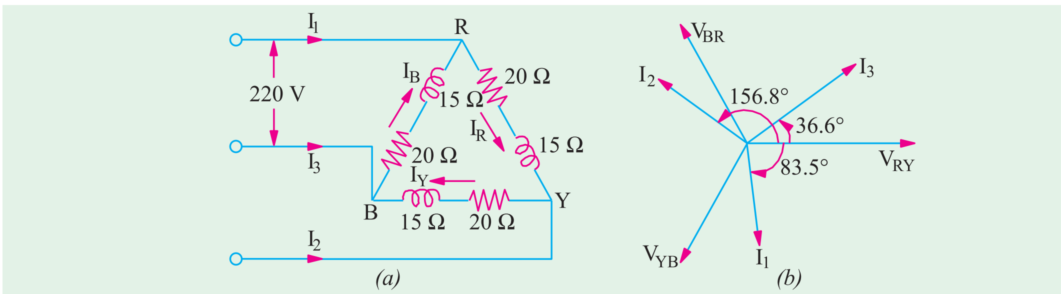


Fig. 19.27

$\mathbf{V_{RY}}$ 20 0 ; $\mathbf{V_{YB}}$ 220 120

V_{BR} 220 120 ; **Z** 15 j20 125 53°8

$$\mathbf{I_R} \quad \frac{V_{RY}}{Z} \quad \frac{220 \quad 0^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 53^\circ 8 \quad (5.28 \quad j7.04)\ A$$

$$\mathbf{I_Y} \quad \frac{V_{YB}}{Z} \quad \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 173^\circ 8 \quad (\ 8.75 \quad j1.05)\ A$$

$$\mathbf{I_B} \quad \frac{V_{BR}}{Z} \quad \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 66^\circ 55 \quad (3.56 \quad j8.1)$$

(*a*) Current in line No. 1 is

$\mathbf{I_1 = I_R - I_B}$ = (5.28 – j 7.04) – (4.56 + j8.1) = (1.72 – j15.14) = 15.23 ∠ – 83.5°

$\mathbf{I_2 = I_Y - I_R}$ = (–8.75 – j 1.05) – (5.28 – j7.04) = (–14.03 + j 6.0) = 15.47 ∠ – 156.8°

$\mathbf{I_3 = I_B - I_Y}$ = (3.56 – j 8.1) – (– 8.75 – j1.05) = (12.31 + j 9.15) = 15.26 ∠ 36.8°

(*b*) Using conjugate of voltage, we get for *R*-phase

$\mathbf{P_{VA} = V_{RY} . I_R}$ = (220 – j0) (5.28 – 7.04) = (1162 – j 1550) voltampere

Real power per phase = **1162 W**

(*c*) Phasor sum of three line currents

= $\mathbf{I_1 + I_2 + I_3}$ = (1.72 – j 15.14) + (– 14.03 + j 6.0) + (12.31 + j 9.15) = 0

As expected, phasor sum of 3 line currents drawn by a balanced load is zero because these are equal in magnitude and have a phase difference of 120° amount themselves.

Example 19.14 *A 3-φ, Δ-connected alternator drives a balanced 3-φ load whose each phase current is 10 A in magnitude. At the time when I_a = 10 ∠ 30°, determine the following, for a phase sequence of abc.*

(*i*) *Polar expression for I_b and I_c and* **(*ii*)** *polar expressions for the three line current.*

Show the phase and line currents on a phasor diagram.

Solution. (*i*) Since it is a balanced 3-phase system, I_b lags I_a by 120° and I_c lags I_a by 240° or leads it by 120°.

$$\therefore \quad I_b = I_a \angle -120° = 10\angle(30°-120°) = 10\angle -90°$$

$$I_c = I_a \angle 120° = 10\angle(30°+120°) = 10\angle 150°$$

The 3-phase currents have been represented on the phasor diagram of Fig. 19.28 (*b*).

As seen from Fig. 19.28 (*b*), the line currents lag behind their nearest phase currents by 30°.

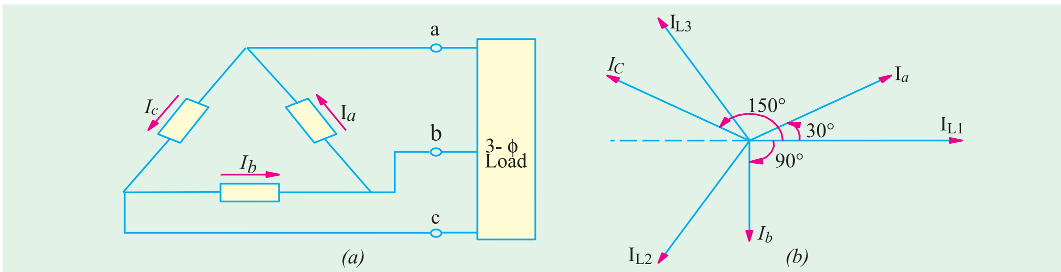


Fig. 19.28

$$\therefore \qquad I_{L1} \quad \sqrt{3}\ I_a \quad (30° \quad 30°) \quad \mathbf{17.3 \quad 0°}$$

$$I_{L2} \quad \sqrt{3}\ I_b \quad (\ 90° \quad 30°) \quad \mathbf{17.3 \qquad 120°}$$

$$I_{L3} \quad \sqrt{3}\ I_C \quad (150° \quad 30°) \quad \mathbf{17.3 \quad 120°}$$

These line currents have also been shown in Fig. 19.28 (*b*).

Example 19.15. *Three similar coils, each having a resistance of 20 ohms and an inductance of 0.05 H are connected in* ***(i)*** *star* ***(ii)*** *mesh to a 3-phase, 50-Hz supply with 400-V between lines. Calculate the total power absorbed and the line current in each case. Draw the vector diagram of current and voltages in each case.* **(Elect. Technology, Punjab Univ. 1990)**

Solution. $X_L \quad 2 \quad 50 \quad 0.05 \quad 15 \quad , Z_{ph} \quad \sqrt{15^2 \quad 20^2} \quad \mathbf{25\,\Omega}$

(*i*) Star Connection. [Fig. 19.29 (*a*)]

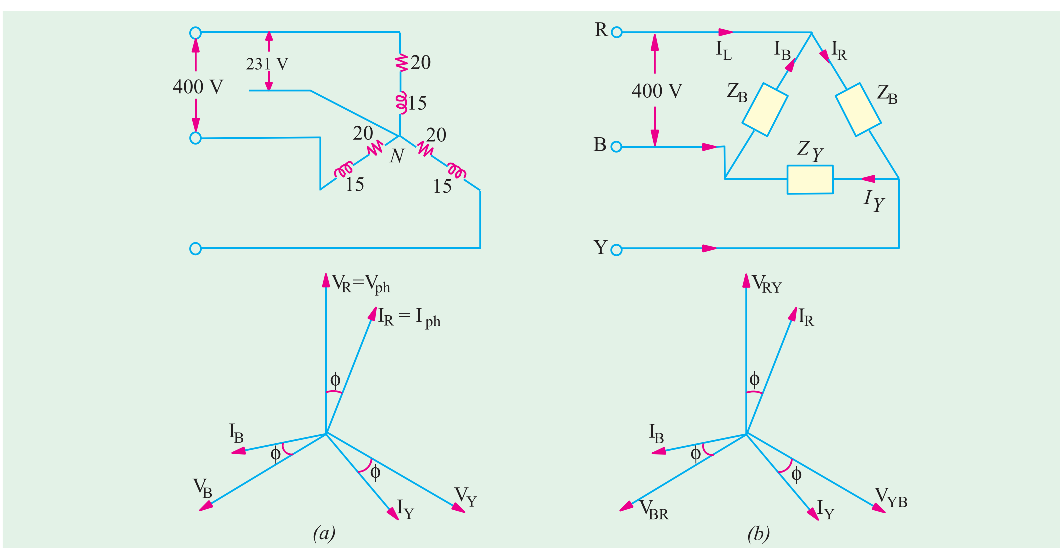


Fig. 19.29

$$V_{ph} = 400/\sqrt{3} = 231 \text{ V};\ I_{ph} = V_{ph}/Z_{ph} = 231/25 = \mathbf{9.24\,\Omega}$$

$$I_L \quad I_{ph} \quad \mathbf{9.24\ A};\ P \quad \sqrt{3} \quad 400 \quad 9.24 \quad (20/25) \quad \mathbf{5120\ W}$$

(*ii*) Delta Connection [Fig. 19.29 (*b*)]

$$V_{ph} \quad V_L \quad 400\,V;\ I_{ph} \quad 400/25 \quad 16\ A;\ I_L \quad \sqrt{3} I_{ph} \quad \sqrt{3} \quad 16 \quad \mathbf{27.7\ A}$$

$$P = \sqrt{3} \times 400 \times 27.7 \times (20/25) = \mathbf{15{,}360\ W}$$

Note. It may be noted that line current as well as power are three times the star values.

Example 19.16. *A Δ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star.* **(Electrical Machines, A.M.I.E. Sec. B. 1992)**

Solution. (*i*) Delta Connection.

$$V_{ph} = V_L = 400\ V;\ I_L = 20\ A;\ I_{ph} = 20/\sqrt{3}\ \text{A}$$

(*i*) $\therefore \quad Z_{ph} \quad \dfrac{400}{20/\sqrt{3}} \quad 20\sqrt{3} \quad \mathbf{34.64\ \Omega}$

Now $P = \sqrt{3}\, V_L I_L \cos\phi \quad \therefore \quad \cos\phi = 10{,}000/\sqrt{3} \times 400 \times 20 =$ **0.7217**

(*ii*) Star Connection

$$V_{ph} \quad \frac{400}{\sqrt{3}},\ I_{ph} \quad \frac{400/\sqrt{3}}{\sqrt{3}} \quad \frac{20}{3} A,\ I_L \quad I_{ph} \quad \frac{20}{3}\ \text{A}$$

Power factor remains the same since impedance is the same.

Power consumed $= \sqrt{3} \times 400 \times (20/3) + 0.7217 =$ **3,330 W**

Note. The power consumed is 1/3 of its value of Δ-connection.

Example 19.17. *Three similar resistors are connected in star across 400-V, 3-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors delta-connected.*

Solution. Star Connection

$$I_L = I_{ph} = 5\ A;\ V_{ph} = 400/\sqrt{3} = 231\text{V} \ \therefore R_{ph} = 231/5 = \mathbf{46.2\ \Omega}$$

Delta Connection

$$I_L = 5 \text{ A... (given)};\ I_{ph} = 5/\sqrt{3} \text{ A};\ R_{ph} = \mathbf{46.2\ \Omega}$$

... found above

$$V_{ph} \quad I_{ph} \quad R_{ph} \quad 5 \quad 46.2/\sqrt{3} \quad \mathbf{133.3\ V}$$

Note. Voltage needed is 1/3rd the star value.

Example 19.18. *A balanced delta connected load, consisting of there coils, draws* $10\sqrt{3}$ *A at 0.5 power factor from 100 V, 3-phase supply. If the coils are re-connected in star across the same supply, find the line current and total power consumed.*

(Elect. Technology, Punjab Univ. Nov.)

Solution. Delta Connection

$$V_{ph} \quad V_L \quad 100V;\ I_L \quad 10\sqrt{3}\ A;\ I_{ph} \quad 10\sqrt{3}/\sqrt{3} \quad \mathbf{10\ A}$$

$Z_{ph} = V_{ph} / I_{ph} = 100/10 = 10\,\Omega$; $\cos\phi = 0.5$ (given); $\sin\phi = 0.866$

$\therefore\ R_{ph}\quad Z_{ph}\cos\quad 10\quad 0.5\quad 5\quad ;\ X_{ph}\quad Z_{ph}\sin\quad 10\quad 0.866\quad \mathbf{8.66}$

Incidentally, total power consumed = $\sqrt{3}V_L I_L\cos\phi = \sqrt{3}\times 100\times 10\sqrt{3}\times 0.5 = 1500\text{ W}$

Star Connection

$V_{ph}\quad V_L/\sqrt{3}\quad 100/\sqrt{3};\ Z_{ph}\quad 10\quad ;I_{ph}\quad V_{ph}/Z_{ph}\quad 100/\sqrt{3}\quad 10\quad 10\sqrt{3}\text{ A}$

Total power absorbed = $\sqrt{3}\times 100\times(10\sqrt{3})\times 0.5 =$ **500 W**

It would be noted that the line current as well as the power absorbed are one-third of that in the delta connection.

Example 19.19. *Three identical impedances are connected in delta to a 3 ϕ supply of 400 V. The line current is 35 A and the total power taken from the supply is 15 kW. Calculate the resistance and reactance values of each impedance.* **(Elect. Technology, Punjab Univ.,)**

Solution. $V_{ph} = V_L = 400$ V; $I_L = 35$ A $\therefore\ I_{ph} = 35/\sqrt{3}$ A

$$Z_{ph} = V_{ph}/I_{ph} = 400\times\sqrt{3}/35 = 19.8\text{ A}$$

Now,

$$\text{Power } P = \sqrt{3}V_L I_L\cos\phi\ \therefore\ \cos\phi = \frac{P}{\sqrt{3}V_L I_L} = \frac{15{,}000}{\sqrt{3}\times 400\times 35} = 0.619;\ \text{But } \sin\phi = 0.786$$

$\therefore\ R_{ph}\quad Z_{ph}\cos\quad 19.8\quad 0.619\quad \mathbf{12.25}\ ;\ X_{ph}\quad Z_{ph}\sin$ and $X_{ph}\quad 19.8\quad 0.786\quad \mathbf{15.5}$

Example 19.20. *Three 100 Ω non-inductive resistances are connected in (a) star (b) delta across a 400-V, 50-Hz, 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting open-circuited, what would be the value of total power taken from the mains in each of the two cases ?*

(Elect. Engg. A.M.Ae. S.I June, 1993)

Solution. (*i*) Star Connection [Fig. 19.30 (*a*)]

$V_{ph}\quad 400/\sqrt{3}\,V$

$P = \sqrt{3}\,V_L I_L\cos\phi$

$= \sqrt{3}\times 400\times 4\times 1/\sqrt{3} =$ **1600 W**

(*ii*) Delta Connection Fig. 19.30 (*b*)

$V_{ph} = 400\text{ V};\ R_{ph} = 100\,\Omega$

$I_{ph} = 400/100 = 4\text{ A}$

$I_L = 4\times\sqrt{3}$ A

$P = \sqrt{3}\times 400\times 4\times\sqrt{3}\times 1$ = **4800 W**

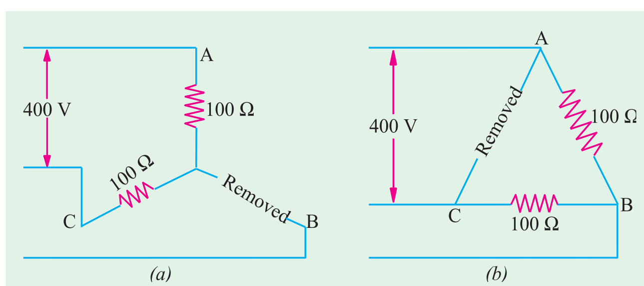


Fig. 19.30

When one of the resistors is disconnected

(*i*) Star Connection [Fig. 19.28 (*a*)]

The circuit no longer remains a 3-phase circuit but consists of two 100 Ω resistors in series across a 400-V supply. Current in lines *A* and *C* is = 400/200 = 2 A

Power absorbed in both = 400 × 2 = 800 W

Hence, by disconnecting one resistor, the power consumption is reduced by half.

(*ii*) Delta Connection [Fig. 19.28 (*b*)]

In this case, currents in *A* and *C* remain as usual 120° out of phase with each other.

Current in each phase = 400/100 = 4 A

Power consumption in both = $2 \times 4^2 \times 100$ = **3200 W**

(or $P = 2 \times 4 \times 400 = 3200$ W)

In this case, when one resistor is disconnected, the power consumption is reduced by one-third.

Example 19.21. *A 200-V, 3-ϕ voltage is applied to a balanced Δ-connected load consisting of the groups of fifty 60-W, 200-V lamps. Calculate phase and line currents, phases voltages, power consumption of all lamps and of a single lamp included in each phase for the following cases :*

(a) *under normal conditions of operation*

(b) *after blowout in line* $R'R$ ***(c)*** *after blowout in phase YB*

Neglect impedances of the line and internal resistances of the sources of e.m.f.

Solution. The load circuit is shown in Fig. 19.31 where each lamp group is represented by two lamps only. It should be kept in mind that lamps remain at the line voltage of the supply irrespective of whether the Δ-connected load is balanced or not.

(*a*) Normal operating conditions [Fig. 19.31 (*a*)]

Since supply voltage equals the rated voltage of the bulbs, the power consumption of the lamps equals their rated wattage.

Power consumption/lamp = 60 W; Power consumption/phase = 50 × 60 = **3,000 W**

Phase current = 3000/200 = **15 A** ; Line current = $15 \times \sqrt{3}$ = **26 A**

(*b*) Line Blowout [Fig. 19.31 (*b*)]

When blowout occurs in line *R*, the lamp group of phase *Y-B* remains connected across line voltage $V_{YB} = V_{y'B'}$. However, the lamp groups of other two phases get connected in series across the same voltage V_{YB}. Assuming that lamp resistances remain constant, voltage drop across YR = V_{YB} 200/2 = **100 V** and that across RB = **100 V**.

Hence, phase currents are as under :

I_{YB} = 3000/200 = **15 A**, $I_{YB} = I_{RB}$ = 15/2 = **7.5 A**

The line currents are :

$$I_{RR'} \quad 0, \ I_{YY'} \quad I_{BB'} \quad I_{YB} \quad I_{YR} \quad 15 \quad 7.5 \quad \mathbf{22.5\ A}$$

Power in phase *YR* = 100 × 7.5 = 750 W; Power/lamp = 750/50 = **15 W**

Power in phase *YB* = 200 × 15 = 3000 W ; Power/lamp = 3000/50 = **60 W**

Power in phase *RB* = 100 × 7.5 = 750 W ; Power/lamp = 750/50 = **15 W**

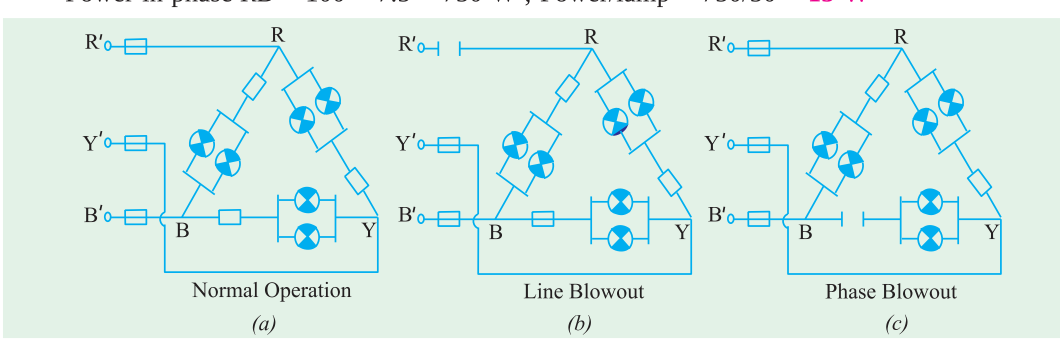


Fig. 19.31

(*c*) Phase Blowout [Fig. 19.31 (*c*)]

When fuse in phase *Y-B* blows out, the phase voltage becomes zero (though voltage across the open remains 200 V). However, the voltage across the other two phases remains the same as under normal operating conditions.

Hence, different phase currents are :

$$I_{RY} \quad \mathbf{15\,A,}\ I_{BR} \quad \mathbf{15\ A,}\ I_{YB} \quad \mathbf{0}$$

The line currents become

$$I_{RR} \quad 15\sqrt{3} \quad \mathbf{26\,A;}\ I_{YY} \quad \mathbf{15\,A,}\ I_{BB} \quad \mathbf{15\,A}$$

Power in phase *RY* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *RB* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *YB* = 0; power/lamp = 0.

Example 19.22. *The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil.*

If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

Solution. Star Connection

$\cos\phi$ k = W/kVA = 11/20 $\quad I_L$ = 25 A $\qquad P$ = 11 kW = 11,000 W

Now $\quad P = \sqrt{3}\ V_L I_L\ \cos\phi \quad \therefore\ 11{,}000 = \sqrt{3} \times V_L \times 25 \times 11/20$

$\therefore\ V_L \quad \mathbf{462\ V}; \qquad V_{ph} = 462/\sqrt{3} = \mathbf{267\ V}$

$$\text{kVAR} \quad \sqrt{\text{kVA}^2 \quad \text{kW}^2} \quad \sqrt{20^2 \quad 11^2} \quad \mathbf{16.7;}\ Z_{ph} \quad 267/2 \quad 10.68$$

$\therefore\ R_{ph} = Z_{ph} \times \cos\phi = 10.68 \times 11/20 = \mathbf{5.87\ \Omega}$

$\therefore\ X_{ph} \quad Z_{ph} \quad \sin \quad 10.68 \quad 0.838 \quad \mathbf{897\Omega}$

Delta Connection

$$V_{ph} = V_L = 462\ V \ \text{ and } \ Z_{ph} = 10.68\,\Omega$$

$$\therefore\ I_{ph} \quad 462/10.68\,\text{A},\ I_L \quad \sqrt{3} \quad 462/10.68 \quad 75\,\text{A}$$

$$P = \sqrt{3} \times 462 \times 75 \times 11/20 = \mathbf{33{,}000\ W}$$

Example 19.23. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 4, 5 and 6 Ω respectively in the three phases. Estimate the current flowing in each phase and the neutral current. Find the total power absorbed.* **(I.E.E. London)**

Solution. Here, V_{ph} = 230 V [Fig. 19.32 (*a*)]

Current in 4-Ω resistor = 230/4
= 57.5 A

Current in 5-Ω resistor = 230/5
= 46 A

Current in 6-Ω resistor = 230/6
= 38.3 A

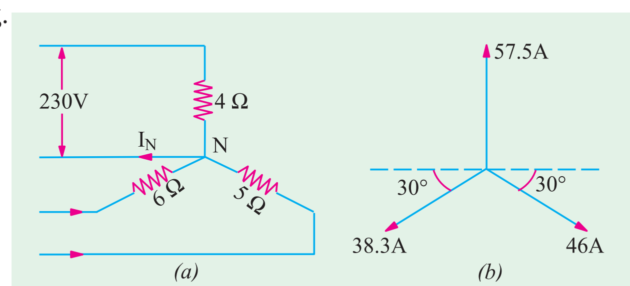


Fig. 19.32

These currents are mutually displaced by 120°. The neutral current I_N is the vector sum* of these three currents. I_N can be obtained by splitting up these three phase currents into their X-components and Y-components and then by combining them together, in diagram 19.32 (*b*).

X-component = 46 cos 30° – 38.3 cos 30° = **6.64 A**

Y-component = 57.5 – 46 sin 30° – 38.3 sin 30° = 15.3 A $I_N \quad \sqrt{6.64^2 \quad 15.3^2}$ **16.71 A**

The power absorbed = 230 (57.5 + 46 + 38.3) = **32.610 W**

Example 19.24. *A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps in use require 70, 84 and 33 A in each of the three lines, what should be the current in the neutral wire ? If a 3-phase motor is now started, taking 200 A from the line at a power factor of 0.2, what would be the current in each line and the neutral current ? Find also the total power supplied to the lamps and the motor.* **(Elect. Technology, Aligarh Univ.)**

Solution. The lamp and motor connections are shown in Fig. 19.33.

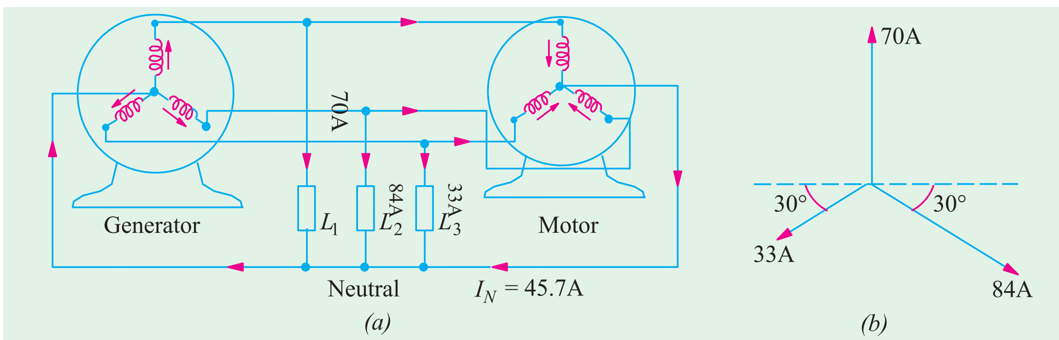


Fig. 19.33

When motor is not started

The neutral current is the vector sum of lamp currents. Again, splitting up the currents into their X- and Y-components, we get

X-component = 84 cos 30° – 33 cos 30° = 44.2 A

Y-component = 70 – 84 sin 30° – 33 sin 30° = 11.5 A

$$\therefore \qquad I_N = \sqrt{44.2^2 + 11.5^2} = \mathbf{45.7\ A}$$

When motor is started

A 3-phase motor is a balanced load. Hence, when it is started, it will change the line currents but being a balanced load, it contributes nothing to the neutral current. Hence, ***the neutral current remains unchanged even after starting the motor.***

Now, the motor takes 200 A from the lines. It means that each line will carry motor current (which lags) as well as lamp current (which is in phase with the voltage). The current in each line would be the vector of sum of these two currents.

Motor p.f. = 0.2 ; sin ϕ = 0.9799 ... from tables

Active component motor current = 200 × 0.2 = 40 A

---

* Some writers disagree with this statement on the ground that according to Kirchhoff's Current Law, at any junction, $\mathbf{I_N + I_R + I_Y + I_B = 0}$ $\therefore$ $\mathbf{I_N = -(I_R + I_Y + I_B)}$

Hence, according to them, numerical value of $\mathbf{I}_N$ is the same but its phase is changed by 180°.

Reactive component of motor current = 200 × 0.9799 = 196 A

(*i*) Current in first line = $\sqrt{(40+70)^2+196^2}$ = **224.8 A**

(*ii*) Current in second line = $\sqrt{(40 \quad 84)^2 \quad 196^2}$ **232A**

(*iii*) Current in third line = $\sqrt{(40+33)^2+196^2}$ = **210.6 A**

Power supplied to lamps = 230 (33 + 84 + 70) = **43,000 W**

Power supplied to motor = $\sqrt{3}\times 200\times 400\times 0.2$ = **27,700 W**

19.11. Star and Delta connected Lighting Loads

In Fig. 19.34 (*a*) is shown a *Y*-connected lighting network in a three storey house. For such a load, it is essential to have neutral wire in order to ensure uniform distribution of load among the three phases despite random switching on and off or burning of lamps. It is seen from Fig. 1934 (*a*),

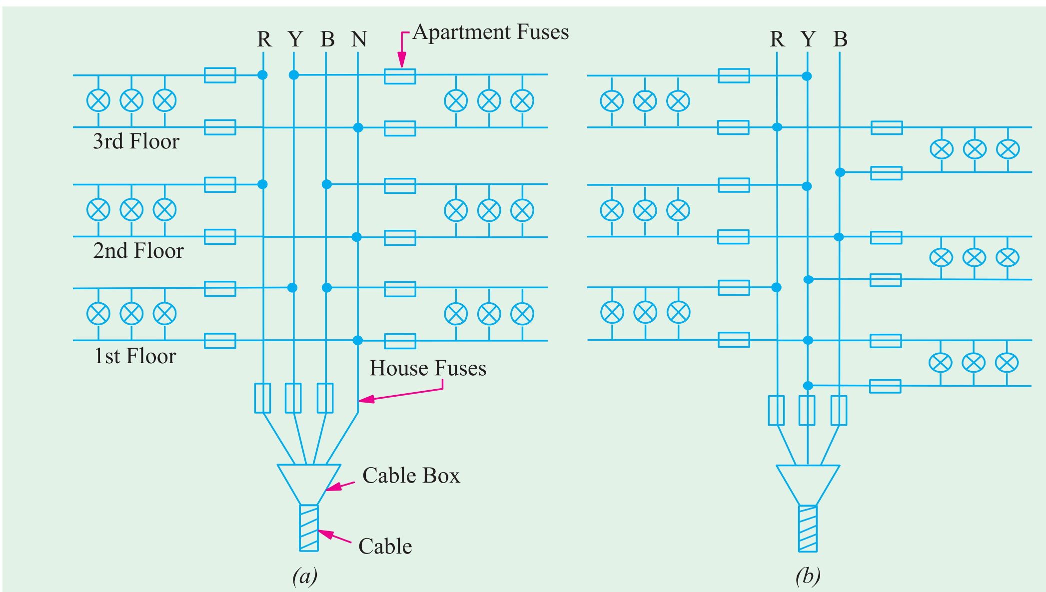


Fig. 19.34

that network supplies two flats on each floor of the three storey residence and there is balanced distribution of lamp load among the three phases. There are house fuses at the cable entry into the building which protect the two mains against short-circuits in the main cable. At the flat entry, there are apartment (or flat) fuses in the single-phase supply which protect the two mains and other flats in the same building from short-circuits in a given building. There is no fuse (or switch) on the neutral wire of the mains because blowing of such a fuse (or disconnection of such a switch) would mean a break in the neutral wire. This would result in unequal voltages across different groups of lamps in case they have different power ratings or number. Consequently, filaments in one group would burn dim whereas in other groups they would burn too bright resulting in their early burn-out.

The house-lighting wire circuit for Δ-connected lamps is shown in Fig. 19.34 (*b*).

19.12. Power Factor Improvement

The heating and lighting loads supplied from 3-phase supply have power factors, ranging from 0.95 to unity. But motor loads have usually low lagging power factors, ranging from 0.5 to 0.9. Single-phase motors may have as low power factor as 0.4 and electric wedding units have even lower power factors of 0.2 or 0.3.

The power factor is given by $\cos \frac{\text{kW}}{\text{kVA}}$ or $\text{kVA} \frac{\text{kW}}{\cos}$

In the case of single-phase supply, $\text{kVA} = \frac{VI}{1000}$ or $I = \frac{1000\ k\text{VA}}{V}$ $\quad \therefore \quad I \propto \text{kVA}$

In the case of 3-phase supply $\text{kVA} = \frac{\sqrt{3}V_L I_L}{1000}$ or $I_L = \frac{1000\ \text{kVA}}{\sqrt{3} \times V_L}$ $\quad \therefore \quad I \propto \text{kVA}$

In each case, the kVA is directly proportional to current. The chief disadvantage of a low p.f. is that the current required for a given power, is very high. This fact leads to the following undesirable results.


Switchgear

(*i*) Large kVA for given amount of power

All electric machinery, like alternators, transformers, switchgears and cables are limited in their current-carrying capacity by the permissible temperature rise, which is proportional to I^2. Hence, they may all be fully loaded with respect to their rated kVA, without delivering their full power. Obviously, it is possible for an existing plant of a given kVA rating to increase its earning capacity (which is proportional to the power supplied in kW) if the overall power factor is improved *i.e.* raised.

(*ii*) Poor voltage regulation

When a load, having allow lagging power factor, is switched on, there is a large voltage drop in the supply voltage because of the increased voltage drop in the supply lines and transformers. This drop in voltage adversely affects the starting torques of motors and necessitates expensive voltage stabilizing equipment for keeping the consumer's voltage fluctuations within the statutory limits. Moreover, due to this excessive drop, heaters take longer time to provide the desired heat energy, fluorescent lights flicker and incandescent lamps are not as bright as they should be. Hence, all supply undertakings try to encourage consumers to have a high power factor.

Example 19.25. *A 50-MVA, 11-kV, 3-φ alternator supplies full load at a lagging power factor of 0.7. What would be the percentage increase in earning capacity if the power factor is increased to 0.95 ?*

Solution. The earning capacity is proportional to the power (in MW or kW) supplied by the alternator.

MW supplied at 0.7 lagging = 50 × 0.7 = 35

MW supplied at 0.95 lagging = 50 × 0.95 = 47.5

increase in MW = 12.5

The increase in earning capacity is proportional to 12.5

∴ Percentage increase in earning capacity = (12.5/35) × 100 = **35.7**

19.13. Power Correction Equipment

The following equipment is generally used for improving or correcting the power factor :

(*i*) Synchronous Motors (or capacitors)

These machines draw leading kVAR when they are over-excited and, especially, when they are running idle. They are employed for correcting the power factor in bulk and have the special advantage that the amount of correction can be varied by changing their excitation.

(*ii*) Static Capacitors

They are installed to improve the power factor of a group of a.c. motors and are practically loss-free (*i.e.* they draw a current leading in phase by 90°). Since their capacitances are not variable, they tend to over-compensate on light loads, unless arrangements for automatic switching off the capacitor bank are made.

(*iii*) Phase Advancers

They are fitted with individual machines.

However, it may be noted that the economical degree of correction to be applied in each case, depends upon the tariff arrangement between the consumers and the supply authorities.

Example 19.26. *A 3-phase, 37.3 kW, 440-V, 50-Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate the total kVA rating of capacitors required to raise the full-load power factor at 0.95 lagging. What will be the capacitance per phase if the capacitors are* **(a)** *delta-connected and* **(b)** *star-connected ?*

Solution. It is helpful to approach such problems from the 'power triangle' rather than from vector diagram viewpoint.

Motor power input P = 37.3/0.89 = 41.191 kW

Power Factor 0.85 (lag)

$$\cos\phi_1 = 0.85 : \phi_1 = \cos^{-1}(0.85) = 31.8^\circ ; \tan\phi_1 = \tan 31.8^\circ = 0.62$$

Motor $\text{kVAR}_1 = P \tan\phi_1 = 41.91 \times 0.62 = 25.98$

Power Factor 0.95 (lag)

Motor power input P = 41.91 kW ... as before

It is the same as before because capacitors are loss-free *i.e.* they do not absorb any power.

$$\cos\phi_2 = 0.95 \quad \therefore \phi_2 = 18.2^\circ ; \quad \tan 18.2^\circ = 0.3288$$

Motor $\text{kVAR}_2 = P \tan\phi_2 = 41.91 \times 0.3288 = 13.79$

The difference in the values of kVAR is due to the capacitors which supply ***leading*** kVAR to partially neutralize the ***lagging*** kVAR of the motor.

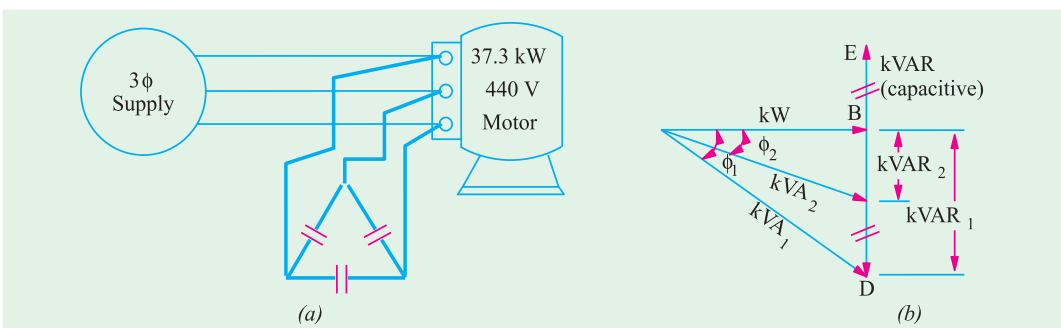


Fig. 19.35

$\therefore$ leading kVAR supplied by capacitors is

= $\text{kVAR}_1 - \text{kVAR}_2$ = 25.98 – 13.79 = **12.19** ... *CD* in Fing. 19.35 (*b*)

Since capacitors are loss-free, their kVAR is the same as kVA

$\therefore$ kVA/capacitor = 12.19/3 = 4.063 $\therefore$ VAR/capacitor = 4,063

(a) In Δ-connection, voltage across each capacitor is 440 V

Current drawn by each capacitor I_C = 4063/440 = 9.23 A

Now, $$I_c = \frac{V}{X_c} = \frac{V}{1/\omega C} = \omega VC$$

$\therefore$ $$C = I_c/\omega V = 9.23/2\pi \times 50 \times 440 = 66.8 \times 10^{-6}\text{ F} = \mathbf{66.8\ \mu F}$$

(b) In star connection, voltage across each capacitor is = $440/\sqrt{3}$ volt

Current drawn by each capacitor, $I_c \quad \dfrac{4063}{440/\sqrt{3}} \quad 16.0$ A

$$I_c \quad \frac{V}{X_c} \quad VC \quad \text{or} \quad 16 = \frac{440}{\sqrt{3}} \times 2\pi \times 50 \times C$$

$\therefore$ $C = 200.4 \times 10^{-6}\text{F} = \mathbf{200.4\ \mu F}$

Note. Star value is three times the delta value.

Example 19.27. *If the motor of Example 19.24 is supplied through a cable of resistance 0.04 Ω per core, calculate*

(i) *the percentage reduction in cable Cu loss and*

(ii) *the additional balanced lighting load which the cable can supply when the capacitors are connected.*

Solution. Original motor $kVA_1 = P/\cos\phi_1 = 41.91/0.85 = 49.3$

Original line current, $$I_{L1} = \frac{kVA_1 \times 1000}{\sqrt{3} \times 440} = \frac{49.3 \times 1000}{\sqrt{3} \times 440} = 64.49\text{ A}$$

$\therefore$ Original Cu loss/conductor = $64.69^2 \times 0.04$ = 167.4 W

From Fig . 19.34, it is seen that the new kVA *i.e.*kVA_2 when capacitors are connected is given by $kVA_2 = kW/\cos\phi_2$ = 41.91/0.95 = 44.12

New line current $$I_{L2} = \frac{44{,}120}{\sqrt{3} \times 440} = 57.89\text{ A}$$

New Cu loss $= 57.89^2 \times 0.04 = 134.1$ W

(i) $\therefore$ percentage reduction $= \dfrac{167.4 - 134.1}{167.4} \times 100 = \mathbf{19.9}$

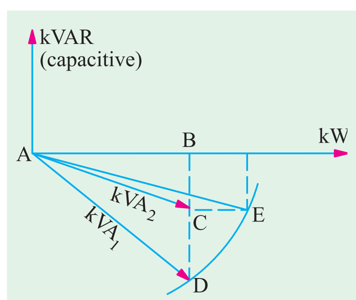


Fig. 19.36

The total kVA which the cable can supply is 49.3 kVA. When the capacitors are connected, the kVA supplied is 44.12 at a power factor of 0.94 lagging. The lighting load will be assumed at unity power factor. The kVA diagram is shown in Fig. 19.34. We will tabulate the different loads as follows. Let the additional lighting load be x kW.

Load	*kVA*	$\cos\phi$	*kW*	$\sin\phi$	*kVAR*
Motor	49.3	0.85 lag	41.91	0.527	–25.98
Capacitors	12.19	0 lead	0	1.0	+12.19
Lighting	–	1.0	x	0	0
			1.91 + x)		–13.79

From Fig. 19.36 it is seen that

AF = 41.91 + x and EF = 13.79 AE = resultant kVA = 49.3

Also $AF^2 + EF^2 = AE^2$ or $(41.91 + x)^2 + 13.79^2 = 49.3^2$ $\therefore$ x = **5.42 kW**

Example 19.28. *Three impedance coils, each having a resistance of 20 Ω and a reactance of 15 Ω, are connected in star to a 400-V, 3-φ, 50-Hz supply. Calculate **(i)** the line current **(ii)** power supplied and **(iii)** the power factor.*

If three capacitors, each of the same capacitance, are connected in delta to the same supply so as to form parallel circuit with the above impedance coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

Solution. $V_{ph} \;\; 100/\sqrt{3}V, Z_{ph} \;\; \sqrt{20^2 \;\; 15^2} \;\; 25$

$\cos \;_1 \;\; R_{ph}/Z_{ph} \;\; 20/25 \;\; 0.8 \text{lag}; \;_1 \;\; 0.6 \text{ lag}$

where ϕ_1 is the power factor angle of the coils.

When capacitors are not connected

(*i*) $I_{ph} = 400/25 \times \sqrt{3} = 9.24\text{ A}$ $\therefore I_L$ = **9.24 A**

(*ii*) $P = \sqrt{3}\, V_L I_L \cos\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.8 =$ **5.120 W**

(*iii*) Power factor = **0.8 (lag)**

$\therefore$ Motor VAR$_1$ = $\sqrt{3} V_L I_L \sin\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.6 =$ **3,840**

When capacitors are connected

Power factor, $\cos\phi_2$ = 0.95, ϕ_2 = 18.2° ; tan 18.2° = 0.3288

Since capacitors themselves do not absorb any power, power remains the same *i.e.* 5,120 W even whin capacitors are connected. The only thing that changes is the VAR.

Now VAR$_2$ = $P \tan\phi_2$ = 5120 × 0.3288 = 1684

Leading VAR supplied by the three capacitors is

= VAR$_1$ – VAR$_2$ = 3840 – 1684 = 2156 BD or CE in Fig 19.37 (*b*)

VAR/ Capacitor = 2156/3 = 719

For delta connection, voltage across each capacitor is 400 V $\therefore$ I_c = 719/400 = 1.798 A

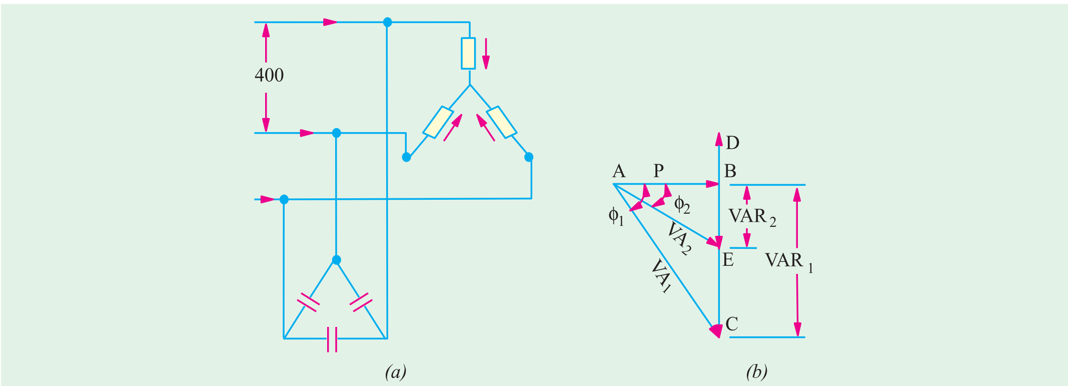


Fig. 19.37

Also $I_c = \dfrac{V}{I/\omega C} = \omega VC \therefore C = 1.798/\pi \times 50 \times 400 = 14.32 \times 10^{-6}\text{ F}$ = **14.32 μF**

19.14. Parallel Loads

A combination of balanced 3-phase loads connected in parallel may be solved by any one of the following three methods :

1. All the given loads may be converted into equivalent Δ-loads and then combined together according to the law governing parallel circuits.

2. All the given loads may be converted into equivalent *Y*-loads and treated as in **(1)** above.

3. The third method, which requires less work, is to work in terms of volt-amperes. The special advantage of this approach is that voltameters can be added regardless of the kind of connection involved. The real power of various loads can be added arithmetically and VARs may be added algebraically so that total voltamperes are given by

$$VA = \sqrt{W^2 + VAR^2} \quad \text{or} \quad S = \sqrt{P^2 + Q^2}$$

where *P* is the power in water and *Q* represents reactive voltamperes.

Example 19.29. *For the power distribution system shown in Fig. 19.38, find*

(*a*) *total apparent power, power factor and magnitude of the total current I_T without the capacitor in the system*

(*b*) *the capacitive kVARs that must be supplied by C to raise the power factor of the system to unity ;*

(*c*) *the capacitance C necessary to achieve the power correction in part* **(*b*)** *above*

(*d*) *total apparent power and supply current I_T after the power factor correction.*

Solution. (*a*) We will take the inductive *i.e.* lagging kVARs as negative and capacitive *i.e.* leading kVARs as positive.

Total *Q* = – 16 + 6 – 12 = – 22 kVAR (lag); Total *P* = 30 + 4 + 36 = 70 kW

$\therefore$ apparent power $S = \sqrt{(-22)^2 + 70^2}$ = **73.4 kVA;** p.f. = cos ϕ = P/S = 70/73.4 = **0.95**

$$S = VI_T \text{ or } 73.4 \times 10^3 = 400 \times I_T \quad \therefore \quad I_T = \mathbf{183.5\ A}$$

(*b*) Since total lagging kVARs are **– 22**, hence, for making the power factor unity, 22 leading kVARs must be supplied by the capacitor to neutralize them. In that case, total *Q* = 0 and *S* = *P* and p.f. is unity.

(*c*) If I_C is the current drawn by the capacitor, then $22 \times 10^3 = 400 \times I_C$

Now, $I_C = V/X_C = V\omega C$

$= 400 \times 2\pi \times 50 \times C$

$\therefore 20 \quad 10^3 \quad 400 \quad (400 \quad 2 \quad 50 \quad C)$;

$\therefore$ *C* = **483 μF**

(*d*) Since *Q* = 0,

hence, $S = \sqrt{10^2 + 70^2}$ = **70 kVA**

Now, $VI_T = 70 \times 10^3$;

$I_T = 70 \times 10^3 / 400$ = 175 A.

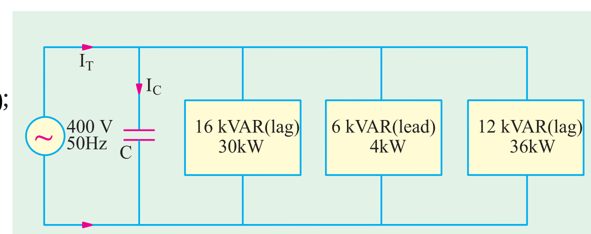


Fig. 19.38

It would be seen that after the power correction, lesser amount of current is required to deliver the same amount of real power to the system.

Example 19.30. *A symmetrical 3-phase, 3-wire supply with a line voltage of 173 V supplies two balanced 3-phase loads; one Y-connected with each branch impedance equal to (6 + j8) ohm and the other Δ-connected with each branch impedance equal to (18 + j24) ohm. Calculate*

(*i*) *the magnitudes of branch currents taken by each 3-phase load* *(ii) the magnitude of the total line current and*

(iii) the power factor of the entire load circuit

Draw the phasor diagram of the voltages and currents for the two loads.

(Elect. Engineering-I, Bombay Univ.)

Solution. The equivalent *Y*-load of the given Δ-load (Art.19.10) is = (18 + *j*24)/3 = (6 + *j*8) Ω.

With this, the problem now reduces to one of solving two equal *Y*-loads connected in parallel across the 3-phase supply as shown in Fig. 19.39 (*a*). Phasor diagram for the combined load for one phase only is given in Fig. 19.39 (*b*).

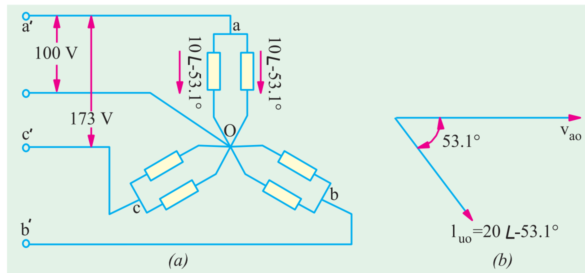


Fig. 19.39

Combined load impedance

$$= (6 + j8)/2 = 3 + j4$$

$$= 5\angle 53.1° \text{ ohm}$$

$$V_{ph} = 173/\sqrt{3} = 100 \text{ V}$$

Let $V_{ph} = 100\angle 0°$

$$\therefore \quad I_{ph} = \frac{100\angle 0°}{5\angle 53.1°} = 20\angle -53.1°$$

Current in each load = $10\angle -53.1$ A

(*i*) branch current taken by each load is **10 A**; (*ii*) line current is **20 A**;

(*iii*) combined power factor = cos 53.1° = **0.6 (lag).**

Example 19.31. *Three identical impedances of* $30\angle 30°$ *ohms are connected in delta to a 3-phase, 3-wire, 208 V volt abc system by conductors which have impedances of (0.8 + j 0.63) ohm. Find the magnitude of the line voltage at the load end.*

(Elect. Engg. Punjab Univ. May 1990)

Solution. The equivalent Z_y, of the given Z_Δ is $30\angle 30/3 = 10\angle 30°$ = (8.86 + *j*5). Hence, the load connections become as shown in Fig. 19.40.

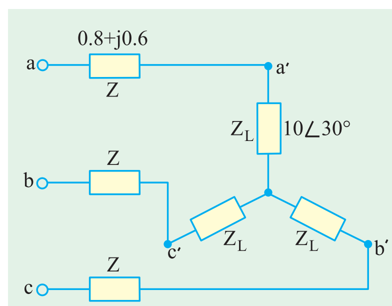


Fig. 19.40

$$Z_{an} = (0.8 + j0.6) + (8.86 + j5)$$

$$= 9.66 + j5.6 = 11.16\angle 30.1°$$

$$V_{an} = V_{ph} = 208/\sqrt{3} = 120 \text{ V}$$

Let $V_{an} = 120\angle 0°$

$$\therefore \quad I_{an} = 120\angle 0°/11.16\angle 30.1° = 10.75\angle -30.1°$$

Now, $Z_{aa'} = 0.8 + j0.6 = 1\angle 36.9°$

Voltage drop on line conductors is

$$V'_{aa} = I_{an} Z'_{aa} = 10.75\angle -30.1° \times 1\angle 36.9° = 10.75\angle 6.8° = 10.67 + j1.27$$

$$\therefore \quad V'_{an} = V_{an} - V'_{aa} = (120 + j0) - (10.67 + j1.27) = \mathbf{109.3 \quad 2.03°}$$

Example 19.32. *A balanced delta-connected load having an impedance* Z_L *= (300 + j210) ohm in each phase is supplied from 400-V, 3-phase supply through a 3-phase line having an impedance of* Z_s *= (4 + j8) ohm in each phase. Find the total power supplied to the load as well as the current and voltage in each phase of the load.*

(Elect. Circuit Theory, Kerala Univ.)

Solution. The equivalent *Y*-load of the given Δ-load is

$= (300 + j210)/3 = (100 + j70)\,\Omega$

Hence, connections become as shown in Fig. 19.41

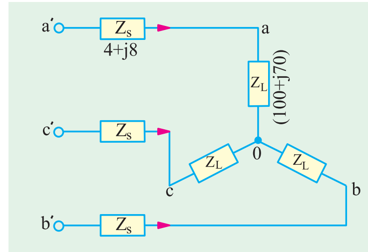


Fig. 19.41

$\mathbf{Z}_{a0}$ (4 j8) (100 j70) 104 j78 130 36.9

$\mathbf{V}_{a0}$ $400/\sqrt{3}$ 231V,

$\mathbf{I}_{a0}$ 231 0 /130 36.9 1.78 36.9

Now, $Z'_{a0} = (4 + j8) = 8.94\angle 63.4°$

Line drop $\mathbf{V}_{aa}$ $\mathbf{I}_{aa}\mathbf{Z}_{aa}$ 1.78 36.9 8.94 63.4 15.9 26.5 14.2 j7.1

$\mathbf{V}_{a0}$ $\mathbf{V}_{a0}$ $\mathbf{V}_{aa}$ (231 j0) (14.2 j7.1)

$= (216.8 - j7.1) = 216.9\angle -1°52'$

Phase voltage at load end, V_{a0} = **216.9 V**

Phase current at load end, I_{a0} = **1.78 A**

Power supplied to load = 3 × 1.782 × 100 = **951 W**

Incidentally, line voltage at load end $V_{ac} = 216.9 \times \sqrt{3}$ = **375.7 V***

Example 19.33. *A star connected load having R = 42.6 ohms/ph and X_L = 32 ohms/ph is connected across 400 V, 3 phase supply, calculate:*

(i) *Line current, reactive power and power loss*

(ii) *Line current when one of load becomes open circuited.*

[Nagpur University, Summer 2001]

Solution.

(i) Z = 42.6 + j32

$|Z| = 53.28$ ohms, Impedance angle, $\theta = \cos^{-1}\left(\frac{42.6}{53.28}\right) = \cos^{-1} 0.80$

$\theta = 36.9°$

Line Current = phase current, due to star-connection

$$= \frac{\text{Voltage/phase}}{\text{Impedance/phase}} \quad \frac{400/\sqrt{3}}{53.28} = 4.336 \text{ amp}$$

Due to the phase angle of 36.9° lagging,

Reactive Power for the three-phases

$= 3\ V_{ph}\ I_{ph} \sin\phi = 3 \times 231 \times 4.336 \times 0.6 = 1803$ VAR

Total Power-loss = $3\ V_{ph}\ I_{ph} \cos\phi = 3 \times 231 \times 4.336 \times 0.8$ = 2404 watts

(ii) One of the Loads is open-circuited.

The circuit is shown in Fig. 19.42 (*b*).

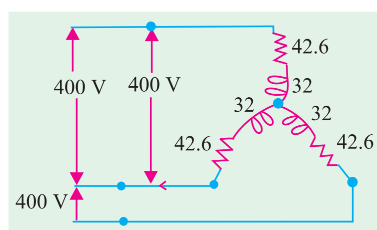


Fig. 19.42 (*a*)

* It should be noted that total line drop is not the numerical sum of the individual line drops because they are 120° out of phase with each other. By a laborious process $\mathbf{V}_{ac} = \mathbf{V}'_{ac} - \mathbf{V}'_{aa} - \mathbf{V}'_{ac}$.

Between A and B, the Line voltage of 400 V drives a current through two "phase-impedances" in series.

Total Impedance between A and B = (42.6 + j 32) × 2 ohms

Hence, the line current I for the two Lines A and B

$$= \frac{400}{2 \times 53.25} = 3.754 \text{ amp}$$

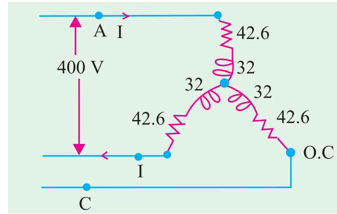


Fig. 19.42 (*b*) One phase open circuited

Note : Third Line 'C' does not carry any current.

Example 19.34. *Three non-inductive resistances, each of 100 ohms, are connected in star to a three-phase, 440-V supply. Three inductive coils, each of reactance 100 ohms connected in delta are also connected to the supply. Calculate:* ***(i)*** *Line-currents, and (ii) power factor of the system*

[Nagpur University, November 1998]

Solution. (*a*) Three resistances are connected in star. Each resistance is of 100 ohms and 254 – V appears across it. Hence, a current of 2.54 A flows through the resistors and the concerned power-factor is unity. Due to star-connection,

Line-current = Phase-current = 2.54A

(*b*) Three inductive reactance are delta connected.

Line-Voltage = Phase – Voltage = 440 V

Phase Current = 440/100 = 4.4 A

Line current = 1.732 × 4.4 = 7.62 A

The current has a zero lagging power-factor.

Total Line Current = 2.54 - j 7.62 A

= 8.032 A, in each of the lines.

Power factor = 2.54/8.032 = 0.32 Lag.

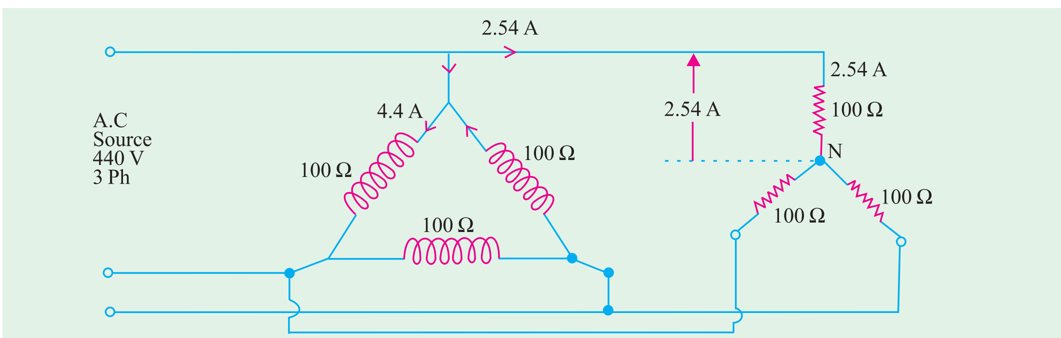


Fig. 19.43

Example 19.35. *The delta-connected generator of Fig 19.44 has the voltage;* V_{RY} = *220* $\angle 0°$, V_{YB}*=220* $\angle -120°$ *and* V_{BR} *= 220* $\angle -240°$ *Volts.*

The load is balanced and delta-connected. Find:

(a) *Impedance per phase,* ***(b)*** *Current per phase,* ***(c)*** *Other line – currents* I_Y *and* I_B*.*

[Nagpur University, November 1997]

Solution. Draw phasors for voltages as mentioned in the data. V_{RY} naturally becomes a reference-phasor, along which the phasor I_R also must lie, as shown in Fig. 19.44 (*b*) & (*c*). I_R is the line voltage which is related to the phase-currents I_{RY} and – I_{BR}. In terms of magnitudes,

$$|I_{RY}| = |I_{BR}| = |I_R|/\sqrt{3} = 10/\sqrt{3} = 5.8 \text{ Amp}$$

Thus, I_{RY} leads V_{RY} by 30°. This can take place only with a series combination of a resistor and a capacitor, as the simplest impedance in each phase

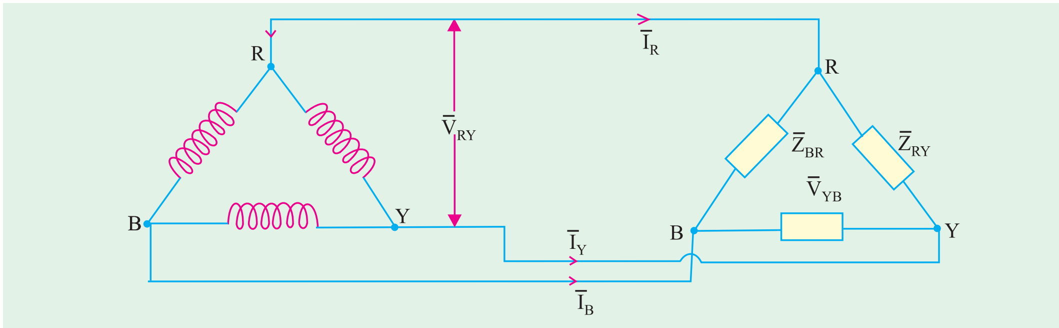


Fig 19.44 (*a*)

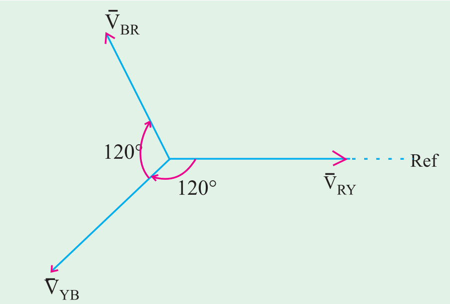


Fig. 19.44 (*b*)

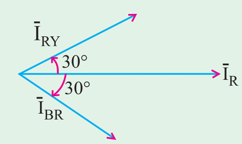


Fig. 19.44 (*c*)

(*a*) $|Z| = 220/5.8 = 38.1$ ohms

Resistance per phase = 38.1 × cos 30° = 33 ohms

Capacitive Reactance/phase = 38.1 × sin 30° = 19.05 ohms

(*b*) Current per phase = 5.8 amp, as calculated above.

(*c*) Otherline currents: Since a symmetrical three phase system is being dealt with, three currents have a mutual phase-difference of 120°. Hence

$I_R = 10 \angle 0°$ as given, $I_Y = 10 \angle -120°$ amp; $I_B = 10 \angle -240°$ amp.

Example 19.36. *A balanced 3-phase star-connected load of 8 + j 6 ohms per phase is connected to a three-phase 230 V supply. Find the line-current, power-factor, active power, reactive-power, and total volt-amperes.*

[Rajiv Gandhi Technical University, Bhopal, April 2001]

Solution. When a statement is made about three–phase voltage, when not mentioned otherwise, the voltage is the line-to-line voltage. Thus, 230 V is the line voltage, which means, in star-system, phase-voltage is 230/1.732, which comes to 132.8 V.

$|Z| = \sqrt{8^2 + 6^2} = 10$ ohms

Line current = Phase current

= 132.8/10 = 13.28 amp

Power – factor = R/Z = 0.8, Lagging

Total Active Power = P = 1.732 × Line Voltage × Line Current × P.f.

Or = 3. Phase Voltage × Phase-current × P.f

= 3 × 132.8 × 13.28 × 0.8 = 4232 watts

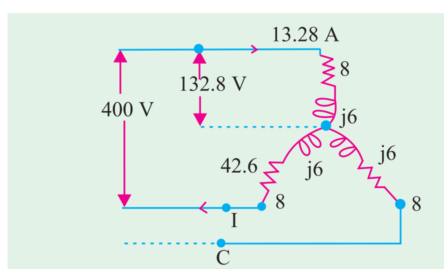


Fig. 19.45

Total Reactive Power = Q

= 3 × Phase-voltage × Phase-current × sin ϕ

= 3 × 132.8 × 13.28 × 0.60 = 3174 VAR

Total Volt-amps = S = $\sqrt{P^2 + Q^2}$ = 5290 VA

Or S = $\sqrt{3} \times 230 \times 13.28$ = 5290 VA

Example 19.37. *A balanced three-phase star connected load of 100 kW takes a leading current of 80 amp, when connected across a three-phase 1100 V, 50 Hz, supply. Find the circuit constants of the load per phase.* **[Nagpur University, April 1996]**

Solution. Voltage per phase = 1100/1.732 = 635 V

Impedance = 635/80 = 7.94 ohms.

Due to the leading current, a capacitor exists.

Resistance R can be evaluated from current and power consumed

$3 I^2 R = 100 \times 1000$, giving R = 5.21 ohms

$X_c = (7.94^2 - 5.21^2)^{0.5}$ = 6 ohms

At 50 Hz, C = 1/(314 × 6) = 531 microfarads.

Tutorial Problem No. 19.1

1. Each phase of a delta-connected load comprises a resistor of 50 Ω and capacitor of 50 μF in series. Calculate (*a*) the line and phase currents (*b*) the total power and (*c*) the kilovoltamperes when the load is connected to a 440-V, 3-phase, 50-Hz supply. **[(*a*) 9.46 A; 5.46 A (*b*) 4480 W (*c*) 7.24 kVA]**

2. Three similar-coils, *A*, *B* and *C* are available. Each coil has 9 Ω resistance and 12 Ω reactance. They are connected in delta to a 3-phase, 440-V, 50-Hz supply. Calculate for this load:

(*a*) the line current (*b*) the power factor

(*c*) the total kilovolt-amperes (*d*) the total kilowatts

If the coils are reconnected in star, calculate for the new load the quantities named at (*a*), (*b*); (*c*) and (*d*) above. **[50.7 A; 0.6; 38.6 kVA; 23.16 kW; 16.9 A; 0.6; 12.867 kVA; 7.72 kW]**

3. Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.

[577.3 V; 333.3 V; 10.2 kVAR; 15.1 Ω; 16.3 Ω]

4. The load in each branch of a delta-connected balanced 3-ϕ circuit consists of an inductance of 0.0318 H in series with a resistance of 10 Ω. The line voltage is 400 V at 50 Hz. Calculate (*i*) the line current and (*ii*) the total power in the circuit. **[(*i*) 49 A (*ii*) 24 kW] *(London Univ.)***

5. A 3-phase, delta-connected load, each phase of which has *R* = 10 Ω and *X* = 8 Ω, is supplied from a star-connected secondary winding of a 3-phase transformer each phase of which gives 230 V. Calculate

(*a*) the current in each phase of the load and in the secondary windings of the transformer

(*b*) the total power taken by the load

(*c*) the power factor of the load. **[(*a*) 31.1 A; 54 A (*b*) 29 kW (*c*) 0.78]**

6. A 3-phase load consists of three similar inductive coils, each of resistance 50 Ω and inductance 0.3 H. The supply is 415 V, 50 Hz, Calculate (*a*) the line current (*b*) the power factor and (*c*) the total power when the load is (*i*) star-connected and (*ii*) delta-connected.

[(*i*) 2.25 A, 0.47 lag, 762 W (*ii*) 6.75 A, 0.47 lag, 2280 W] (*London Univ.*)

7. Three 20 Ω non-inductive resistors are connected in star across a three phase supply the line voltage of which is 480 V. Three other equal non-inductive resistors are connected in delta across the same supply so as to take the same-line current. What are the resistance values of these other resistors and what is the current- flowing through each of them? **[60 Ω; 8A] (*Sheffield Univ. U.K.*)**

8. A 415-V, 3-phase, 4-wire system supplies power to three non-inductive loads. The loads are 25 kW between red and neutral, 30 kW between yellow and neutral and 12 kW between blue and neutral.

Calculate (*a*) the current in each-line wire and (*b*) the current in the neutral conductor.

[(*a*) 104.2 A, 125 A, 50 A (*b*) 67 A] (*London Univ.*)

9. Non-inductive loads of 10, 6 and 4 kW are connected between the neutral and the red, yellow and blue phases respectively of a three-phase, four-wire system. The line voltage is 400 V. Find the current in each line conductor and in the neutral. **[(*a*) 43.3 A, 26A, 173. A, 22.9] (*App. Elect. London Univ.*)**

10. A three-phase, star-connected alternator supplies a delta-connected load, each phase of which has a resistance of 20 Ω and a reactance of 10 Ω. Calculate (*a*) the current supplied by the alternator (*b*) the output of the alternator in kW and kVA, neglecting the losses in the lines between the alternator and the load. The line voltage is 400 V. **[(*a*) 30.95 A (b) 19.2 kW, 21.45 kVA]**

11. Three non-inductive resistances, each of 100 Ω, are connected in star to 3-phase, 440-V supply. Three equal choking coils each of reactance 100 Ω are also connected in delta to the same supply. Calculate:

(*a*) line current (*b*) p.f. of the system. **[(*a*) 8.04 A (*b*) 0.3156] (*I.E.E. London*)**

12. In a 3-phase, 4-wire system, there is a balanced 3-phase motor load taking 200 kW at a power factor of 0.8 lagging, while lamps connected between phase conductors and the neutral take 50, 70 and 100 kW respectively. The voltage between phase conductors is 430 V. Calculate the current in each phase and in the neutral wire of the feeder supplying the load.

[512 A, 5.87 A, 699 A; 213.3 A] (*Elect. Power, London Univ.*)

13. A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three Δ-connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor. **[11.45 kVA, 62.7 μF] (*I.E.E. London*)**

14. Three resistances, each of 500 Ω, are connected in star to a 400-V, 50-Hz, 3-phase supply. If three capacitors, when connected in delta to the same supply, take the same line currents, calculate the capacitance of each capacitor and the line current. **[2.123 μ F, 0.653 A] (*London Univ.*)**

15. A factory takes the following balanced loads from a 440-V, 3-phase, 50-Hz supply:

(*a*) a lighting load of 20 kW (*b*) a continuous motor load of 30 kVA at 0.5 p.f. lagging.

(*c*) an intermittent welding load of 30 kVA at 0.5 p.f. lagging.

Calculate the kVA rating of the capacitor bank required to improve the power factor of loads (*a*) and (*b*) together to unity. Give also the value of capacitor required in each phase if a star-connected bank is employed.

What is the new overall p.f. if, after correction has been applied, the welding load is switched on.

[30 kVAR; 490μ F; 0.945 kg]

16. A three-wire, three-phase system, with 400 V between the line wires, supplies a balanced delta-connected load taking a total power of 30 kW at 0.8 power factor lagging. Calculate (*i*) the resistance and (*ii*) the reactance of each branch of the load and sketch a vector diagram showing the line voltages and line currents. If the power factor of the system is to be raised to 0.95 lagging by means of three delta-connected capacitors, calculate (*iii*) the capacitance of each branch assuming the supply frequency to be 50 Hz.

[(*i*) 10.24 A (*ii*) 7.68 Ω (*iii*) 83.2 μF] (*London Univ.*)

19.15. Power Measurement in 3-phase Circuits

Following methods are available for measuring power in a 3-phase load.

(*a*) Three Wattmeter Method

In this method, three wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.

(*b*) Two Wattmeter Method

(*i*) This method gives true power in the 3-phase circuit without regard to balance or wave form provided in the case of *Y*-connected load. The neutral of the load is isolated from the neutral of the source of power. Or if there is a neutral connection, the neutral wire should not carry any current. This is possible only if the load is perfectly balanced and there are no harmonics present of triple frequency or any other multiples of that frequency.

(ii) This method can also be used for 3-phase, 4-wire system in which the neutral wire carries the neutral current. In this method, the current coils of the wattmeters are supplied from current transformers inserted in the principal line wires in order to get the correct magnitude and phase differences of the currents in the current coils of the wattmeter, because in the 3-phase, 4-wire system, the sum of the instantaneous currents in the principal line wires is not necessarily equal to zero as in 3-phase 3-wire system.

(c) One Wattmeter Method

In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used only when the load is balanced.

19.16. Three Wattmeter Method

A wattmeter consists of ***(i)*** a low resistance current coil which is inserted in series with the line carrying the current and ***(ii)*** a high resistance pressure coil which is connected across the two points whose potential difference is to be measured.

A wattmeter shows a reading which is proportional to the product of the current through its current coil, the p.d. across its potential or pressure coil and cosine of the angle between this voltage and current.

As shown in Fig. 19.46 in this method three wattmeters are inserted in each of the three phases of the load whether Δ-connected or *Y*-connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase-voltage of this phase. Hence, each wattmeter measures the power in a single phase. The algebraic sum of the readings of three wattmeters must give the total power in the load.

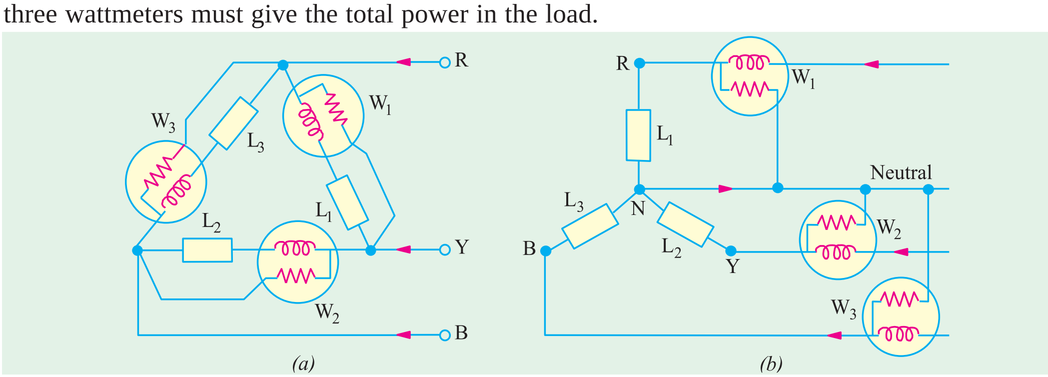


Fig. 19.46

The difficulty with this method is that under ordinary conditions it is not generally feasible to break into the phases of a delta-connected load nor is it always possible, in the case of a *Y*-connected load, to get at the neutral point which is required for connections as shown in Fig. 19.47 (*b*). However, it is not necessary to use three wattmeters to measure power, two wattmeters can be used for the purpose as shown below.

19.17. Two Wattmeter Method-Balanced or Unbalanced Load

As shown in Fig. 19.41, the current coils of the two wattmeters are inserted in ***any two*** lines and the potential coil of each joined to the third line. It can be proved that the sum of the instantaneous powers indicated by W_1 and W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 and L_3. A star-connected load is considered in the following discussion although it can be equally applied to Δ-connected loads because a Δ-connected load can always be replaced by an equivalent *Y*-connected load.

Now, before we consider the currents through and p.d. across each wattmeter, it may be pointed out that ***it is important to take the direction of the voltage through the circuit the same as that taken for the current when establishing the readings of the two wattmeters***.

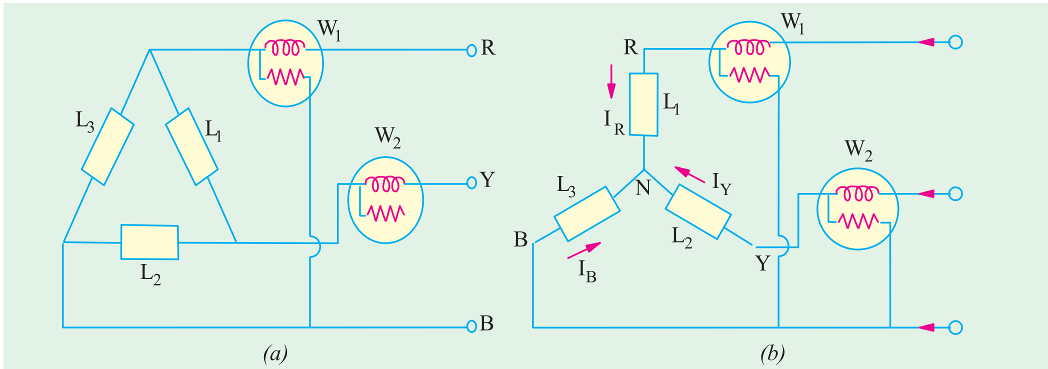


Fig. 19.47

Instantaneous current through $W_1 = i_R$

p.d. across $W_1 = e_{RB} \quad = e_R - e_B$

p.d. across power read by $W_1 = i_R\,(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous p.d. across $W_2 = e_{YB} = (e_Y - e_B)$

Instantaneous power read by $W_2 = i_Y\,(e_Y - e_B)$

$$\therefore \quad W_1 + W_2 = e_R(e_R - e_B) + i_Y(e_Y - e_B) = i_R e_R + i_Y e_Y - e_B(i_R + i_Y)$$

Now, $\quad i_R + i_Y + i_B = 0 \quad$ ... Kirchhoff's Current Law

$$\therefore \quad i_R + i_Y = -i_B$$

or $\quad W_1 + W_2 = i_R . e_R + i_Y . e_y + i_B . e_B = p_1 + p_2 + p_3$

where p_1 is the power absorbed by load L_1, p_2 that absorbed by L_2 and p_3 that absorbed by L_3

$\therefore \quad W_1 + W_2$ = total power absorbed

The proof is true whether the load is balanced or unbalanced. If the load is *Y*-connected, it should have no neutral connection (*i.e.* $3 - \phi$, 3-wire connected) and if it has a neutral connection (*i.e.* $3 - \phi$, 4-wire connected) then it should be exactly balanced so that in each case there is no neutral current i_N otherwise Kirchoff's current law will give $i_N + i_R + i_y + i_B = 0$.

We have considered ***instantaneous*** readings, but in fact, the moving system of the wattmeter, due to its inertia, cannot quickly follow the variations taking place in a cycle, hence it indicates the ***average*** power.

$$\therefore \quad W_1 + W_2 = \frac{1}{T}\int_0^T i_R e_{RB}\,dt + \frac{1}{T}\int_0^T i_Y e_{YB}\,dt$$

19.18. Two Wattmeter Method– *Balanced Load*

If the load is balanced, then power factor of the load can also be found from the two wattmeter readings. The *Y*-connected load in Fig. 19.47 (*b*) will be assumed inductive. The vector diagram for such a balanced *Y*-connected load is shown in Fig. 19.48. We will now consider the problem in terms of r.m.s. values instead of instantaneous values.

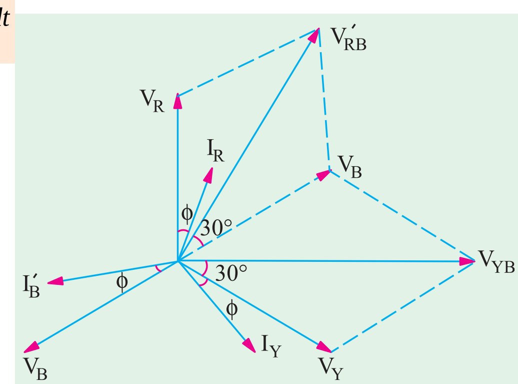


Fig. 19.48

Let V_R, V_Y and V_B be the r.m.s. values of the three phase voltages and I_R, I_Y and I_B the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by ϕ .

Current through wattmeter W_1 [Fig. 19.47 (*b*)] is = I_R.

P.D. across voltage coil of W_1 is

$$V_{RB} = V_R - V_B \qquad \text{... vectorially}$$

This V_{RB} is foundby compounding V_R and V_B reversed as shown in Fig. 19.42. It is seen that phase difference between V_{RB} and I_R = (30° – ϕ).

$\therefore$ Reading of W_1 = I_R V_{RB} cos (30° – ϕ)

Similarly, as seen from Fig. 19.47 (*b*). Current through W_2 = I_Y

P.D. across $W_2 = V_{YB} = V_Y - V_B$ ... vectorially

Again, V_{YB} is found by compounding V_Y and V_B reversed as shown in Fig. 19.48. The angle between I_Y and V_{YB} is (30° + ϕ). Reading of W_2 = $I_Y V_{YB}$ cos (30° + ϕ)

Since load is balanced, V_{RB} = V_{YB} = line voltage V_L; I_Y = I_R = line current, I_L

$$\therefore \quad W_1 = V_L I_L \cos(30° - \Phi) \text{ and } W_2 = V_L I_L \cos(30° + \phi)$$

$$\therefore \quad W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L (\cos(30° + \phi)$$

$$= V_L I_L [\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$$

$$= V_L I_L (2\cos 30° \cos\phi) = \sqrt{3} V_L I_L \cos\phi \; = \text{total power in the 3-phase load}$$

Hence, the sum of the two wattmeter readings gives the total power consumption in the 3-phase load.

It should be noted that phase sequence of *RYB* has been assumed in the above discussion. Reversal of phase sequence will interchange the readings of the two wattmeters.

19.19. Variations in Wattmeter Readings

It has been shown above that for a lagging power factor

$$W_1 = V_L I_L \cos(30° - \varphi) \text{ and } W_2 = V_L I_L \cos(30° + \phi)$$

From this it is clear that individual readings of the wattmeters not only depend ***on the load but upon its power factor also.*** We will consider the following cases:

(*a*) When ϕ = 0 *i.e.* power factor is unity (*i.e.* resistive load) then,

$$W_1 = W_2 = V_L I_L \cos 30°$$

Both wattmeters indicate equal and positive *i.e.* up-scale readings.

(*b*) When ϕ = 60° *i.e.* power factor = 0.5 (lagging)

Then W_2 = $V_L I_L$ cos (30° + 60°) = 0. Hence, the power is measured by W_1 alone.

(*c*) When 90° > ϕ > 60° *i.e.* 0.5 > p.f. > 0, then W_1 is still positive but reading of W_2 is reversed because the phase angle between the current and voltage is more than 90°. For getting the total power, the reading of W_2 is to be subtracted from that of W_1.

ϕ	0°	60°	90°
$\cos\phi$	1	0.5	0
W_1	+ve	+ve	+ve
W_2	+ ve $W_1 = W_2$	0	– ve $W_1 = W_2$

Under this condition, W_2 will read 'down scale' *i.e.* backwards. Hence, to obtain a reading on W_2 it is necessary to reverse either its pressure coil or current coil, usually the

All readings taken after reversal of pressure coil are to be taken as negative.

(*d*) When $\phi = 90°$ (*i.e.* pure inductive or capacitive load), then

$$W_1 = V_L I_L \cos(30° - 90°) = V_L I_L \sin 30°;$$

$$W_2 = V_L I_L \cos(30° + 90°) = -V_L I_L \sin 30°$$

As seen, the two readings are equal but of opposite sign.

$$\therefore \qquad W_1 + W_2 = 0$$

The above facts have been summarised in the above table for a lagging power factor.

19.20. Leading Power Factor*

In the above discussion, lagging angles are taken positive. Now, we will see how wattmeter readings are changed if the power factor becomes leading. For $\phi = +60°$ (lag), W_2 is zero. But for $\phi = -60°$ (lead), W_1 is zero. So we find that for angles of lead, the reading of the two wattmeters are interchanged. Hence, for a ***leading*** power factor.

$$W_1 = V_L I_L \cos(30° + \phi) \text{ and } W_2 = V_L I_L \cos(30° - \phi)$$

19.21. Power Factor–*Balanced Load*

In case the load is balanced (and currents and voltages are sinusoidal) and for a *lagging* power factor:

$$W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi) = \sqrt{3} V_L I_L \cos\phi \qquad \text{... } (i)$$

Similarly $W_1 - W_2 = V_L I_L \cos(30° - \phi) - V_L I_L \cos(30° + \phi)$

$$= V_L I_L (2 \times \sin\phi \times 1/2) = V_L I_L \sin\phi \qquad \text{... } (ii)$$

Dividing **(*ii*)** by **(*i*)**, we have tan $\dfrac{\sqrt{3}(W_1 \quad W_2)}{(W_1 \quad W_2)}$ ** ... **(*iii*)**

Knowing tan ϕ and hence ϕ, the value of power factor cos ϕ can be found by consulting the trigonometrical tables. It should, however, be kept in mind that if W_2 reading has been taken after reversing the pressure coil *i.e.* if W_2 is negative, then the above relation becomes

$$\tan\phi = -\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \qquad \text{... Art 19.22}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - (-W_2)}{W_1 + (-W_2)} = \sqrt{3}\,\frac{W_1 + W_2}{W_1 - W_2}$$

Obviously, in this expression, only *numerical* values of W_1 and W_2 should be substituted. We may express power factor in terms of the ratio of the two wattmeters as under:

$$\text{Let} \quad \frac{\text{smaller reading}}{\text{larger reading}} = \frac{W_2}{W_1} = r$$

---

* For a leading p.f., conditions are just the opposite of this. In that case, W_1 reads negative (Art. 19.22).

** For a leading power factor, this expression becomes

$$\tan\phi = -\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \qquad \text{... Art 19.22}$$

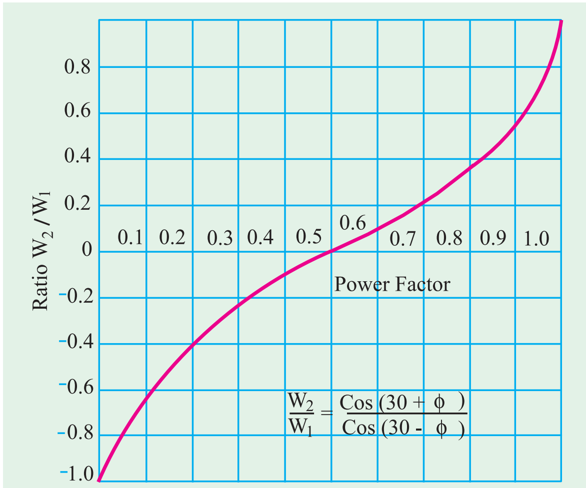


Fig. 19.49

Then from equation ***(iii)*** above,

$$\tan\phi = \frac{\sqrt{3}[1-(W_2/W_1)]}{1+(W_2/W_1)} = \frac{\sqrt{3}(1-r)}{1+r}$$

Now $\sec^2\phi = 1+\tan^2\phi$

or $\frac{1}{\cos^2\phi} = 1+\tan^2\phi$

$$\therefore \quad \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

$$= \frac{1}{\sqrt{1+3\left(\frac{1-r}{1+r}\right)^2}}$$

$$= \frac{1+r}{2\sqrt{1-r+r^2}}$$

If r is plotted against cos ϕ, then a curve called watt-ratio curve is obtained as shown in Fig. 19.49.

19.22. Balanced Load – *leading power factor*

In this case, as seen from Fig. 19.50

$$W_1 = V_L I_L \cos(30+\phi)$$

and $W_2 = V_L I_L \cos(30-\phi)$

$$\therefore \quad W_1 + W_2 = \sqrt{3}V_L I_L \cos\phi \text{ – as found above}$$

$$W_1 - W_2 = -V_L I_L \sin\phi$$

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(W_1 - W_2)}{(W_1+W_2)}$$

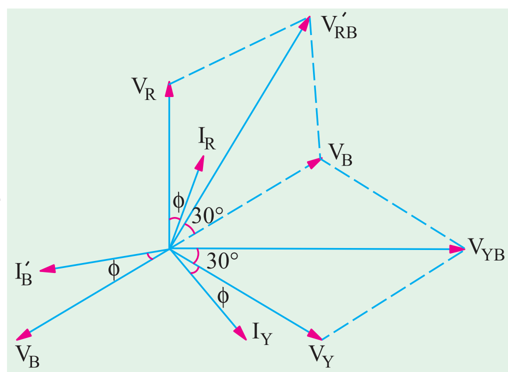


Fig. 19.50

Obviously, if ϕ > 60°, then phase angle between V_{RB} and I_R becomes more than 90°. Hence, W_1 reads 'down-scale' *i.e.* it indicates negative reading. However, W_2 gives positive reading even in the extreme case when ϕ = 90°.

19.23. Reactive Voltamperes with Two Wattmeters

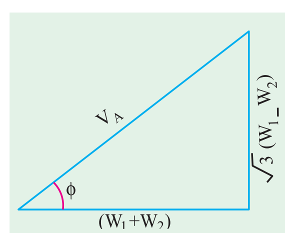


Fig. 19.51

We have seen that $\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1+W_2)}$

Since the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of the reactive power to the active power (in watts), it is clear that $\sqrt{3}\,(W_1 - W_2)$ represents the reactive power (Fig. 19.51). Hence, for a balanced load, the reactive power is given by $\sqrt{3}$ times the difference of the readings of the two wattmeters used to measure the power for a 3-phase circuit by the two wattmeter method. It may also be proved mathematically a follows:

$$= \sqrt{3}(W_1 - W_2) = \sqrt{3}[V_L I_L \cos(30° - \phi) - V_L I_L \cos(30° - \phi)]$$

$$= \sqrt{3} V_L I_L (\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\phi + \sin 30° \sin\phi)$$

$$= \sqrt{3} V_L I_L \sin\phi$$

19.24. Reactive Voltamperes with One Wattmeter

For this purpose, the wattmeter is connected as shown in Fig. 19.52 (*a*) and (*b*). The pressure coil is connected across *Y* and *B* lines whereas the current coil is included in the *R* line. In Fig. 19.48 (*a*), the current coil is connected between terminals *A* and *B* whereas pressure coil is connected between terminals *C* and *D*. Obviously, current flowing through the wattmeter is I_R and p.d. is V_{YB}. The angle between the two, as seen from vector diagram of Fig. 19.48, is (30 + 30 + 30 – φ) = (90 – φ)

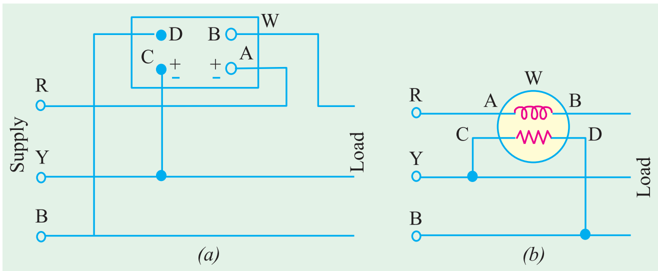


Fig. 19.52

Hence, reading of the wattmeter is $W = V_{YB} I_R \cos(90 - \phi) = V_{YB} I_R \sin\phi$

For a balanced load, V_{YB} equals the line voltage V_L and I_R equals the line current I_L, hence

$$W = V_L I_L \sin\phi$$

We know that the total reactive voltamperes of the load are $Q = \sqrt{3} V_L I_L \sin\phi$.

Hence, to obtain total *VARs*, the wattmeter reading must be multiplied by a factor of $\sqrt{3}$.

19.25. One Wattmeter Method

In this case, it is possible to apply two-wattmeter method by means of one wattmeter without breaking the circuit. The current coil is connected in any one line and the pressure coil is connected alternately between this and the other two lines (Fig. 19.53). The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method. It should be kept in mind that this method is not of as much universal application as the two wattmeter method because it is restricted to fairly balanced loads only. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load upon the motor.

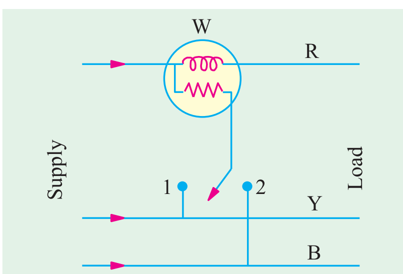


Fig. 19.53

It may be pointed out here that the two wattmeters used in the two-wattmeter method (Art. 19.17) are usually combined into a single instrument in the case of switchboard wattmeter which is then known as a polyphase wattmeter. The combination is affected by arranging the two sets of coils in such a way as to operate on a single moving system resulting in an indication of the total power on the scale.

19.26. Copper Required for Transmitting Power under Fixed Conditions

The comparison between 3-phase and single-phase systems will be done on the basis of a fixed amount of power transmitted to a fixed distance with the same amount of loss and at the same maximum voltage between conductors. In both cases, the weight of copper will be directly proportional to the number of wires (since the distance is fixed) and inversely proportional to the resistance of each wire. We will assume the same power factor and same voltage.

$$P_1 = VI_1 \cos\phi \text{ and } P_3 = \sqrt{3}VI_3 \cos\phi$$

where I_1 = r.m.s. value of current in 1-phase system
I_3 = r.m.s. value of line current in 3-phase system

$$P_1 = P_2 \quad \therefore \quad VI_1 \cos\phi = \sqrt{3}VI_3 \cos\phi \quad \therefore \quad I_1 = \sqrt{3}I_3$$

also $I_1^2 R_1 \times 2 = I_2^3 R_3 \times 3$ or $\dfrac{R_1}{R_3} = \dfrac{3I_3^2}{2I_1^2}$

Substituting the value of I_1, we get $\dfrac{R_1}{R_3} = \dfrac{3I_3^2}{3I_3^2 \times 2} = \dfrac{1}{2}$

$$\therefore \quad \frac{\text{copper } 3-\text{phase}}{\text{copper } 1-\text{phase}} = \frac{\text{No. of wires } 3-\text{phase}}{\text{No. of wires } 1-\text{phase}} \times \frac{R_1}{R_3} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

Hence, we find that for transmitting the same amount of power over a fixed distance with a fixed line loss, we need only three-fourths of the amount of copper that would be required for a single phase or to put it in another way, one-third more copper is required for a 1-phase system than would be necessary for a three-phase system.

Example 19.38. *Phase voltage and current of a star-connected inductive load is 150 V and 25 A. Power factor of load is 0.707 (lag). Assuming that the system is 3-wire and power is measured using two wattmeters, find the readings of wattmeters.*

(Elect. Instrument & Measurements, Nagpur Univ. 1993)

Solution. V_{ph} 150V; V_L 150 $\sqrt{3}$V; I_{ph} I_L = 25 A

Total power = $\sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707$ = 7954 W

$\therefore \quad W_1 + W_2$ = 7954 *W* ... **(*i*)**

$$\cos\phi = 0.707; \phi = \cos^{-1}(0.707) = 45°; \tan 45° = 1$$

Now, for a lagging power factor, $\tan\phi = \sqrt{3}\,(W_1 - W_2)/(W_1 + W_2)$ or $1 = \sqrt{3}(W_1 - W_2)/7954$

$\therefore \quad (W_1 - W_2)$ = 4592 *W* ... **(*ii*)**

From **(*i*)** and **(*ii*)** above, we get, W_1 = **6273 W; W_2 = 1681 W.**

Example 19.39. *In a balanced 3-phase 400-V circuit, the line current is 115.5 A. When power is measured by two wattmeter method, one meter reads 40 kW and the other zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?*

Solution. Since W_2 = 0, the whole power is measured by W_1. As per Art. 19.18, in such a situation, p.f. = 0.5. However, it can be calculated as under.

Since total power is 40 kW, $\therefore$ 40,000 = $\sqrt{3}$ 400 115.5 cos ; cos 0.5

If the power factor is unity with line currents remaining the same, we have

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = 0 \text{ or } W_1 = W_2$$

Also, $(W_1 + W_2) = \sqrt{3} \times 400 \times 115.5 \times 1$ = 80000 W = 80 kW

As per Art. 19.19, at unity p.f., $W_1 = W_2$. Hence, each wattmeter reads = 80/2 = **40 kW.**

Example 19.40. *The input power to a three-phase motor was measured by two wattmeter method. The readings were 10.4 KW and – 3.4 KW and the voltage was 400 V. Calculate **(a)** the power factor **(b)** the line current.* **(Elect. Engg. A.M.Ae, S.I. June 1991)**

Solution. As given in Art. 19.21, when W_2 reads negative, then we have

$\tan\phi = \sqrt{3}(W_1 + W_2)/(W_1 - W_2)$. Substituting numerical values of W_1 and W_2, we get

$\tan\phi = \sqrt{3}(10.4 + 3.4)/(10.4 - 3.4) = 1.97; \phi = \tan^{-1}(1.97) = 63.1°$

(*a*) p.f. = $\cos\phi = \cos 63.1°$ = **0.45 (lag)**

(*b*) W = 10.4 – 3.4 = 7 KW = 7,000 W

$7000 = \sqrt{3} I_L \times 400 \times 0.45; I_L$ = **22.4 A**

Example 19.41. *A three-phase, three-wire, 100-V, ABC system supplies a balanced delta connected load with impedance of* $20\angle 45°$ *ohm.*

(a) *Determine the phase and line currents and draw the phase or diagram* ***(b)*** *Find the wattmeter readings when the two wattmeter method is applied to the system.*

(Elect. Machines, A.M.I.E. Sec B.)

Solution. (*a*) The phasor diagram is shown in Fig. 19.54 (*b*).

Let $V_{AB} = 100\angle 0°$. Since phase sequence is *ABC*, $V_{BC} = 100\angle -120°$ and V_{CA} = 100° 120

Phase current $I_{AB} = \dfrac{V_{AB}}{Z_{AB}} = \dfrac{100\angle 0°}{20\angle 45°} = 5\angle -45°$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100\angle -120°}{20\angle 45°} = 5\angle -165°, I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100\angle 120°}{20\angle 45°} = 5\angle 75°$$

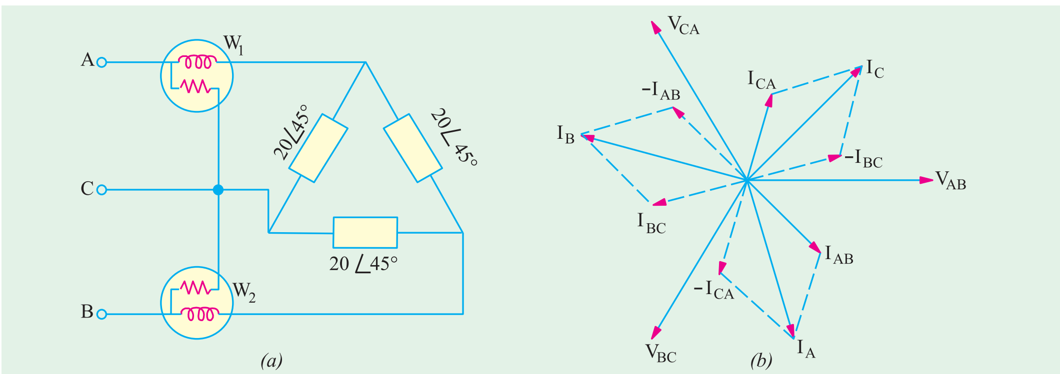


Fig. 19.54

Applying *KCL* to junction A, we have

$$I_A + I_{CA} - I_{AB} = 0 \text{ or } I_A = I_{AB} - I_{CA}$$

$\therefore$ Line current $I_A = 5\angle -45° – 5\angle 75° = 8.66\angle -75°$

Since the system is balanced, I_B will lag I_A by 120° and I_C will lag I_A by 240°.

$\because I_B = 8.66\angle(75° – 120°) = 8.66\angle -195°; I_C = 8.66\angle(–75° – 240°) = 8.66\angle -315° = 8.66\angle 45°$

(*b*) As shown in Fig. 19.54 (*b*), reading of wattmeter W_1 is $W_1 = V_{AC}\ I_C \cos\ \varphi$. Phasor V_{AC} is the reverse of phasor V_{CA}. Hence, V_{AC} is the reverse of phasor V_{CA}. Hence, V_{AC} lags the reference vector by 60° whereas I_A lags by 75°. Hence, phase difference between the two is (75° – 60°) = 15°

$\therefore$ W_1 = 100 × 8.66 × cos 15° = 836.5 W

Similarly $W_2 = V_{BC} I_B \cos\phi = 100 \times 8.66 \times \cos 75°$ = 224.1 W

$\therefore$ $W_1 + W_2$ = 836.5 + 224.1 = 1060.6 W

Resistance of each delta branch = 20 cos 45° = 14.14 Ω

Total power consumed = 3 I^2R = 3 × 5^2 × 14.14 = 1060.6 W

Hence, it proves that the sum of the two wattmeter readings gives the total power consumed.

Example 19.42. *A 3-phase, 500-V motor load has a power factor of 0.4 Two wattmeters connected to measure the power show the input to be 30 kW. Find the reading on each instrument.* **(Electrical Meas., Nagpur Univ. 1991)**

Solution. As seen from Art. 19.21

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \text{... } (i)$$

Now, $\cos\phi = 0.4; \phi = \cos^{-1}(0.4) = 66.6°;$ tan 66.6° = 2.311

$$W_1 + W_2 = 30 \quad \text{... } (ii)$$

Substituting these values in equation (*i*) above, we get

$$2.311 = \frac{\sqrt{3}(W_1 - W_2)}{30} \quad \therefore \quad W_1 - W_2 = 40 \quad \text{... } (iii)$$

From Eq. (*ii*) and (*iii*), we have W_1 = **45 kW** and W_2 = **– 5 kW**

Since W_2 comes out to be negative, second wattmeter reads 'down scale'. Even otherwise it is obvious that p.f. being less than 0.5, W_2 must be negative (Art. 19.19)

Example 19.43. *The power in a 3-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter? Give the connection diagram for the wattmeter circuit. For what p.f. will one of the wattmeter read zero?*

Solution. $\phi = \cos^{-1}(0.66) = 48.7°; \tan\phi = 1.1383$

Since p.f. is leading,

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \therefore \quad 1.1383 = \sqrt{3}(W_1 \quad W_2)/100$$

$\therefore$ $W_1 - W_2 = -65.7$ and $W_1 + W_2 = 100$ $\therefore$ W_1 = **17.14 kW;** W_2 = **82.85 kW**

Connection diagram is similar to that shown in Fig. 19.47 (*b*). One of the wattmeters will read zero when p.f. = **0.5**

Example 19.44. *Two wattmeters are used for measuring the power input and the power factor of an over-excited synchronous motor. If the readings of the meters are (– 2.0 kW) and (+ 7.0 kW) respectively, calculate the input and power factor of the motor.*

(Elect. Technology, Punjab Univ., June, 1991)

Solution. Since an over-excited synchronous motor runs with a leading p.f., we should use the relationship derived in Art. 19.22.

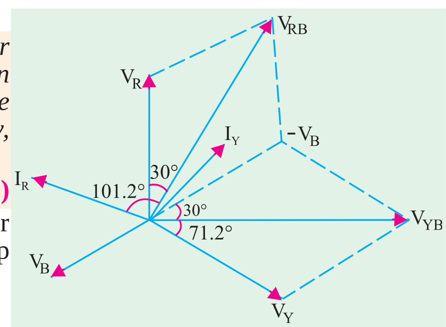


Fig. 19.55

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

Moreover, as explained in the same article, it is W_1 that gives negative reading and not W_2.

Hence, $$W_1 = -2\,\text{kW}$$

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(-2-7)}{-2+7} = \sqrt{3} \times \frac{9}{5} = 3.1176$$

$$\therefore \quad \phi = \tan^{-1}(3.1176) = 71.2° \text{ (lead)}$$

$$\therefore \quad \cos\phi = \cos 71.2° = \mathbf{0.3057\ (lead)} \text{ and}$$

$$\text{Input} = W_1 + W_2 = -2 + 7 = \mathbf{5\ kW}$$

Example 19.45. *A 440-V, 3-phase, delta-connected induction motor has an output of 14.92 kW at a p.f. of 0.82 and efficiency 85%. Calculate the readings on each of the two wattmeters connected to measure the input. Prove any formula used.*

If another star-connected load of 10 kW at 0.85 p.f. lagging is added in parallel to the motor, what will be the current draw from the line and the power taken from the line?

(Elect. Technology-I, Bombay Univ.)

Solution. Motor input = 14,920/0.85 = 17,600 W $\therefore$ $W_1 + W_2$ = 17.6 kW ... *(i)*

$\cos\phi = 0.82; \phi = 34.9°, \tan 34.9° = 0.6976;\ 0.6976 = \sqrt{3}\dfrac{W_1 - W_2}{17.6}$

$\therefore\ W_1 - W_2$ = 7.09 kW ... *(ii)*

From *(i)* and *(ii)* above, we get W_1 = **12.35 kW** and W_2 = **5.26 kW**

Motor **kVA,** $S_m = \dfrac{\text{motor kW}}{\cos\phi_m} = \dfrac{17.6}{0.82} = 21.46$ $\therefore$ $\mathbf{S_m}$ = 21.46 ∠– 34.9° = (17.6 – *j* 12.28) kVA

Load p.f. = 0.85 $\therefore$ $\phi = \cos^{-1}(0.85) = 31.8°$; Load kVA, S_Y = 10/0.85 = 11.76

$\therefore\ S_Y = 11.76\angle -31.8° = (10 - j6.2)$ kVA

Combined kVA, $\mathbf{S = S_m + S_Y}$ = (27.6 – j 18.48) = 32.2 ∠ – 33.8° kVA

$$I = \frac{S}{\sqrt{3}.V} = \frac{33.2\times 10^3}{\sqrt{3}\times 440} = \mathbf{43.56\ A}$$

Power taken = **27.6 kW**

Example 19.46. *The power input to a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the power factor of the motor be changed to 0.866 leading, determine the readings of the two wattmeters, the total input power remaining the same. Draw the vector diagram for the second condition of the load.* **(Elect. Technology, Nagpur Univ. 1992)**

Solution. In the first case both wattmeters read equal and positive. Hence motor must be running at unity power (Art. 19.22).

When p.f. is 0.866 leading

In this case; $W_1 = V_L I_L \cos(30°+\phi)$;

$W_2 = V_L I_L \cos(30°-\phi)$

$\therefore\ W_1 + W_2 = \sqrt{3}V_L I_L \cos\phi$

$W_1 - W_2 = -V_L I_L \sin\phi$

$\therefore\ \tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$

$\phi = \cos^{-1}(0.866) = 30°$

$\tan\phi = 1/\sqrt{3}$

$\therefore\ \dfrac{1}{\sqrt{3}} = \dfrac{-\sqrt{3}(W_1 - W_2)}{100}$

$\therefore\ W_1 - W_2 = -100/3$

and $W_1 + W_2 = 100$

$\therefore\ 2W_1$ = 200/3; W_1 = **33.33 kW;** $\mathbf{W_2}$ **= 66.67 kW**

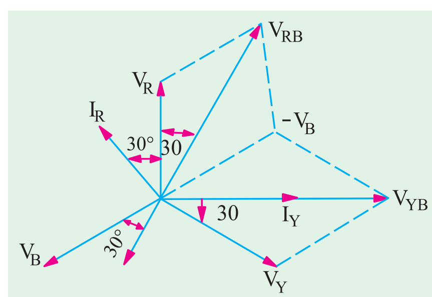


Fig. 19.56

For connection diagram, please refer to Fig. 19.47. The vector or phasor diagram is shown in Fig. 19.56.

Example 19.47 (*a*). *A star-connected balanced load is supplied from a* $3-\phi$ *balanced supply with a line voltage of 416 volts at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the reading on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate*

(i) *power factor of circuit,* ***(ii)*** *the line current,* ***(iii)*** *the capacitance of each capacitor.*

(Elect. Engg. I, Nagpur Univ. 1993)

Solution. ***(i)*** As seen from Art. 19.21 $\tan\phi = -\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = -\dfrac{\sqrt{3}(782 - 1980)}{(782 + 1980)}$ = 0.75; $\phi = 36.9°$, $\cos\phi = 0.8$

(ii) $\sqrt{3} \times 416 \times I_L \times 0.8 = 2762, I_L = 4.8\,\text{A}$

(iii) $Z_{ph} = V_{ph} / I_{ph} = (416\sqrt{3}) / 4.8 = 50\Omega, X_C = Z_{ph} \sin\phi = 50 \times 0.6 = 30\,\Omega$

Now, $X_C = 1/2\pi fC = 1/2\phi \times 50 \times C = 106 \times 10^{-6}\,\text{F}$

Example 19.48. *Each phase of a 3-phase, Δ-connected load consists of an impedance Z = 20 ∠60° ohm. The line voltage is 440 V at 50 Hz. Compute the power consumed by each phase impedance and the total power. What will be the readings of the two wattmeters connected?* **(Elect. and Mech. Technology, Osmania Univ.)**

Solution. $Z_{ph} = 20\Omega; V_{ph} = V_L = 440\text{V}; I_{ph} = V_{ph} / Z_{ph}$ = 440/20 = 22 *A*

Since $\phi = 60°; \cos\phi = \cos 60° = 0.5°;\ R_{ph} = Z_{ph} \times \cos 60° = 20 \times 0.5 = 10\,\Omega$

∴ Power/phase = $I_{ph}^2 R_{ph} = 22^2 \times 10$ = **4,840 W**

Total power = $3 \times 4{,}840$ = **14,520 W** [or *P* = $\sqrt{3} \times 440 \times (\sqrt{3} \times 22) \times 0.5$ = 14,520 W]

Now, $W_1 + W_2 = 14{,}520.$

Also $\tan\phi = \sqrt{3}.\dfrac{W_1 - W_2}{W_1 + W_2}$ ∴ $\tan 60° = \sqrt{3} = \sqrt{3}.\dfrac{W_1 - W_2}{14{,}520}$

∴ $W_1 - W_2$ = 14,520. Obviously, W_2 = 0

Even otherwise it is obvious that W_2 should be zero because p.f. = cos 60° = 0.5 (Art. 19.19).

Example 19.49. *Three identical coils, each having a reactance of 20 Ω and resistance of 20 Ω are connected in **(a)** star **(b)** delta across a 440-V, 3-phase line. Calculate for each method of connection the line current and readings on each of the two wattmeters connected to measure the power.* **(Electro-mechanics, Allahabad Univ. 1992)**

Solution. (*a*) Star Connection

$$Z_{ph} = \sqrt{20^2 + 20^2} = 20\sqrt{2} = 28.3\Omega; V_{ph} = 440 / \sqrt{3} = 254\,\text{V}$$

$I_{ph} = 254 / 28.3 = 8.97 A; I_L$ = **8.97 A;** cos φ = R_{ph} / Z_{ph} = 20/28.3 = 0.707

Total power taken = $\sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 440 \times 8.97 \times 0.707$ = 4830 W

If W_1 and W_2 are wattmeter readings, then $W_1 + W_2$ = 4830 W ... ***(i)***

Now, $\tan\phi = 20/20 = \sqrt{3}(W_1 - W_2)/(W_1 + W_2);\ (W_1 - W_2); = 2790\,\text{W}$ ... ***(ii)***

From ***(i)*** and ***(ii)*** above, W_1 = **3810 W;** W_2 = **1020 W**

(*b*) Delta Connection

$$Z_{ph} = 28.3\Omega, V_{ph} = 440\,V, I_{ph} = 440 / 28.3 = 15.5 A; I_L = 15.5 \times \sqrt{3} = 28.8\,\text{A}$$

$P = \sqrt{3} \times 440 \times 28.8 \times 0.707$ = 14,490 W (it is 3 times the *Y*-power)

∴ $W_1 + W_2$ = 14,490 W ... ***(iii)***

$\tan\phi = 20/20 = \sqrt{3}(W_1 - W_2)/14{,}490; W_1 - W_2$ = 8370 ... ***(iv)***

From Eq. ***(iii)*** and ***(iv)***, we get, W_1 = **11,430 W;** W_2 = **3060 W**

Note: These readings are 3-times the Y-readings.

Example 19.50. *Three identical coils are connected in star to a 200-V, three-phase supply and each takes 500 W. The power factor is 0.8 lagging. What will be the current and the total power if the same coils are connected in delta to the same supply? If the power is measured by two wattmeters, what will be their readings? Prove any formula used.* **(Elect. Engg. A.M. A. S.I. Dec. 1991)**

Solution. When connected in star as shown in Fig. 19.57 (*a*), $V_{ph} = 200/\sqrt{3}$ = 115.5 V

Now, $V_{ph} I_{ph} \cos\phi$ = power per phase or 115.5 × I_{ph} × 0.8 = 500

$\therefore \; I_{ph} = 5.41 A; Z_{ph} = V_{ph}/I_{ph} = 115.5/5.41 = 21.34\,\Omega$

$R = Z_{ph} \cos\phi = 21.34 \times 0.8 = 17\Omega;\; X_L = Z_{ph} \sin\phi = 21.34 \times 0.6 = 12.8\,\Omega$

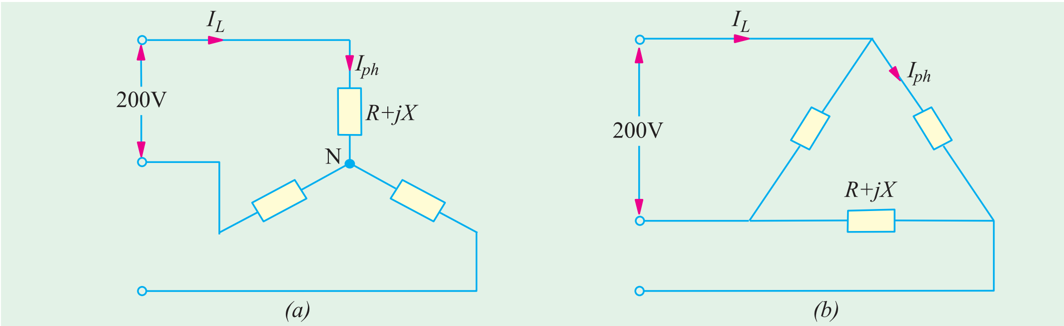


Fig. 19.57

The same three coils have been connected in delta in Fig. 19.57 (*b*). Here, $V_{ph} = V_L = 200\,\text{V}$.

$$I_{ph} = 200/21.34 = 9.37 A; I_L = \sqrt{3} I_{ph} = 9.37 \times 1.732 = \mathbf{16.23\ A}$$

Total power consumed = $\sqrt{3} \times 200 \times 16.23 \times 0.8$ = **4500 W**

It would be seen that when the same coils are connected in delta, they consume three times more power than when connected in star.

Wattmeter Readings

Now, $$W_1 + W_2 = 4500; \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \cos^{-1}(0.8) = 36.87°; \tan\phi = 0.75$$

$$0.75 = \frac{\sqrt{3}(W_1 - W_2)}{4500} \quad \therefore \; (W_1 - W_2) = 1950\,\text{W}$$

$\therefore$ $$W_1 = (4500 + 1950)/2 = 3225\,\text{W};\; \mathbf{W_2 = 1275\ W.}$$

Example 19.51. *A 3-phase, 3-wire, 415-V system supplies a balanced load of 20 A at a power factor 0.8 lag. The current coil of wattmeter I is in phase R and of wattmeter 2 in phase B. Calculate* ***(i)*** *the reading on 1 when its voltage coil is across R and Y* ***(ii)*** *the reading on 2 when its voltage coil is across B and Y and* ***(iii)*** *the reading on 1 when its voltage coil is across Y and B. Justify your answer with relevant phasor diagram.* **(Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. *(i)* As seen from phasor diagram of Fig. 19.57 (*a*)

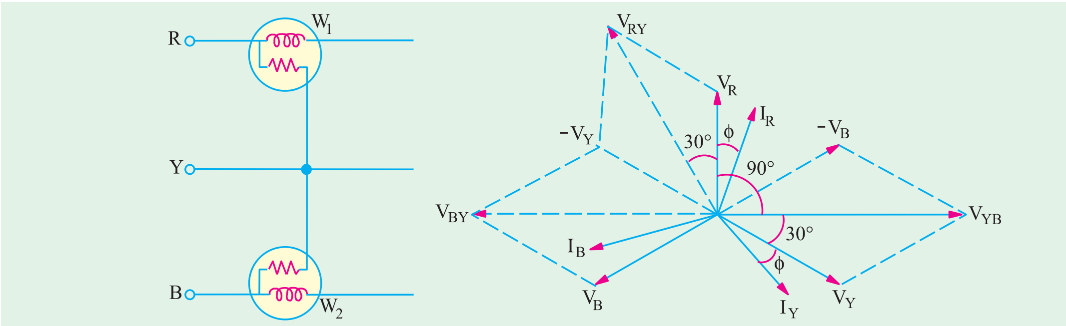


Fig. 19.57 (*a*)

$$W_1 = V_{RY} I_A \cos(30 + \phi) = \sqrt{3} \times 415 \times 20 \times \cos(36.87° + 30°) = \mathbf{5647\ W}$$

(ii) Similarly, $W_2 = V_{BY} I_B \cos(30 - \phi)$

It should be noted that voltage across W_2 is V_{BY} and not V_{YB}. Moreover, $\phi = \cos^{-1}(0.8) = 36.87°$,

$$\therefore \quad W_2 = \sqrt{3} \times 415 \times 20 \times \cos(30° - 36.87°) = \mathbf{14{,}275\ W}$$

(iii) Now, phase angle between I_R and V_{YB} is $(90° - \phi)$

$$\therefore \quad W_2 = V_{YB} I_R \cos(90° - \phi) = \sqrt{3} \times 415 \times 20 \times \sin 36.87° = 8626 \text{ VAR}$$

Example 19.52. *A wattmeter reads 5.54 kW when its current coil is connected in R phase and its voltage coil is connected between the neutral and the R phase of a symmetrical 3-phase system supplying a balanced load of 30 A at 400 V. What will be the reading on the instrument if the connections to the current coil remain unchanged and the voltage coil be connected between B and Y phases? Take phase sequence RYB. Draw the corresponding phasor diagram.* **(Elect. Machines, A.M.I.E., Sec. B, 1992)**

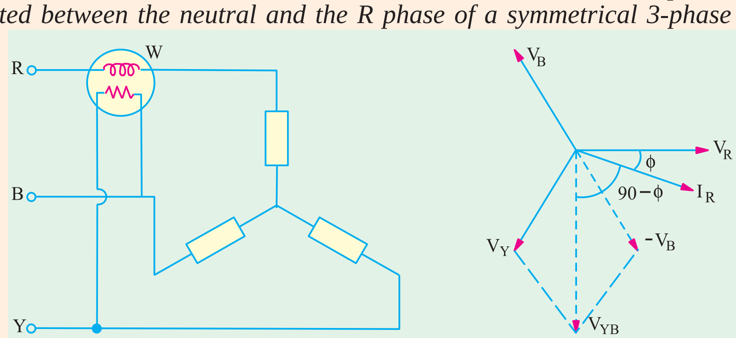


Fig. 19.57 (*b*)

Solution. As seen from Fig. 19.57 (*b*).

$$W_1 = V_R I_R \cos\phi \text{ or } 5.54 \times 10^3 = (400/\sqrt{3}) \times 30 \times \cos\phi; \quad \therefore \cos\phi = 0.8, \sin\phi = 0.6$$

In the second case (Fig. 19.57 (*b*))

$$W_2 = V_{YB} I_R \cos(90° - \phi) = 400 \times 30 \times \sin\phi = 400 \times 30 \times 0.6 = \mathbf{7.2\ kW}$$

Example 19.53. *A 3-phase, 3-wire balanced load with a lagging power factor is supplied at 400 V (between lines). A I-phase wattmeter (scaled in kW) when connected with its current coil in the R-line and voltage coil between R and Y lines gives a reading of 6 kW. When the same terminals of the voltage coil are switched over to Y- and B-lines, the current-coil connections remaining the same, the reading of the wattmeter remains unchanged. Calculate the line current and power factor of the load. Phase sequence is* $R \to Y \to B$. **(Elect. Engg-1, Bombay Univ. 1985)**

Solution. The current through the wattmeter is I_R and p.d. across its pressure coil is V_{RY}. As seen from the phasor diagram of Fig. 19.58, the angle between the two is $(30° + \phi)$.

$$\therefore \quad W_1 = V_{RY} I_R \cos(30° + \phi) = V_L I_L \cos(30° + \phi) \quad \text{... } (i)$$

In the second case, current is I_R but voltage is V_{YB}. The angle between the two is $(90° - \phi)$

$$\therefore \quad W_2 = V_{YB} I_R \cos(90° - \phi) = V_L I_L \cos(90° - \phi)$$

Since $W_1 = W_2$ we have

$$V_L I_L \cos(30° + \phi) = V_L I_L \cos(90° - \phi)$$

$$\therefore \quad 30° + \phi = 90° - \phi$$

or $2\phi = 60°$ $\therefore$ $\phi = 30°$

$\therefore$ load power factor = cos 30° = **0.866 (lag)**

Now $W_1 = W_2 = 6$ kW.

Hence, from (*i*) above, we get

$$6000 = 400 \times I_L \cos 60°; I_L = \mathbf{30\ A}$$

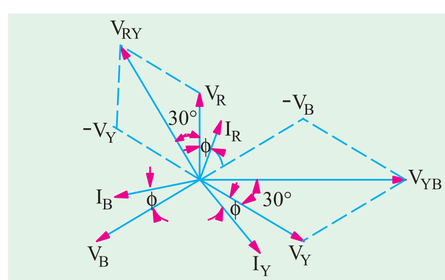


Fig. 19.58

Example 19.54. *A 3-phase, 400 V circuit supplies a Δ-connected load having phase impedances of* $Z_{AB} = 25\angle 0°; Z_{BC} = 25\angle 30°$ and $V_{CA} = 25\angle -30°$.

*Two wattmeters are connected in the circuit to measure the load power. Determine the wattmeter readings if their current coils are in the lines **(a)** A and B ; **(b)** B and C; and **(c)** C and A. The phase sequence is ABC. Draw the connections of the wattmeter for the above three cases and check the sum of the two wattmeter readings against total power consumed.*

Solution. Taking V_{AB} as the reference voltage, we have $Z_{AB} = 400\angle 0°; Z_{BC} = 400\angle -120°$ and $Z_{CA} = 400\angle 120°$.

The three phase currents can be found as follows:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{400\angle 0°}{25\angle 0°} = 16\angle 0° = (16 + j0)$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{400\angle -120°}{25\angle 30°} = .16\angle -150° = (-13.8 - j8)$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -120°}{25\angle 30°} = 16\angle -150° = (-13.8 - j8)$$

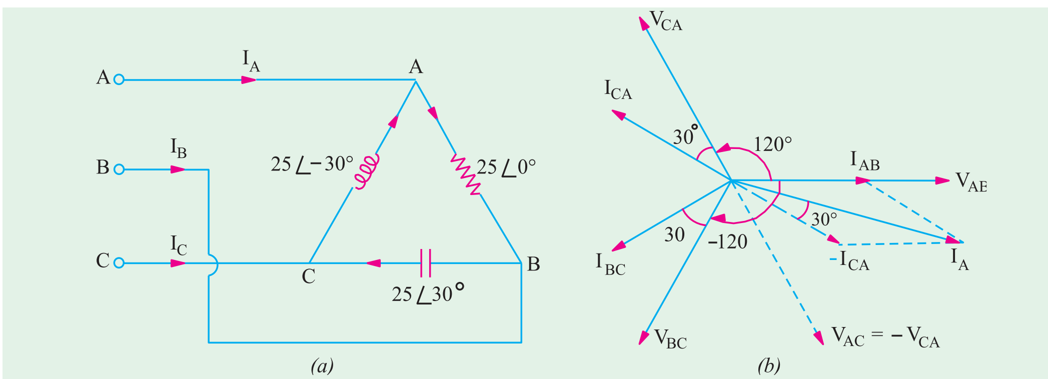


Fig. 19.59

The line currents I_A, I_B and I_C can be found by applying KCL at the three nodes *A*, *B* and *C* of the load.

$$I_A = I_{AB} + I_{AC} = I_{AB} - I_{CA} = (16 + j0) - (-13.8 + j8) = 29.8 - j8 = 30.8\angle -15°$$

$$I_B = I_{BC} - I_{AB} = (-13.8 - j8) - (16 + j0) = -29.8 - j8 = 30.8\angle -165°$$

$$I_C = I_{CA} - I_{BC} = (-13.8 + j8) - (-13.8 - j8) = 0 + j16 = 16\angle 90°$$

The phasor diagram for line and phase currents is shown in Fig. 19.59 (*a*) and (*b*).

(*a*) As shown in Fig. 19.60 (*a*), the current coils of the wattmeters are in the line *A* and *B* and the voltage coil of W_1 is across the lines *A* and *C* and that of W_2 is across the lines *B* and *C*. Hence, current through W_1 is I_A and voltage across it is V_{AC}. The power indicated by W_1 may be found in the following two ways:

(*i*) $P_1 = |V_{AC}|.|I_A| \times$ (cosine of the angle between V_{AC} and I_A).

= 400 × 30.8 × cos (30° + 15°) = 8710 W

(*ii*) We may use current conjugate (Art.) for finding the power

$$P_{VA} = V_{AC}.I_A = -400\angle 120° \times 30.8\angle 15°$$

∴ P_1 = real part of P_{VA} = – 400 × 30.8 × cos 135° = 8710 W

P_2 = real part of $[V_{BC} Z_B] = 400\angle 120° \times 30.8\angle -165°$

$= 400 \times 30.8 \times \cos(-45°) = 8710$ W

∴ $P_1 + P_2 = 8710 + 8710 = 17{,}420$ W.

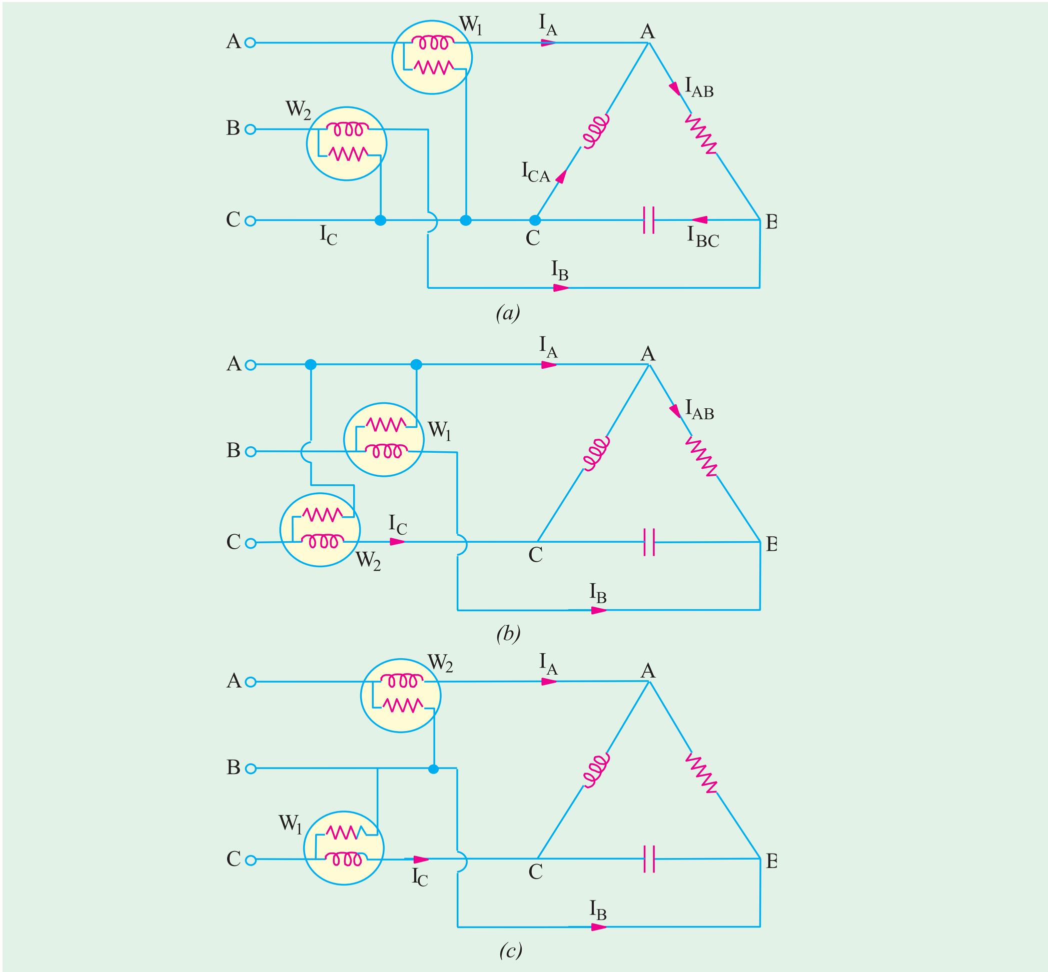


Fig. 19.60

(*b*) As shown in Fig. 19.60 (*b*), the current coils of the wattmeters are in the lines *B* and *C* whereas voltage coil of W_1 is across the lines *B* and *A* and that of W_2 is across lines *C* and *A*.

(*i*) $\therefore$ $P_1 = |V_{BA}|.|I_B|$ (cosine of the angle between V_{BA} and I_B)

$= 400 \times 30.8 \times \cos 15° = 11{,}900$ W

(*ii*) Using voltage conjugate (which is more convenient in this case), we have

$$P_{VA} = V_{BA}^{*}.I_B = -400\angle 0° \times 30.8 \angle \times -165°$$

$\therefore$ P_1 = real part of P_{VA} = – 400 × 30.8 × cos (– 165°) = 11,900 W

P_2 = real part of $[V_{CA}{*}I_C]$ = $[400\ \angle -120° \times 16\angle 90°] = 400 \times 16 \times \cos(-30°)$ = 5,540 W.

$\therefore$ $P_1 + P_2$ = 11,900 + 5,540 = 17,440 W.

(*c*) As shown in Fig. 19.60 (*c*), the current coils of the wattmeters are in the lines *C* and *A* whereas the voltage coil of W_1 is across the lines *C* and *B* and that of W_2 is across the lines *A* and *B*.

(*i*) P_1 = real part of $[V_{CB} * I_C]$ $[(\ 400\ \ 120\ )\ \ 16\ \ 90\ ]$

$=\ \ 400\ \ 16\ \ \cos\ \ 210\ \ \ 5540\,\text{W}$

P_2 = real part of $[V_{AB} * I_A] = [400\angle °0 \times 30.8\angle -15° = 400 \times 30.8 \times \cos\angle -15°$ = 11,900 W

$\therefore$ $P_1 + P_2$ = 5,540 + 11,900 = 17,440 W

Total power consumed by the phase load can be found directly as under :-

P_T = real part of $[V_{AB}I_{AB}* + V_{BC}I_{BC}* + V_{CA}I_{CA}*]$
= real part of

$$[(400\angle 0°)(16\angle -0°) + (400\angle -120°)(16\angle 150°) + (400\angle 120°)(16\angle -150°)]$$

= 400 × 16 × real part of $(1\angle 0° + 1\angle 30° + 1\angle -30°)$
= 400 × 16 (cos 0° + cos° + cos (– 30°) = 17,485 W

Note. The slight variation in the different answers is due to the approximation made.

Example 19.55. *In a balanced 3-phase system load I draws 60 kW and 80 leading kVAR whereas load 2 draws 160 kW and 120 lagging kVAR. If line voltage of the supply is 1000 V, find the line current supplied by the generator. (Fig. 19.61)*

Solution. For load 1 which is a leading load, $\tan\phi_1 = Q_1 / P_1$ = 80/60 = – 1.333; ϕ_1 = 53.1°, cos ϕ_1 = 0.6. Hence, line current of this load is

$$I_1 = 60{,}000/\sqrt{3}\times 1000\times 0.6 = 57.8 \text{ A}$$

For load 2, tan ϕ_2 = 120/160 = 0.75; ϕ_2 = 26.9°, cos ϕ_2 = 0.8. The line current drawn by this load is

$$I_2 = 160{,}000/\sqrt{3}\times 1000\times 0.8 = 115.5 \text{ A}$$

If we take the phase voltage as the reference voltage *i.e.* $V_{ph} = (1000/\sqrt{3})\angle 0° = 578\angle 0°$; then I_1 leads this voltage by 53.1° whereas I_2 lags it by 36.9°. Hence, $I_1 = 57.8\angle 53.1°$ and I_2 = 115.5 36.9

$$\therefore\ I_{L1} = I_1 + I_2 = 57.8\ \angle 53.1° + 115.5$$
$$\angle -36.9° = 171.7\angle 42.3° \text{ A}.$$

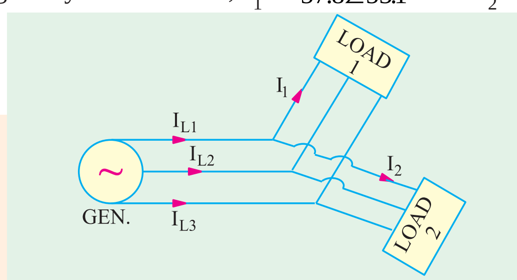


Fig. 19.61

Example 19.56. *A single-phase motor drawing 10A at 0.707 lagging power factor is connected across lines R and Y of a 3-phase supply line connected to a 3-phase motor drawing 15A at a lagging power factor of 0.8 as shown in Fig. 19.62(a). Assuming RYB sequence, calculate the three line currents.*

Solution. In the phasor diagram of Fig. 19.61 (*b*) are shown the three phase voltages and the one line voltage V_{RY} which is ahead of its phase voltage V_R. The current I_1 drawn by single-phase motor lags V_{RY} by $\cos^{-1}$ 0.707 or 45°. It lags behind the reference voltage V_R by 15° as shown. Hence, $I_1 = 10\angle -15° = 9.6 - j2.6$ A. The 3-phase motor currents lag behind their respective phase voltages by $\cos^{-1}$ 0.8 or 36.9°. Hence, $I_{R1} = 15\angle -36.9° = 12 - j9$.

$$I_{Y1} = 1.5\angle(-120° - 36.9°) = 15\angle -156.9° = -13.8 - j5.9$$
$$I_B = 15\angle(120° - 36.9°) = 15\angle 83.1°$$

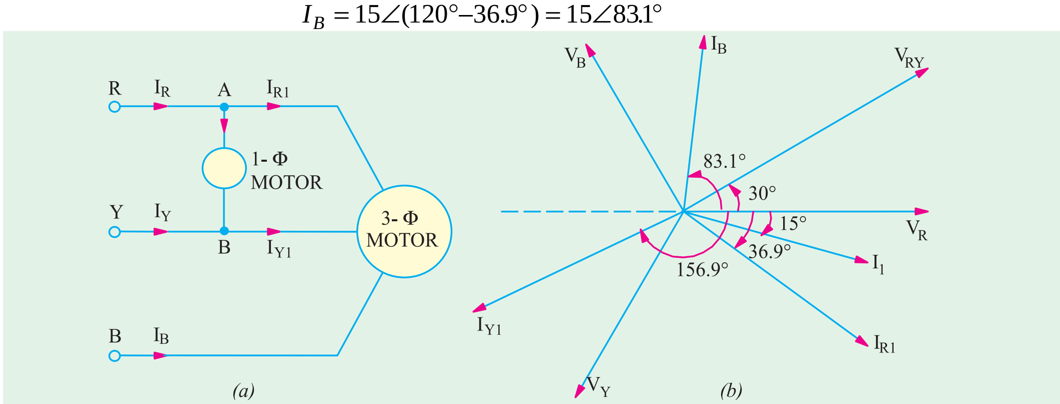


Fig. 19.62

Applying Kirchhoff's laws to point A of Fig. 19.62 (a), we get

$$I_R = I_1 + I_{R1} = 9.6 - j2.6 + 12 - j9 = 21.6 - j11.6 = 24.5\angle -28.2°$$

Similarly, applying KCL to point B, we get

$$I_Y + I_1 = I_{Y1} \text{ or } I_Y = I_{Y1} - I_1 = -13.8 - j5.9 - 9.6 + j2.6 = -23.4 - j3.3 = 23.6\angle -172°.$$

Example 19.57. *A 3-ϕ, 434-V, 50-Hz, supply is connected to a 3-ϕ, Y-connected induction motor and synchronous motor. Impedance of each phase of induction motor is (1.25 + j2.17) Ω The 3-ϕ synchronous motor is over-excited and it draws a current of 120 A at 0.87 leading p.f. Two wattmeters are connected in usual manner to measure power drawn by the two motors. Calculate* ***(i)*** *reading on each wattmeter* ***(ii)*** *combined power factor.*

(Elect. Technology, Hyderabad Univ. 1992)

Solution. It will be assumed that the synchronous motor is *Y*-connected. Since it is over-excited it has a leading p.f. The wattmeter connections and phasor diagrams are as shown in Fig. 19.63.

$$\mathbf{Z_1} = 1.25 + j2.17 = 2.5\angle 60°$$

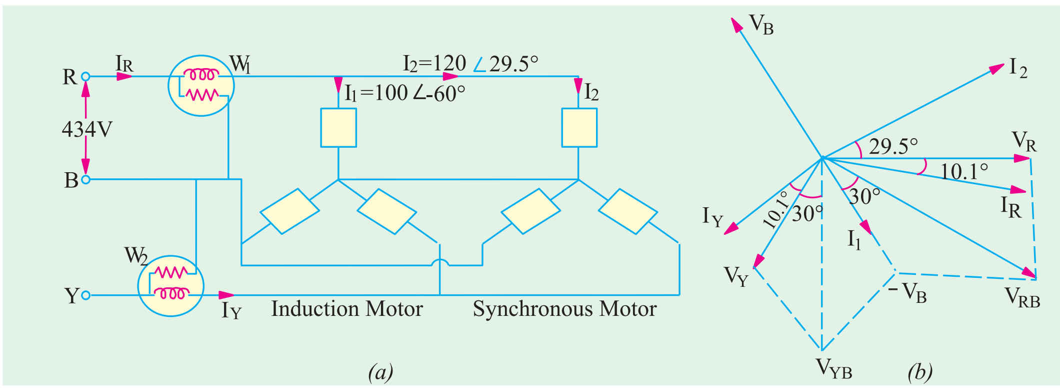


Fig. 19.63

Phase voltage in each case = $434/\sqrt{3}$ = 250 V

I_1 = 250/2.5 = 100 A lagging the reference vector V_R by 60°. Current I_2 = 120 A and leads V_R by an angle = $\cos^{-1}$ (0.87) = 29.5°

∴ $\mathbf{I_1}$ = 100 60 50 j86.6; $\mathbf{I_2}$ 120 29.5 104.6 j59

$\mathbf{I_R = I_1 + I_2}$ = 154.6 – j27.6 = $156.8\angle -10.1°$

(*a*) As shown in Fig. 19.63 (*b*), I_R lags V_R by 10.1°. Similarly, I_Y lags V_Y by 10.1°.

As seen from Fig. 19.63 (*a*), current through W_1 is I_R and voltage across it is $V_{RB} = V_R - V_B$. As seen, V_{RB} = 434 V lagging by 30°. Phase difference between V_{RB} and I_R is = 30 – 10.1 = 19.9°.

∴ reading of W_1 = 434 × 156.8 × cos 19.9° = **63,970 W**

Current I_Y is also (like I_R) the vector sum of the line currents drawn by the two motors. It is equal to 156.8 A and lags behind its respective phase voltage V_Y by 10.1°. Current through W_2 is I_Y and voltage across it is $\mathbf{V_{YB} = V_Y - V_B}$. As seen, V_{YB} = 434 V. Phase difference between V_{YB} and I_Y = 30° + 10.1° = 40.1° (lag).

∴ reading of W_2 = 434 × 156.8 × cos 40.1° = **52,050 W**

(*b*) Combined p.f. = cos 10.1° = **0.9845 (lag)**

Example 19.58. *Power in a balanced 3-phase system is measured by the two-wattmeter method and it is found that the ratio of the two readings is 2 to 1. What is the power factor of the system?* **(Elect. Science-1, Allahabad Univ. 1991)**

Solution. We are given that $W_1 : W_2$ = 2 : 1. Hence, W_1/W_2 = r = 1/2 = 0.5. As seen from Art. 19.21.

$$\cos\phi = \frac{1+r}{2\sqrt{1-r+r^2}} = \frac{1+0.5}{2\sqrt{1-0.5+0.5^2}} = \mathbf{0.866\ lag}$$

Example 19.59. *A synchronous motor absorbing 50 kW is connected in parallel with a factory load of 200 kW having a lagging power factor of 0.8. If the combination has a power factor of 0.9 lagging, find the kVAR supplied by the motor and its power factor.*

(Elect. Machines, A.M.I.E. Sec B)

Solution. Load kVA = 200/0.8 = 250

Load kVAR = 250 × 0.6 = 150 (lag) [cos ϕ = 0.8 sin ϕ = 0.6]

Total combined load = 50 + 200 = 250 kW

kVA of combined load = 250/0.9 = 277.8

Combined kVAR = 277.8 × 0.4356 = 121 (inductive) (combined cos ϕ = 0.9, sin ϕ = 0.4356)

Hence, leading kVAR supplied by synch, motor = 150 – 121 = 29 (capacitive)

kVA of motor alone = $\sqrt{(kW^2 + kVAR^2)} = \sqrt{50^2 + 29^2}$ = 57.8

p.f. of motor = kW/kVA = 50/57.8 = 0.865 (leading)

Example 19.60. *A star-connected balanced load is supplied from a 3-phase balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of load consists of a resistance and a capacitor joined in series and the readings on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate* **(a)** *the power factor of the circuit* **(b)** *the line current and* **(c)** *the capacitance of each capacitor.*

(Elect. Machinery-I, Bombay Univ.)

Solution. $W_1 = 728 \text{ and } W_2 = 1980$

For a leading p.f. $\tan\phi = -\sqrt{3}\dfrac{W_1 - W_2}{W_1 + W_2}$ $\therefore$ $\tan\phi = -\sqrt{3} \times \dfrac{(782 - 1980)}{782 + 1980}$ = 0.75

From tables, $\phi = 36°54'$

(*a*) $\therefore$ $\cos\phi = \cos 36°54'$ = **0.8 (leading)**

(*b*) power = $\sqrt{3} V_L I_L \cos\phi$ or $W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$

or $(782 + 1980) = \sqrt{3} \times 416 \times I_L \times 0.8$ $\therefore$ $I_L = I_{ph}$ = **4.8 A**

(*c*) Now $V_{ph} = 416/\sqrt{3}$ V $\therefore$ $Z_{ph} = 416/\sqrt{3} \times 4.8 = 50\,\Omega$

$\therefore$ In Fig. 19.64, $Z_{ph} = 50\angle -36°54' = 50(0.8 - j0.6) = 40 - j30$

Capacitive reactance X_C = 30; or $\dfrac{1}{2\pi \times 50 \times C} = 30$ $\therefore$ $\boldsymbol{C = 106}$ μF **.**

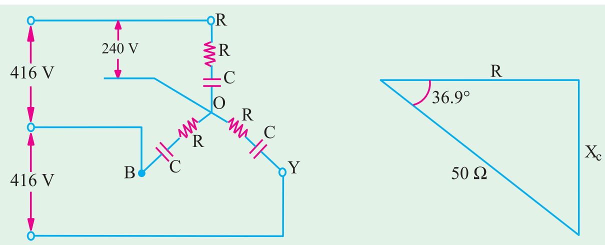


Fig. 19.64

Example 19.61. *The two wattmeters A and B, give readings as 5000 W and 1000 W respectively during the power measurement of 3-ϕ, 3-wire, balanced load system.* **(a)** *Calculate the power and power factor if* **(i)** *both meters read direct and* **(ii)** *one of them reads in reverse.* **(b)** *If the voltage of the circuit is 400 V, what is the value of capacitance which must be introduced in each phase to cause the whole of the power to appear on A. The frequency of supply is 50 Hz.*

(Elect. Engg-I, Nagpur Univ. 1992)

Solution. (*a*) (*i*) Both Meters Read Direct

$$W_1 = 5000\text{ W}; W_2 = 1000\text{ W}; \;\therefore\; W_1 + W_2 = 6000\text{ W}; W_1 - W_2 = 4000\text{ W}$$

$$\tan\phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2) = \sqrt{3}\times 4000/6000 = 1.1547$$

$\therefore$ $\phi = \tan^{-1}(1.1547) = 49.1°$; p.f. = cos 49.1° = 0.655 (lag)

Total power = 5000 + 1000 = 6000 W

(*ii*) One Meter Reads in Reverse

In this case, tan $\phi = \sqrt{3}(W_1 + W_2)/(W_1 - W_2) = \sqrt{3}\times 6000/4000 = 2.598$

$\therefore$ $\phi = \tan^{-1}(2.598) = 68.95°$; p.f. = cos 68.95° = 0.36 (lag)

Total power = W_1 + W_2 = 5000 – 1000 = 4000 W ...Art.

(*b*) The whole of power would be measured by wattmeter W_1 if the load power factor is 0.5 (lagging) or less. It means that in the present case p.f. of the load will have to be reduced from 0.655 to 0.5 In other words, capacitive reactance will have to be introduced in each phase of the load in order to partially neutralize the inductive-reactance.

Now, $\sqrt{3}V_L I_L \cos\phi = 6000$ or $\sqrt{3}\times 400\, I_L \times 0.655 = 6000$

$\therefore$ I_L = 13.2A; $\therefore$ $I_{ph} = 13.2/\sqrt{3}$ = 7.63 A

$$Z_{ph} = V_{ph}/I_{ph} = 400/7.63 = 52.4\,\Omega$$

$$X_L = Z_{ph}\sin\phi = 52.4\times\sin 49.1° = 39.6\,\Omega$$

When p.f. = 0.5

$\sqrt{3}\times 400\times I_L\times 0.5 = 6000; I_L = 17.32A; I_{ph} = 17.32/\sqrt{3} = 10A; Z_{ph} = 400/10 = 40\Omega$

$\cos\phi = 0.5; \phi = 60; \sin 60° = 0.886; X = Z_{ph}\sin\phi = 40\times 0.886 = 35.4\,\Omega$

$\therefore$ $X = X_L - X_C = 35.4$ or $39.6 - X_C = 35.4$; $\therefore$ $X_C = 4.2\,\Omega$.

If C is the required capacitance, then 4.2 = $1/2\pi\times 50\times C$; $\therefore$ $C = 758\,\mu F$.

Tutorial Problems No. 19.2

1. Two wattmeters connected to measure the input to a balanced three-phase circuit indicate 2500 W and 500 W respectively. Find the power factor of the circuit (*a*) when both readings are positive and (*b*) when the latter reading is obtained after reversing the connections to the current coil of one instrument.

[(*a*) 0.655 (*b*) 0.3591] (*City & Guilds, London*)

2. A 400-V, 3-phase induction motor load takes 900 kVA at a power factor of 0.707. Calculate the kVA rating of the capacitor bank to raise the resultant power factor of the installation of 0.866 lagging.

Find also the resultant power factor when the capacitors are in circuit and the motor load has fallen to 300 kVA at 0.5 power factor. **[296 kVA, 0.998 leading] (*City & Guilds, London*)**

3. Two wattmeters measure the total power in three-phase circuits and are correctly connected. One reads 4,800 W while other reads backwards. On reversing the latter, it reads 400 W. What is the total power absorbed by the circuit and the power factor? **[4400 W; 0.49] (*Sheffield Univ. U.K.*)**

4. The power taken by a 3-phase, 400-V motor is measured by the two wattmeter method and the readings of the two wattmeters are 460 and 780 watts respectively. Estimate the power factor of the motor and the line current. **[0.913, 1.96 A] (*City & Guilds, London*)**

5. Two wattmeters, W_1 and W_2 connected to read the input to a three-phase induction motor running unloaded, indicate 3 kW and 1 kW respectively. On increasing the load, the reading on W_1 increases while that on W_2 decreases and eventually reverses.

Explain the above phenomenon and find the unloaded power and power factor of the motor.

[2 kW, 0.287 lag] (*London Univ.*)

6. The power flowing in a 3-ϕ, 3-wire, balanced-load system is measured by the two wattmeter method. The reading on wattmeter *A* is 5,000 W and on wattmeter *B* is – 1,000 W

(*a*) What is the power factor of the system?

(*b*) If the voltage of the circuit is 440, what is the value of capacitance which must be introduced into each phase to cause the whole of the power measured to appear on wattmeter A?

[0.359; 5.43 Ω] (*Meters and Meas. Insts. A.M.I.E.E. London*)

7. Two wattmeters are connected to measure the input to a 400 V; 3-phase, connected motor outputting 24.4 kW at a power factor of 0.4 (lag) and 80% efficiency. Calculate the

(*i*) resistance and reactance of motor per phase
(*ii*) reading of each wattmeters.

[(*i*) 2.55 Ω; 5.85 Ω; (*ii*) 34,915 W; – 4850 W] (*Elect. Machines, A.M.I.E. Sec. B, 1993*)

8. The readings of the two instruments connected to a balanced three-phase load are 128 W and 56 W. When a resistor of about 25 Ω is added to each phase, the reading of the second instrument is reduced to zero. State, giving reasons, the power in the circuit before the resistors were added. **[72 W] (*London Univ.*)**

9. A balanced star-connected load, each phase having a resistance of 10 Ω and inductive reactance of 30 Ω is connected to 400-*V*, 50-Hz supply. The phase rotation is red, yellow and blue. Wattmeters connected to read total power have their current coils in the red and blue lines respectively. Calculate the reading on each wattmeter and draw a vector diagram in explanation. **[2190 W, – 583 W] (*London Univ.*)**

10. A 7.46 kW induction motor runs from a 3-phase, 400-V supply. On no-load, the motor takes a line current of 4. A at a power factor of 0.208 lagging. On full load, it operates at a power factor of 0.88 lagging and an efficiency of 89 per cent. Determine the readings on each of the two wattmeters connected to read the total power on (*a*) no load and (*b*) full load. **[1070 W, – 494 W; 5500 W; 2890 W]**

11. A balanced inductive load, connected in star across 415-V, 50-Hz, three-phase mains, takes a line current of 25A. The phase sequence is *RYB*. A single-phase wattmeter has its current coil connected in the *R* line and its voltage coil across the line *YB*. With these connections, the reading is 8 kW. Draw the vector diagram and find (*i*) the kW (*ii*) the kVAR (*iii*) the kVA and (*iv*) the power factor of the load.

[(*i*) 11.45 kW (*ii*) 13.87 kVAR (*iii*) 18 kVA (iv) 0.637] (*City & Guilds, London*)

19.27. Double Subscript Notation

In symmetrically-arranged networks, it is comparatively easier and actually more advantageous, to use single-subscript notation. But for unbalanced 3-phase circuits, it is essential to use double subscript notation, in order to avoid unnecessary confusion which is likely to result in serious errors.

Suppose, we are given two coils are whose induced e.m.fs. are 60° out of phase with each other [Fig. 19.65 (*a*)]. Next, suppose that it is required to connect these coils in additive series *i.e.* in such

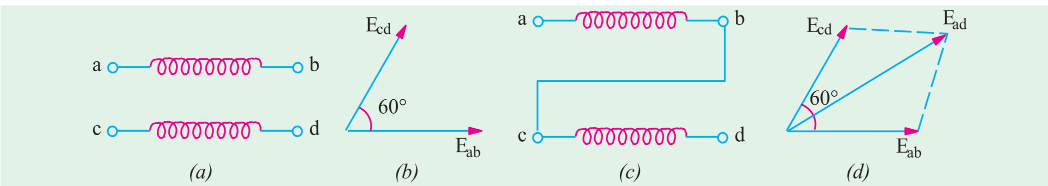


Fig. 19.65

a way that their e.m.fs. add at an angle of 60°. From the information given, it is impossible to know whether to connect terminal '*a*' to terminal '*c*' or to terminal '*d*'. But if additionally it were given that e.m.f. from terminal '*c*' to terminal '*d*' is 60° out of phase with that from terminal '*a*' to terminal '*b*', then the way to connect the coils is definitely fixed, as shown in Fig. 19.59 (*b*) and 19.60 (*a*). The double-subscript notation is obviously very convenient in such cases. The order in which these subscripts are written indicates the direction along which the voltage acts (or current flows). For example the e.m.f. '*a*' to '*b*' [Fig. 19.59 (*a*)], may be written as E_{ab} and that from '*c*' to '*d*' as E_{cd}..... The e.m.f. between '*a*' and '*d*' is E_{ad} where $\mathbf{E}_{ad} = \mathbf{E}_{ab} + \mathbf{E}_{ad}$ and is shown in Fig. 19.59 (*b*).

Example 19.62. *If in Fig. 19.60 (a), terminal 'b' is connected to 'd', find E_{ac} if E = 100 V.*

Solution. Vector diagram is shown in Fig. 19.60 (*b*)

Obviously, $\mathbf{E}_{ac} = \mathbf{E}_{ab} + \mathbf{E}_{dc} = \mathbf{E}_{ab} \times (-\mathbf{E}_{cd})$

Hence, $\mathbf{E}_{cd}$ is reversed and ***added*** to $\mathbf{E}_{ab}$ to get $\mathbf{E}_{ac}$ as shown in Fig. 19.60 (*b*). The magnitude of resultant vector is

$$E_{ac} = 2 \times 100 \cos 120°/2 = 100\,\text{V};\ \mathbf{E}_{ac} = 100\angle -60°$$

Example 19.62(*a*). *In Fig. 19.66 (a) with terminal 'b' connected to 'd', find E_{ca}.*

Solution. $\mathbf{E}_{ca} = \mathbf{E}_{cd} + \mathbf{E}_{ba} = \mathbf{E}_{cd} + (-\mathbf{E}_{ab})$

As shown in Fig. 19.67, vector $\mathbf{E}_{ab}$ is reversed and then combined with $\mathbf{E}_{cd}$ to get $\mathbf{E}_{ca}$.

Magnitude of $\mathbf{E}_{ca}$ is given by 2 × 100 × cos 60° = 100 V but it leads $\mathbf{E}_{ab}$ by 120°.

$$\therefore \quad \mathbf{E}_{ca} = 100\angle 120°$$

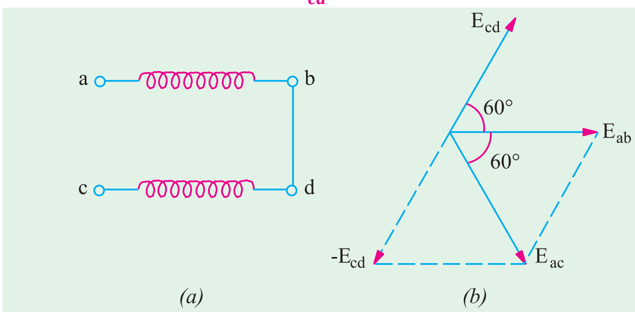


Fig. 19.66

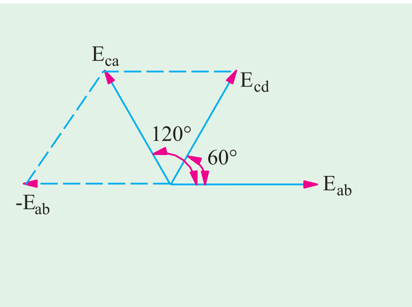


Fig. 19.67

In Fig. 19.68 (*b*) is shown the vector diagram of the e.m.fs induced in the three phases 1, 2, 3 (or *R*, *Y*, *B*) of a 3-phase alternator [Fig.19.68 (*a*)]. According to double subscript notation, each phase e.m.f. may be written as $\mathbf{E}_{01}$, $\mathbf{E}_{02}$ and $\mathbf{E}_{03}$, the order of the subscripts indicating the direction in which the e.m.fs. act. It is seen that while passing from phase 1 to phase 2 through the external circuit, we are in opposition to $\mathbf{E}_{02}$.

$$\mathbf{E}_{12} = \mathbf{E}_{20} + \mathbf{E}_{01} = (-\mathbf{E}_{02}) + \mathbf{E}_{01} = \mathbf{E}_{01} - \mathbf{E}_{02}$$

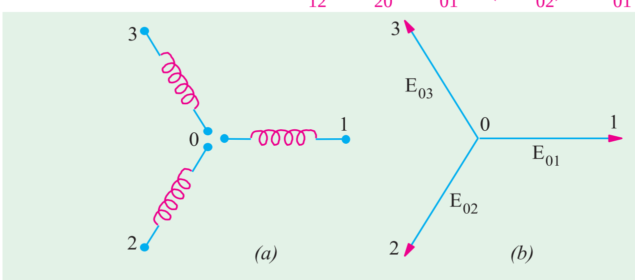


Fig. 19.68

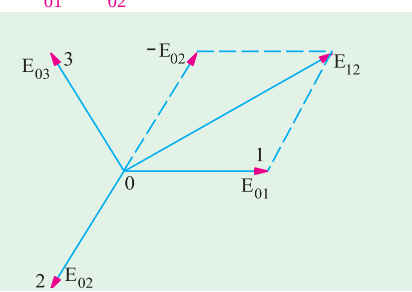


Fig. 19.69

It means that for obtaining $\mathbf{E}_{12}$, $\mathbf{E}_{20}$ has to be reversed to obtain $-\mathbf{E}_{02}$ which is then combined with $\mathbf{E}_{01}$ to get $\mathbf{E}_{12}$ (Fig. 19.69). Similarly,

$$\mathbf{E}_{23} = \mathbf{E}_{30} + \mathbf{E}_{02} = (-\mathbf{E}_{03}) + \mathbf{E}_{02} = \mathbf{E}_{02} - \mathbf{E}_{03}$$

$$\mathbf{E}_{31} = \mathbf{E}_{10} + \mathbf{E}_{03} = (-\mathbf{E}_{01}) + \mathbf{E}_{03} = \mathbf{E}_{03} - \mathbf{E}_{01}$$

By now it should be clear that double-subscript notation is based on lettering every junction and terminal point of diagrams of connections and on the use of two subscripts with all vectors representing voltage or current. The subscripts on the vector diagram, taken from the diagram of connections, indicate that the positive direction of the current or voltage is from the first subscript to the second. For example, according to this notation $\mathbf{I}_{ab}$ represents a current whose + ve direction is from *a* to *b* in the branch *ab* of the circuit in the diagram of connections. In the like manner, $\mathbf{E}_{ab}$ represents the e.m.f. which produces this current. Further, $\mathbf{I}_{ba}$ will represent a current flowing from *b* to *a*, hence its vector will be drawn equal to but in a direction opposite to that of $\mathbf{I}_{ab}$ *i.e.* $\mathbf{I}_{ab}$ and $\mathbf{I}_{ba}$ differ in phase by 180° although they do not differ in magnitude.

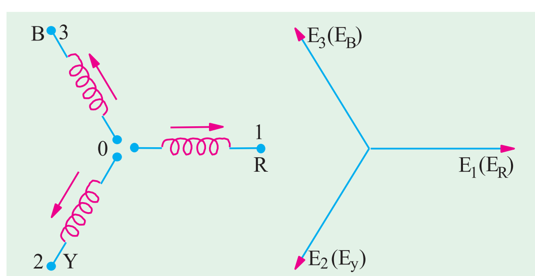


Fig. 19.69 (*a*)

In single subscript notation (*i.e.* the one in which single subscript is used) the + ve directions are fixed by putting arrows on the circuit diagrams as shown in Fig. 19.69 (*a*). According to this notation

$$\mathbf{E}_{12} = -\mathbf{E}_2 + \mathbf{E}_1 = \mathbf{E}_1 - \mathbf{E}_2;\ \mathbf{E}_{23} = -\mathbf{E}_3 + \mathbf{E}_2 = \mathbf{E}_2 - \mathbf{E}_3 \text{ and } \mathbf{E}_{31} = -\mathbf{E}_1 + \mathbf{E}_3 = \mathbf{E}_3 - \mathbf{E}_1$$

or $\mathbf{E}_{RY} = \mathbf{E}_R - \mathbf{E}_Y;\ \mathbf{E}_{YB} = \mathbf{E}_Y - \mathbf{E}_B;\ \mathbf{E}_{BR} = \mathbf{E}_B - \mathbf{E}_R$

Example 19.63. *Given the phasors* $V_{12} = 10\angle 30°; V_{23} = 5\angle 0°;\ V_{14} = 6\angle -60°;$ $V_{45} = 10\angle 90°$. *Find* ***(i)*** V_{13} ***(ii)*** V_{34} *and* ***(iii)*** V_{25}.

Solution. Different points and the voltage between them have been shown in Fig. 19.70.

(i) Using KVL, we have

$$V_{12} + V_{23} + V_{31} = 0 \text{ or } V_{12} + V_{23} - V_{13} = 0$$

or $V_{13} = V_{12} + V_{23} = 10\angle 30° + 5\angle 0° = 8.86 + j5 + 5$

$= 13.86 + j5 = 14.7\angle 70.2°$

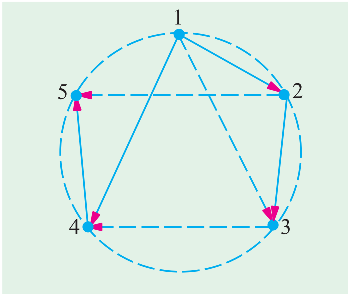


Fig. 19.70

(ii) Similarly, $V_{13} + V_{34} + V_{41} = 0$ or $V_{13} + V_{34} - V_{14} = 0$

or $V_{34} = V_{14} - V_{13} = 6\angle -60° - 14.7\angle 70.2°$

$= 3 - j5.3 - 13.86 - j5 = -10.86 - j10.3 = 15\angle 226.5°$

(iii) Similarly, $V_{23} + V_{34} + V_{45} + V_{52} = 0$

or $V_{23} + V_{34} + V_{45} - V_{52} = 0$

or $V_{25} = V_{23} + V_{34} + V_{45} = 5\angle 0° + 15\angle 226.5° + 10\angle 90°$

$= 5 - 10.86 - j10.3 + j10 = -5.86 - j0.3 = 5.86 - \angle 2.9°$.

Example 19.64. *In a balanced 3-phase Y-connected voltage source having phase sequence abc,* $V_{an} = 230\angle 30°$. *Calculate analytically* ***(i)*** V_{bn} ***(ii)*** V_{cn} ***(iii)*** V_{ab} ***(iv)*** V_{bc} *and* ***(v)*** V_{ca}. *Show the phase and line voltages on a phasor diagram.*

Solution. It should be noted that V_{an} stands for the voltage of terminal a with respect to the neutral point *n*. The usual positive direction of the phase voltages are as shown in Fig. 19.71 (*a*). Since the 3-phase system is balanced, the phase differences between the different phase voltages are 120°.

(i) $V_{bn} = \angle -120° = 230\ \angle(30° - 120°) = 230\angle -90°$

(ii) $V_{cn} = V_{an}\angle 120° = 230\ \angle(30° + 120°) = 230\angle 150°$ ... Fig. 19.71 (*b*)

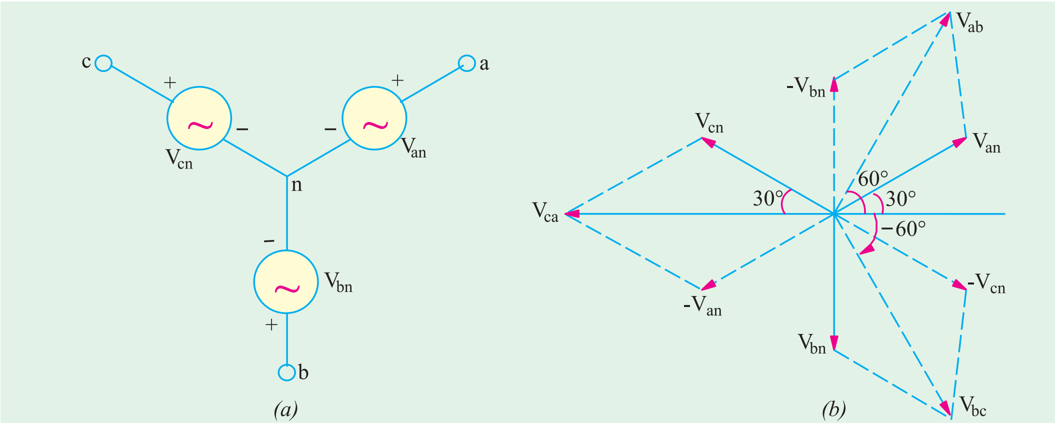


Fig. 19.71

(iii) It should be kept in mind that V_{ab} stands for the voltage of point a with respect to point *b*. For this purpose, we start from the reference point *b* in Fig.19.71 (*a*) and go to point *a* and find the sum of the voltages met on the way. As per sign convention given in Art, 19.27 as we go from *b* to *n*, there is a fall in voltage of by an amount equal to V_{bn}. Next as we go from *n* to *a*, there is increase of voltage given by V_{an}.

$\therefore\ V_{ab} = -V_{bn} + V_{an} = V_{an} - V_{bn} = 230\angle 30° - 230\angle -90°$

$= 230(\cos 30° + j\sin 30°) - 230(0 - j\sin 90°)$

$$= 230 \quad \frac{\sqrt{3}}{2} \quad j\frac{1}{2} \quad j230 \quad 230 \quad \frac{\sqrt{3}}{2} \quad j\frac{3}{2} \quad = 230\sqrt{3} \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 400 \quad 60$$

(iv) $V_{bc} = V_{bn} - V_{cn} = 230\angle -90° - 230\angle 150° = -j230 - 230$

$$\frac{\sqrt{3}}{2} \quad j\frac{1}{2} \quad 230\sqrt{3} \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 400 \quad 60$$

(v) $V_{ca} \quad V_{cn} \quad V_{an} \quad 230 \quad 150 \quad 230 \quad 30 \quad \frac{\sqrt{3}}{2} \quad j\frac{1}{2} \quad 230 \quad \frac{\sqrt{3}}{2} \quad j\frac{1}{2} \quad 400 \quad 400 \quad 180$

These line voltages along with the phase voltages have been shown in the phasor diagram of Fig. 19.71 (*b*).

Example 19.65. *Three non-inductive resistances, each of 100 Ω are connected in star to a 3-phase, 440-V supply. Three equal choking coils are also connected in delta to the same supply; the resistance of one coil being equal to 100 Ω. Calculate* ***(a)*** *the line current and* ***(b)*** *the power factor of the system.* **(Elect. Technology-II, Sambal Univ.)**

Solution. The diagram of connections and the vector diagram of the *Y*-and Δ-connected impedances are shown in Fig. 19.72.

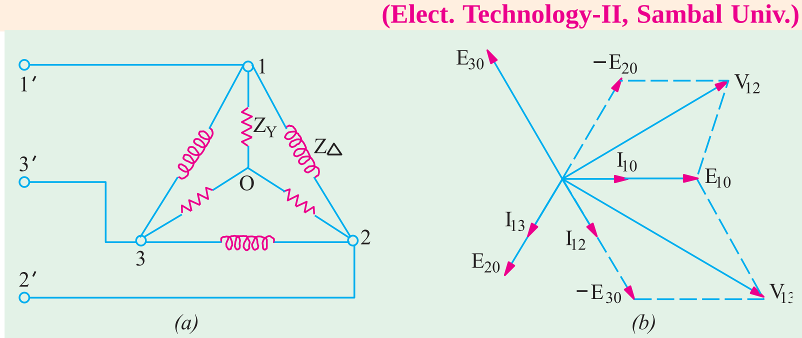


Fig. 19.72

The voltage $\mathbf{E}_{10}$ between line 1 and neutral is taken along the *X*-axis. Since the load is balanced, it will suffice to determine the current in one line only. Applying Kirchhoff's Law to junction 1, we have

$$\mathbf{I}'_{11} = \mathbf{I}_{10} + \mathbf{I}_{12} + \mathbf{I}_{13}$$

Let us first get the vector expressions for $\mathbf{E}_{10}$, $\mathbf{E}_{20}$ and $\mathbf{E}_{30}$

$$\mathbf{E}_{10} = \frac{440}{\sqrt{3}}(1 + j0) = 254 + j0, , \mathbf{E}_{20} = 254 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad = -127 - j220$$

$$\mathbf{E}_{30} = 254 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad = -127 + j220$$

Let us now derive vector expressions for $\mathbf{V}_{12}$ and $\mathbf{V}_{31}$.

$\mathbf{V}_{10} = \mathbf{E}_{10} + \mathbf{E}_{02} = \mathbf{E}_{10} - \mathbf{E}_{20} = (254 + j0) - (-127 - j220) = 381 + j220$

$\mathbf{V}_{13} = \mathbf{E}_{10} + \mathbf{E}_{03} = \mathbf{E}_{10} - \mathbf{E}_{30} = (254 - j0) - (-127 + j220) = 381 - j220$

$$\mathbf{I}_{10} = \frac{\mathbf{E}_{10}}{\mathbf{Z}_y} \quad \frac{254 \quad j0}{100} \quad 2.54 \quad j0, \ \mathbf{I}_{12} = \frac{\mathbf{V}_{13}}{\mathbf{Z}} \quad \frac{381 \quad j220}{j100} \quad 2.2 \quad j3.81 \quad 4.4 \quad 60$$

$$\mathbf{I}_{13} = \frac{V_{13}}{Z_\Delta} = \frac{381 - j220}{j100} = -2.2 - j3.81 = 4.4\angle -120°$$

(a) $\mathbf{I}'_{11} = (2.54 + j0) + (2.2 - j3.81) + (-2.2 - j3.81) = (2.54 - j7.62) = 8.03 \angle -71.6°$

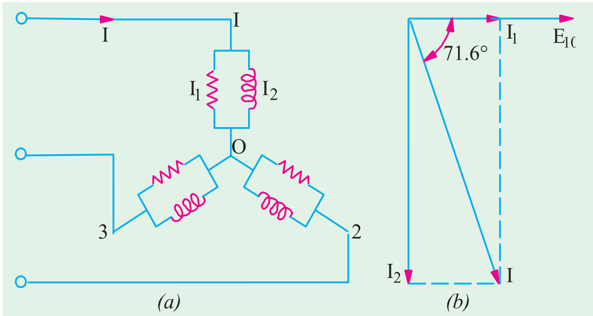


Fig. 19.73

(*b*) p.f. = cos 71.6º = **0.316 (lag)**

Alternative Method

This question may be easily solved by Δ/*Y* conversion. The star equivalent of the delta reactance is 100/3 Ω per phase.

As shown is Fig. 19.73, there are now two parallel circuits across each phase, one consisting of a resistance of 100 Ω and the other of a reactance of 100/3 Ω.

Taking E_{10} as the reference vector, we have

$$\mathbf{E}_{10} = (254 + j0)$$

$$I_1 = \frac{254 + j0}{100} = 2.54 + j0; \quad I_2 = \frac{254 + j0}{j100/3} = -j7.62$$

Line current **I** = (2.54 + *j*0) + (–*j*7.62) = (2.54 – *j*7.62) = 8.03 ∠– 71.6º ... Fig. 19.73 (*b*)

19.28. Unbalanced Loads

Any polyphase load in which the impedances in one or more phases differ from the impedances of other phases is said to be an unbalanced load. We will now consider different methods to handle unbalanced star-connected and delta-connected loads.

19.29. Unbalanced D-c onnected Load

Unlike unbalanced *Y*-connected load, the unbalanced Δ-connected load supplied from a balanced 3-phase supply does not present any new problems because the voltage across each load phase is fixed. It is independent of the nature of the load and is equal to line voltage. In fact, the problem resolves itself into three independent single-phase circuits supplied with voltages which are 120° apart in phase.

The different phase currents can be calculated in the usual manner and the three line currents are obtained by taking the vector difference of phase currents in pairs.

If the load consists of three different pure resistances, then trigonometrical method can be used with advantage, otherwise symbolic method may be used.

Example 19.66. *A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which* Z_{AB} *= 25 ∠90º,* Z_{BC} *= 15 ∠30°,* Z_{CA} *= 20 ∠0º ohms. Find the line currents and total power.*
(Advanced Elect. Machines A.M.I.E. Sec. B, Summer 1991)

Solution. As explained in Art. 19.2, a 3-phase system has only two possible sequences : *ABC* and *CBA*. In the *ABC* sequence, the voltage of phase *B* lags behind voltage of phase *A* by 120º and that of phase *C* lags behind phase *A* voltage by 240º. In the *CBA* phase which can be written as A →*C* →*B*, voltage of *C* lags behind voltage *A* by 120º and that of *B* lags behind voltage *A* by 240º. Hence, the phase voltage which can be written as

$$E_{AB} = E \angle 0º \; ; \; E_{BC} = E \angle -120º$$

and $$E_{CA} = E \angle -240º \quad \text{or} \quad E_{CA} = \angle 120º$$

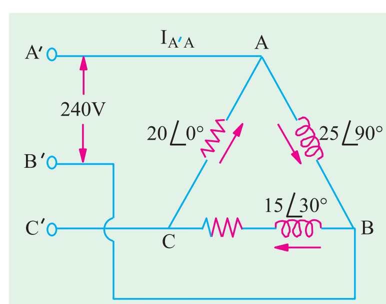


Fig. 19.74

$$\therefore \qquad I_{AB} = \frac{E_{AB}}{Z_{AB}} = \frac{240\angle 0^\circ}{25\angle 90^\circ} = 9.6\angle -90^\circ = -j9.6 \text{ A}$$

$$I_{BC} = \frac{E_{BC}}{Z_{BC}} = \frac{240\angle 120^\circ}{15\angle 30^\circ} = 16\angle 90^\circ = j16 \text{ A}$$

$$I_{CA} = \frac{E_{CA}}{Z_{CA}} = \frac{240\angle -120^\circ}{20\angle 0^\circ} = 12\angle -120^\circ = 12(0.5 - j0.866) = (-6 - j10.4) \text{ A}$$

The circuit is shown in Fig. 19.74.

Line current $I_{A'A} = I_{AB} + I_{AC} = I_{AB} - I_{CA} = -j9.6 - (-6 - j10.4) = 6 + j0.08$

Line current $I_{B'B} = I_{BC} - I_{AB} = j16 - (-j9.6) = j25.6$ A

$I_{C'C} = I_{CA} - I_{BC} = (-6 - j10.4) - j\,16 = (-5 - j26.4)$ A

Now, $R_{AB} = 0$; $R_{BC} = 15 \cos 30 = 13\,\Omega$; $R_{CA} = 20\,\Omega$

Power

$W_{AB} = 0$; $W_{BC} = I_{BC}^2\, R_{BC} = 16^2 \times 13 = 3328$ W; $W_{CA} = I_{CA}^2 \times R_{CA} = 27^2 \times 20 = 14{,}580$ W.

Total Power = 3328 + 14580 = 17,908 W.

Example 19.67. *In the network of Fig. 19.75, $E_{na} = 230\angle 0^\circ$ and the phase sequence is abc. Find the line currents I_a, I_b and I_c as also the phase currents I_{AB}, I_{BC} and I_{CA}.*

E_{na}, E_{nb}, E_{nc} is a balanced three-phase voltage system with phase sequence abc.

(Network Theory, Nagpur Univ. 1993)

Solution. Since the phase sequence is *abc*, the generator phase voltages are:

$$E_{na} = 320\angle 0^\circ; E_{nb} = 230\angle -120^\circ; E_{nc} = 230\angle 120^\circ$$

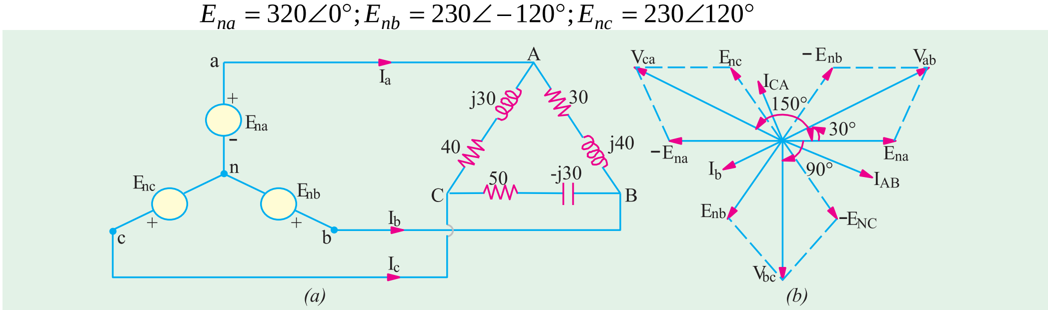


Fig. 19.75

As seen from the phasor diagram of Fig. 19.75 (*b*), the line voltages are as under :-

$V_{ab} = E_{na} - E_{nb}$; $V_{bc} = E_{nb} - E_{nc}$; $V_{ca} = E_{nc} - E_{na}$

$\therefore$ $V_{ab} = \sqrt{3} \times 230\sqrt{30}^\circ = 400\angle 30^\circ$ *i.e* it is ahead of the reference generator phase voltage E_{na} by 30°.

$V_{bc} = \sqrt{3} \times 230\angle 90^\circ = 400\angle -90^\circ$. This voltage is 90° behind E_{na} but 120° behind V_{ab}.

$V_{ca} = \sqrt{3} \times 230\angle 150^\circ = 400\angle 150^\circ$ or $\angle -210^\circ$. This voltage leads reference voltage E_{na} by 150° but leads V_{ab} by 120°.

These voltages are applied across the unbalanced Δ-connected lead as shown in Fig. 19.75 (*a*).

$Z_{AB} = 30 + j40 = 50\angle 53.1^\circ; Z_{BC} = 50 - j30 = 58.3\angle -31^\circ,$

$Z_{CA} = 40 + j30 = 50\angle 36.9^\circ$

$$I_{AB} = \frac{V_{ab}}{Z_{AB}} = \frac{400\angle 30^\circ}{50\angle 53.1^\circ} = 8\angle -23.1^\circ = 7.36 - j3.14$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{400\angle -90^\circ}{58.3\angle -31^\circ} = 6.86\angle -59^\circ = 3.53 - j5.88$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{400\angle 150°}{50\angle 36.9°} = 8\angle 113.1° = 3.14 + j7.36$$

$$I_a = I_{AB} - I_{CA} = 7.36 - j3.14 + 3.14 - j7.36 = 10.5 - j10.5 = 14.85\angle -45°$$

$$I_b = I_{BC} - I_{AB} = 3.53 - j5.88 - 7.36 + j3.14 = -3.83 - j2.74 = 4.71\angle -215.6°$$

$$I_C = I_{CA} - I_{BC} = -3.14 + j7.36 - 3.53 + j5.88 = -6.67 + j13.24 = 14.8\angle 116.7°$$

Example 19.68. *For the unbalanced* Δ - connected *load of Fig. 19.76 (a), find, the phase currents, line currents and the total power consumed by the load when phase sequence is* **(a)** *abc and* **(b)** *acb.*

Solution. (a) Phase sequence abc (Fig. 19.76).

Let $\mathbf{V}_{ab} = 100\angle 0° = 100 + j0$

$$\mathbf{V}_{bc} = 100 \quad 120 \quad 100 \quad \frac{1}{2}\ j\frac{\sqrt{3}}{2} \quad 50 \quad j86.6$$

$$\mathbf{V}_{ca} = 100 \quad 102 \quad 100 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 50 \quad j86.6$$

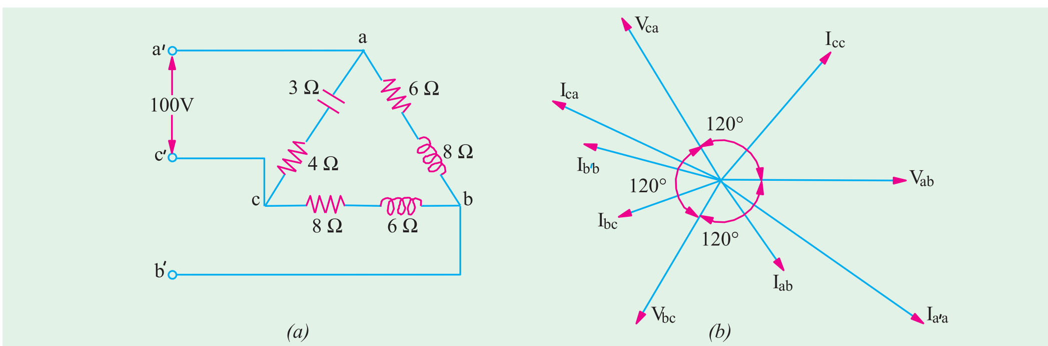


Fig. 19.76

(i) Phase currents

Phase current, $\mathbf{I} \quad \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} \quad \frac{100 \quad j0}{6 \quad j8} \quad 6 \quad j8 \quad 10 \quad 53 \ 8$

Similarly, $\mathbf{I}_{bc} \quad \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} \quad \frac{30 \quad j86.6}{8 \quad j6} \quad 9.2 \quad j3.93 \quad 10 \quad 156 \ 52$

$\mathbf{I}_{ca} \quad \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} \quad \frac{50 \quad j86.6}{4 \quad j3} \quad 18.39 \quad j7.86 \quad 20 \quad 156 \ 52$

(ii) Line Currents

Line Current $\mathbf{I}_{a\ a} \quad \mathbf{I}_{ab} \quad \mathbf{I}_{ac} \quad \mathbf{I}_{ab} \quad \mathbf{I}_{ca} \quad (6 \quad j8) \quad (\ 18.39 \quad j7.86)$

$= 24.39 - j15.86 = 29.1\angle -33°2'$

Similarly, $\mathbf{I}_{b\ b} \quad \mathbf{I}_{bc} \quad \mathbf{I}_{ba} \quad \mathbf{I}_{bc} \quad \mathbf{I}_{ab}$

$(\ 9.2 \quad j3.93) \quad (6 \quad j8) \quad 15.2 \quad j4.07 \quad 15.73 \quad 165 \ 30$

$\mathbf{I}_{c\ c} \quad \mathbf{I}_{ca} \quad \mathbf{I}_{cb} \quad \mathbf{I}_{ca} \quad \mathbf{I}_{bc} \quad (\ 18.39 \quad j7.86) \quad (\ 9.2 \quad j3.93)$

$= 9.19 + j11.79 = 14.94\angle 52°3'$

Check $\Sigma\mathbf{I} = 0 + j0$

(*iii*) Power

$$W_{ab} = I_{ab}^2 R_{ab} = 10^2 \times 6 = 600\text{ W}$$

$$W_{bc} = I_{bc}^2 R_{bc} = 10^2 \times 8 = 800\text{ W}$$

$$W_{ca} = I_{ca}^2 R_{ca} = 20^2 \times 4 = 1600\text{ W}$$

Total = 3000 W

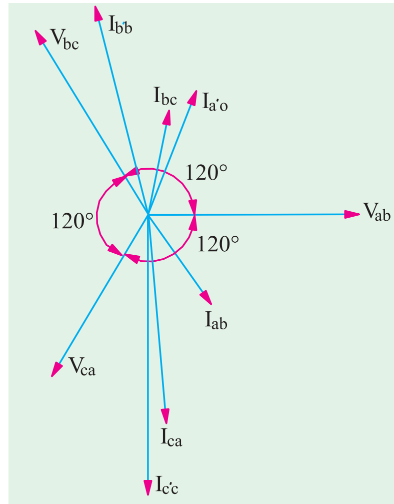


Fig. 19.77

(*b*) Phase sequence acb (Fig. 19.77)

Here, $\mathbf{V}_{ab} \quad 100 \quad 0 \quad 100 \quad j9$

$\mathbf{V}_{bc} \quad 100 \quad 120 \quad 50 \quad 86.6$

$\mathbf{V}_{ca} \quad 100 \quad 120 \quad 50 \quad j86.6$

(*i*) Phase Currents

$$\mathbf{I}_{ab} \quad \frac{100}{6 \quad j8} \quad 6 \quad j8 \quad 10 \quad 53\ 8$$

$$\mathbf{I}_{bc} \quad \frac{50 \quad j86.6}{8 \quad j6} = (1.2 + j9.93) = 10\angle 83°8'$$

$$\mathbf{I}_{ca} \quad \frac{50 \quad j86.6}{7 \quad j3} = (2.4 - j19.86) = 20 - 83°8'$$

(*ii*) Line Currents

$$\mathbf{I}_{a'a} = \mathbf{I}_{ab} + \mathbf{I}_{ac} = \mathbf{I}_{ab} - \mathbf{I}_{ca}$$

$$= (6 - j8) - (2.4 - j19.86) = (3.6 + j11.86) = 12.39\angle 73°6'$$

$\mathbf{I}_{bb} \quad (1.2 \quad j9.93) \quad (6 \quad j8) \quad (\ 4.8 \quad j17.93) \quad 18.56 \quad 105$

$\mathbf{I}_{cc} \quad (2.4 \quad j19.86) \quad (1.2 \quad j9.93) \quad (1.2 \quad j29.79) \quad 29.9 \quad 87\ 42$

It is seen that $\Sigma \mathbf{I} = 0 + j0$

(*iii*) Power

$$W_{ab} = 10^2 \times 6 = 600\text{W}$$

$$W_{bc} = 10^2 \times 8 = 800\text{ W}$$

$$W_{ca} = 20^2 \times 4 = 1600\text{ W}$$

Total = 3000 W — as before

It will be seen that the effect of phase reversal on an unbalanced Δ-connected load is as under:

(*i*) phase currents change in angle only, their magnitudes remaining the same

(*ii*) consequently, phase powers remain unchanged

(*iii*) line currents change both in magnitude and angle.

The adjoining tabulation emphasizes the effect of phase sequence on the line currents drawn by an unbalanced 3-phase load.

Line	*Ampere Sequence a b c*	*Sequence c b a*
a	$29.1\angle -33°2'$	$12.39\angle 73.1°$
b	$15.73\angle 165°$	$18.56\angle 105°$
c	$14.94\angle 52°3'$	$29.9\angle -87.7°$

Example 19.69. *A balanced 3-phase supplies an unbalanced 3-phase delta-connected load made up of to resistors 100 Ω and a reactor having an inductance of 0.3 H with negligible resistance. V_L = 100 V at 50 Hz. Calculate (a) the total power in the system.*

(Elect. Engineering-I, Madras Univ.)

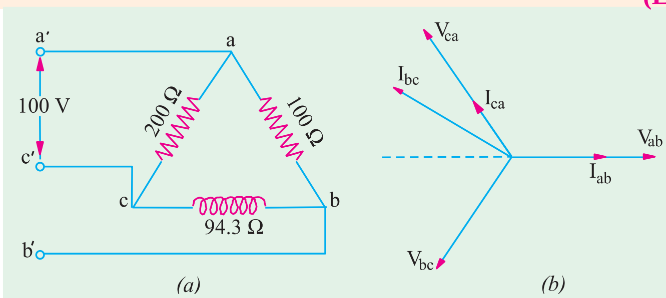


Fig. 19.78

Solution. The Δ-connected load and its phasor diagram are shown in Fig. 19.78 (*a*).

$$X_L = \omega L = 314.2 \times 0.2$$
$$= 94.3\Omega$$

Let $\mathbf{V}_{ab}$ 100 0 100 j0

$\mathbf{V}_{bc}$ 100 120

$= -50 - j86.6$

$\mathbf{V}_{ca}$ 100 120 50 j86.6

$\mathbf{I}_{ab}$ $\frac{V_{ab}}{Z_{ab}}$ $\frac{100 \quad 0}{100 \quad 0}$ 13 0 1 j0

$\mathbf{I}_{bc}$ $\frac{100 \quad 0}{100 \quad 0}$ 1.06 210 0.92 j0.53

$\mathbf{I}_{ca}$ $\frac{100 \quad 120}{200 \quad 0}$ 0.5 120 0.25 j0.43

Watts in branch $ab = V_{ab}^2 / R_{ab} = 100^2 / 100$ = 100 W; VARs = 0

Watts in branch $bc = 0$; VARs = 100 × 1.06 = 106 (lag)

Watts in branch $ca = V_{ca}^2 / R_{ca} = 100^2 / 200$ = 50 W; VRAs = 0

(*a*) Total power = 100 + 50 = 150 W; VARs = **106 (lag)**

19.30. Four-wire Star-connected Unbalanced Load

It is the simplest case of an unbalanced load and may be treated as three separate single-phase systems with a common return wire. It will be assumed that impedance of the line wires and source phase windings is zero. Should such an assumption be unacceptable, these impedances can be added to the load impedances. Under these conditions, source and load line terminals are at the same potential.

Consider the following two cases:

(*i*) Neutral wire of zero impedance

Because of the presence of neutral wire (assumed to behaving zero impedance), the star points of the generator and load are tied together and are at the same potential. Hence, the voltages across the three load impedances are ***equalized*** and each is equal to the voltage of the corresponding phase of the generator. In other words, due to the provision of the neutral, each phase voltage is a ***forced*** voltage so that the three phase voltages are balanced when line voltages are balanced even though phase impedances are unbalanced. However, it is worth noting that a break or open ($\mathbf{Z}_N = \infty$) in the neutral wire of a 3-phase, 2-wire system with ***unbalanced*** load always causes large (in most cases inadmissible) changes in currents and phase voltages. It is because of this reason that no fuses and circuit breakers are ever used in the neutral wire of such a 3-phase system.

The solution for currents follows a pattern similar to that for the unbalanced delta.

Obviously, the vector sum of the currents in the three lines is not zero but is equal to neutral current.

(*ii*) Neutral wire with impedance Z_N

Such a case can be easily solved with the help of Node-pair Voltage method as detailed below. Consider the general case of a *Y*- to -*Y* system with a neutral wire of impedance Z_n as shown in Fig. 19.79 (*a*). As before, the impedance of line wires and source phase windings would be assumed to be zero so that the line and load terminals, *R*,*Y*, *B* and R', Y', B' are the same respective potentials.

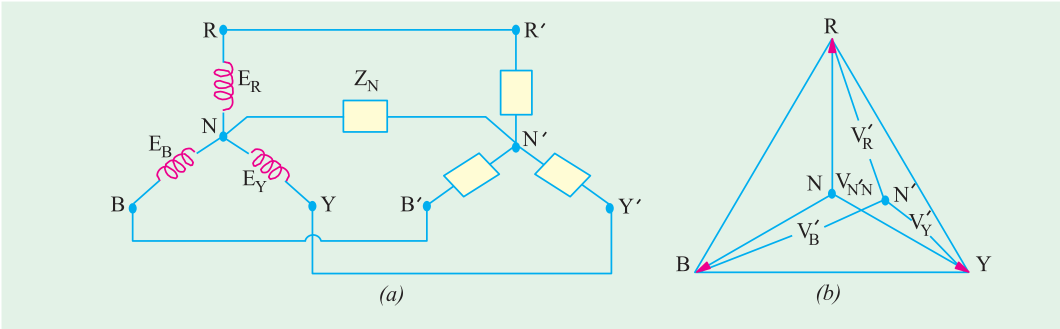


Fig. 19.79

According to Node-pair Voltage method, the above star-to-star system can be looked upon as multi-mesh network with a single pair of nodes *i.e.* neutral points *N* and *N*'. The node potential *i.e.* the potential difference between the supply and local neutrals is given by

$$\boldsymbol{V'_{NN}} = \frac{\boldsymbol{E_R Y_R + E_Y Y_Y + E_B Y_B}}{\boldsymbol{Y_R + Y_Y + Y_B + Y_N}}$$

where $\mathbf{Y}_R$, $\mathbf{Y}_Y$ and $\mathbf{Y}_B$ represent the ***load phase*** admittances. Obviously, the load neutral N' does not coincide with source neutral *N*. Hence, load phase voltages are no longer equal to one another even when phase voltages are as seen from Fig. 19.79 (*b*).

The load phase voltage are given by

$$\boldsymbol{V'_R = E_R - V'_{NN;} V'_Y = E_Y - V_{NN}} \text{ and } \boldsymbol{V_{B'} = E_B - V'_{NN}}$$

The phase currents are

$$\mathbf{I}_R = \mathbf{V}'_R\mathbf{Y}_R,\ \mathbf{I}'_Y = \mathbf{V}'_Y\mathbf{Y}_Y \text{ and } \mathbf{I}_B = \mathbf{V}'_B\mathbf{Y}_B$$

The current in the neutral wire is $\boldsymbol{I_N = V_N Y_N}$

Note. In the above calculations, $I_R \quad I_R{}' \quad I_{RR}$

Similarly, $I_Y = I'_Y = I'_{YY}$ and $I'_B = I_{BB}$.

Example 19.70. *A 3-phase, 4-wire system having a 254-V line-to-neutral has the following loads connected between the respective lines and neutral;* $Z_R = 10\angle 0°$ *ohm;* $Z_Y = 10\angle 37°$ *ohm and* $Z_B = 10\angle -53°$ *ohm. Calculate the current in the neutral wire and the power taken by each load when phase sequence is* ***(i)*** *RYB and* ***(ii)*** *RBY.*

Solution. (*i*) Phase sequence RYB (Fig. 19.80)

$\mathbf{V}_{RN} \quad 254 \quad 0$; $\mathbf{V}_{YN} \quad 254 \quad 120$; $\mathbf{V}_{BN} \quad 254 \quad 120$

$$\mathrm{I}_R \quad \mathrm{I}_{RN} \quad \frac{\mathrm{V}_{RN}}{\mathrm{R}_R} \quad \frac{254 \quad 0}{10 \quad 0} \quad 25.4 \quad 0 \text{*}$$

$$\mathbf{I}_r = \mathbf{I}_{YN} = \frac{254\angle -120°}{10\angle 37°} = 25.4\angle -157° = 25.4(-0.9205 - j0.3907) = -23.38 - j9.95$$

* This method is similar to Millman's Theorem of Art. 19.32.

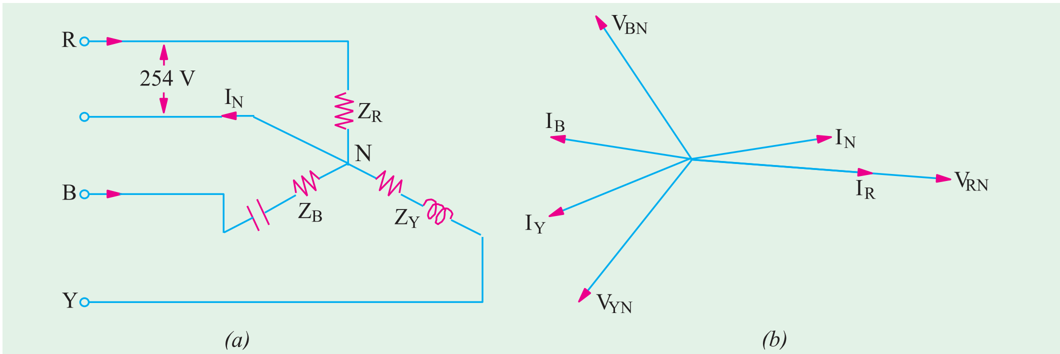


Fig. 19.80

$\boldsymbol{I}_B = \boldsymbol{I}_{BN} = \dfrac{254\angle 120°}{10\angle -53°} = 25.4\angle 173° = 25.4(-0.9925 + j0.1219) = -25.2 + j3.1$

$\boldsymbol{I}_N = -(\boldsymbol{I}_R + \boldsymbol{I}_Y + \boldsymbol{I}_B) = -[25.4 + (-23.38 - j9.55) + (-25.21 + j3.1)] = 23.49 + j6.85$

$= \mathbf{24.46\angle 16°15'}$

Now $R_R = 10\,\Omega;\ R_Y = 10\cos 37° = 8\Omega;\ R_B = 10\cos 53° = 6\Omega$

$W_R \quad 25.4^2 \quad 10 \quad \mathbf{6.452\,W;}\ W_Y \quad 25.4^2 \quad 8 \quad \mathbf{5{,}162\,W}$

$W_B \quad 25.4^2 \quad 6 \quad \mathbf{3{,}871\ W}$

***(ii)* Phase sequence RBY** [Fig. 19.81]

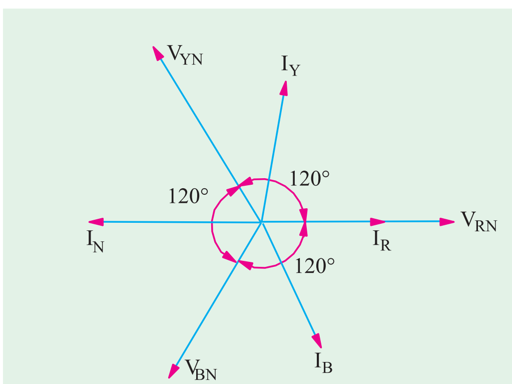


Fig. 19.81

$\boldsymbol{V}_{RN} \quad 254 \quad 0\ ;\ V_{YN} \quad 254 \quad 120$

$\boldsymbol{V}_{BN} \quad 254 \qquad 120$

$\boldsymbol{I}_R \quad \dfrac{254 \quad 0}{10 \quad 0} \quad 25.4 \quad 0$

$\boldsymbol{I}_Y \quad \dfrac{254 \quad 120}{10 \quad 37} \quad 25.4 \quad 83$

$= (3.1 + j25.2)\boldsymbol{I}_B$

$\boldsymbol{I}_B \quad \dfrac{254 \qquad 120}{10 \qquad 53} \quad 25.4 \qquad 67 \quad (9.95 \quad j23.4)$

$\mathbf{I}_N \quad (\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad (38.45 \quad j1.8)$

$= -38.45 - j1.8 = 38.5\angle -177.3°$.

Obviously, power would remain the same because magnitude of branch currents is unaltered. From the above, we conclude that phase reversal in the case of a 4-wire unbalanced load supplied from a balanced voltage system leads to the following changes:

(*i*) it changes the angles of phase currents but not their magnitudes.

(*ii*) however, power remains unchanged.

(*iii*) it changes the magnitude as well as angle of the neutral current $\mathbf{I}_N$.

Example 19.71. *A* $3-\phi$ *, 4-wire, 380-V supply is connected to an unbalanced load having phase impedances of:* $Z_R = (8 + j6)\ \Omega$ *,* $Z_y = (8 - j6)\ \Omega$ *and* $Z_B = 5\,\Omega$ *. Impedance of the neutral wire is* $Z_N = (0.5 + j1)\,\Omega$ *.*

Ignoring the impedances of line wires and internal impedances of the e.m.f. sources, find the phase currents and voltages of the load.

Solution. This question will be solved by using Node-pair Voltage method discussed in Art. 19.30. The admittances of the various branches connected between nodes *N* and *N'* in Fig. 19.82 (*a*).

$\mathbf{Y}_R = 1/\mathbf{Z}_R = 1/(8 + j6) = (0.08 - j0.06)$
$\mathbf{Y}_Y = 1/\mathbf{Z}_Y = 1/(8 - j6) = (0.08 + j0.06)$
$\mathbf{Y}_B = 1/2/\mathbf{Z}_B = 1/(5 + j0) = 0.2$
$\mathbf{Y}_N = 1/\mathbf{Z}_N = 1/(0.5 + j1) = (0.4 - j0.8)$

Let $\mathbf{E}_R = (380/\sqrt{3})\angle 0° = 220\angle 0° = 220 + j0$

$\mathbf{E}_Y = 220\angle -120° = 220(-0.5 - j0.866) = -110 - j190$

$\mathbf{E}_B = 220\angle 120° = 220(-0.5 + j0.866) = -110 + j190$

The node voltage between N' and N is given by

$$V'_{NN} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B + Y_N}$$

$$= \frac{200(0.08 - j0.06) + (-110 - j190)(0.08 + j0.06) + (-110 + j190) \times 0.2}{(0.08 - j0.06) + (0.08 + j0.06) + 0.2 + (0.4 - j0.8)}$$

$$= \frac{-1.8 + j3}{0.76 - j0.8} = -3.41 + j0.76$$

The three load phase voltages are as under:

$\mathbf{V}_R \quad \mathbf{E}_R \quad \mathbf{V}'_{NN} \quad 220 \quad 3.41 \quad j0.76 \quad 223.41 \quad j0.76$

$\mathbf{V}_Y \quad \mathbf{E}_Y \quad \mathbf{V}'_{NN} \quad (\quad 110 \quad j90) \quad 3.41 \quad j0.76 \quad 106.59 \quad j190.76$

$\mathbf{V}_B \quad \mathbf{E}_B \quad \mathbf{V}_{NN} \quad (\quad 110 \quad j90) \quad 3.41 \quad j0.76 \quad 106.59 \quad j190.76$

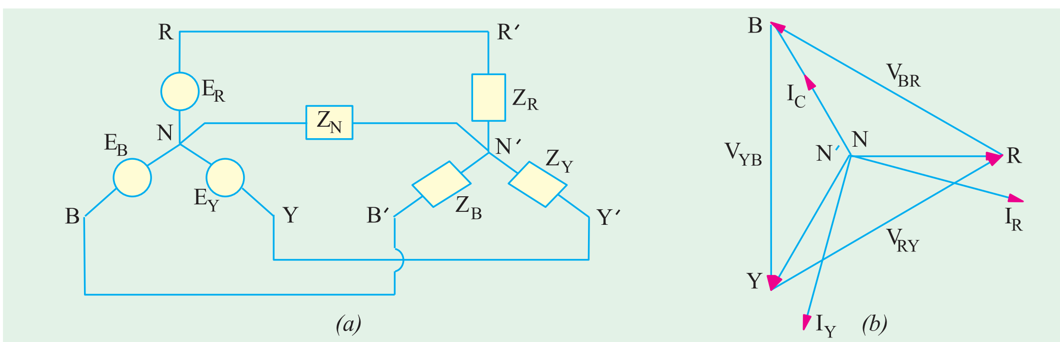


Fig 19.82

$\mathbf{I}_R \quad \mathbf{V}_R\mathbf{Y}_R \quad (223.41 \quad j0.76)(0.08 \quad j0.06) \quad 17.83 \quad j13.1 \quad 22.1 \quad 36.3$ A

$\mathbf{I}_Y \quad \mathbf{V}_Y\mathbf{Y}_Y \quad (\quad 106.59 \quad j190.76)(0.08 \quad j0.06) \quad 2.92 \quad j21.66 \quad 21.86 \quad 82.4$ A

$\mathbf{I}_B \quad \mathbf{V}_B\mathbf{Y}_B \quad (\quad 106.59 \quad j190.76) \quad 0.2 \quad 21.33 \quad j37.85 \quad 43.45 \quad 119.4$ A

$\mathbf{I}_N \quad \mathbf{V}_N\mathbf{Y}_N \quad (\quad 3.41 \quad j0.76)(0.4 \quad j0.8) \quad 0.76 \quad j3.03 \quad 3.12 \quad 104.1$ A

These voltage and currents are shown in the phasor diagram of Fig. 19.82 (b) where displacement of the neutral point has not been shown due to the low value of V'_{NN}.

Note. It can be shown that $I_N = I'_R + I'_Y + I'_B$

19.31. Unbalanced Y-connected Load Without Neutral

When a star-connected load is unbalanced and it has no neutral wire. Then its star point is isolated from the star-point of the generator. The potential of the load star-point is different from that of the generator star-point. The potential of the former is subject to variations according to the imbalance of the load and under certain conditions of loading, the potentials of the two star- point may differ considerably. Such an isolated load star-point or neutral point is called 'floating' neutral point because its potential is always changing and is not fixed.

All Y-connected unbalanced loads supplied from polyphase systems have floating neutral points without a neutral wire. Any unbalancing of the load causes variations not only of the potential of the star-point but also of the voltages across the different branches of the load. ***Hence, in that case, phase voltage of the load is not*** $1/\sqrt{3}$ ***of the line voltage.***

There are many methods to tackle such unbalanced Y-connected loads having isolated neutral points.

1. By converting the Y-connected load to an equivalent unbalanced Δ-connected load by using Y-Δ conversion theorem. The equivalent Δ-connection can be solved in Fig. 19.80. The line currents so calculated are equal in magnitude and phase to those taken by the original unbalanced Y-connected load.

2. By applying Kirchhoff's Laws.

3. By applying Millman's Theorem.

4. By using Maxwell's Mesh or Loop Current Method.

19.32. Millman's Theorem

Fig. 19.83 shows a number of linear bilateral admittances, Y_1, Y_2, ... connected to a common point or node O'. The voltages of the free ends of these admittances with respect to another common point O are V_{10}, V_{20} ... V_{no}. Then, according to this theorem, the voltage of O' with respect to O is given by $\mathbf{V}'_{00} = \dfrac{\mathbf{V}_{10}\mathbf{Y}_1 + \mathbf{V}_{20}\mathbf{Y}_2 + \mathbf{V}_{30}\mathbf{Y}_3 + ..\mathbf{V}_{no}\mathbf{Y}_n}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + ...\mathbf{Y}_n}$

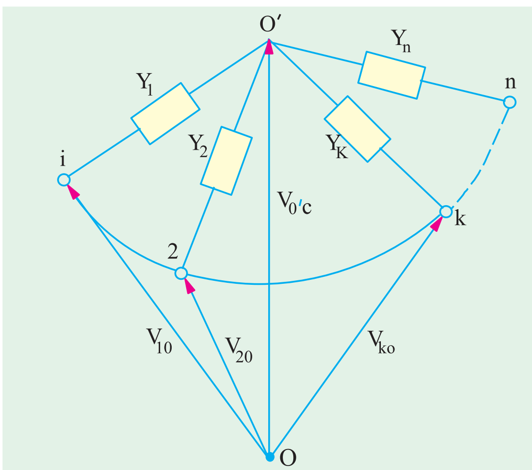


Fig. 19.83

or $$V'_{00} = \frac{\sum_{k=1}^{n} V_{k0} Y_k}{\sum_{k=1}^{n} Y_k}$$

Proof. Consider the closed loop 0' Ok. The sum of p.ds. around it is zero. Starting from O' and going anticlockwise, we have

$$V'_{00} + V_{ok} + V'_{ko} = 0$$

$$\therefore \; V'_{ko} = -V_{ok} - V'_{00} = V_{ko} - V'_{00}$$

Current through Y_k is I'_{ko}

$$= V'_{ko} Y_k = (V_{ko} - V'_{00}) Y_k =$$

By Kirchhoff's Current Law, sum of currents meeting at point O' is zero.

$$\therefore \; I'_{10} + I'_{20} + ... I'_{ko} .... + I'_{no} = 0$$

$$(\mathbf{V}'_{10} - \mathbf{V}'_{00})\mathbf{Y}_1 + (\mathbf{V}_{20} - \mathbf{V}'_{00})\mathbf{Y}_2 + ...(\mathbf{V}_{ko} - \mathbf{V}'_{00})\mathbf{Y}_k + ... = 0$$

or $$\mathbf{V}'_{10}\mathbf{Y}_1 + \mathbf{V}_{20}\mathbf{Y}_2 + ...\mathbf{V}_{ko}\mathbf{Y}_k + ... = \mathbf{V}'_{00}(\mathbf{Y}_1 + \mathbf{Y}_2 + ... + \mathbf{Y}_k + ...)$$

$$V'_{00} = \frac{V_{10}Y_1 + V_{20}Y_2 + ...}{Y_1 + Y_2 + ...}$$

19.33. Application of Kirchhoff's Laws

Consider the unbalanced Y-connected load of Fig. 19.84. Since the common point of the three load impedances is not at the potential of the neutral, it is marked 0' instead of N*. Let us assume the

* For the sake of avoiding printing difficulties, we will take the load star point as 0 instead of 0' for this article.

phase sequence V_{ab}, V_{bc}, V_{ca} *i.e.* V_{ab} leads V_{bc} and V_{bc} leads V_{ca}. Let the three branch impedances be Z_{ca}, Z_{ab} and Z_{ac}, however, since double subscript notation is not necessary for a *Y*-connected impedances in order to indicate to which phase it belongs, single-subscript notation may be used with advantage. Therefore Z_{oa}, Z_{ab}, Z_{oc} can be written as Z_a, Z_b, Z_c respectively. It may be pointed out that double-subscript notation is essential for mesh-connected impedances in order to make them definite.

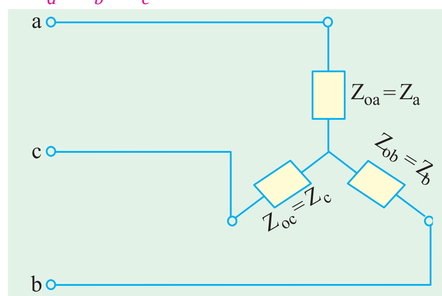


Fig. 19.84

From Kirchhoff's laws, we obtain

$$V_{ab} = I_{ao}Z_a + I_{ab}Z_b \quad \dots (1)$$

$$V_{bc} = I_{bo}Z_b + I_{oc}Z_c \quad \dots (2)$$

$$V_{ca} = I_{co}Z_c + I_{oa}Z_a \quad \dots (3)$$

and $I_{aa} + I_{ba} + I_{co} = 0$ – point law ... **(4)**

Equation **(1)**, **(3)** and **(4)** can be used for finding I_{bo}.

Adding **(1)** and **(3)**, we get

$$V_{ab} + V_{ca} = I_{ao}Z_a + I_{ao}Z_b + I_{co}Z_c + I_{oc}Z_a$$

$$= I_{ob}Z_b + I_{co}Z_c + I_{oa}Z_a - I_{oa}Z_a = I_{ob}Z_b + I_{co}Z_c \quad \dots (5)$$

Substituting $\mathbf{I}_{oa}$ from equation **(4)** in equation **(3)**, we get

$$V_{ca} = I_{co}Z_c + (I_{bo} + I_{co})Z_a = I_{co}(Z_c + Z_a) + I_{bo}Z_a \quad \dots (6)$$

Putting the value of I_{co} from equation (5) in equation (6), we have

$$V_{ca} = (Z_c + Z_a)\frac{(V_{ab} + V_{ca}) - I_{ob}Z_b}{Z_c} + I_{bo}Z_a$$

$$V_{ca}Z_c = -I_{ob}Z_bZ_c - I_{ob}Z_bZ_a + I_{bo}Z_aZ_c + V_{ab}Z_a + V_{ab}Z_c + V_{ca}Z_c + V_{ca}Z_a$$

$$\therefore \quad I_{ob} = \frac{(V_{ab} + V_{ca})Z_a + V_{ab}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$$

Since $V_{ab} + V_{bc} + V_{ca} = 0$ $\quad \therefore \quad I_{ob} = \dfrac{V_{ab}Z_c - V_{bc}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$ ... **(7)**

From the symmetry of the above equation, the expressions for the other branch currents are,

$$I_{oc} = \frac{V_{bc}Z_a - V_{ca}Z_b}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \dots (8) \qquad I_{oa} = \frac{V_{ca}Z_b - V_{ab}Z_c}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \dots (9)$$

Note. Obviously, the three line currents can be written as

$$I_{ao} = -I_{oa} = \frac{V_{ab}Z_c - V_{ca}Z_b}{\sum Z_aZ_b} \quad \dots (10) \qquad I_{bo} = -I_{ob}\frac{V_{bc}Z_a - V_{ab}Z_c}{\sum Z_aZ_b} \quad \dots (11)$$

$$I_{co} = -I_{oc}\frac{V_{ca}Z_b - V_{bc}Z_a}{\sum Z_aZ_b} \quad \dots (12)$$

Example 19.72. *If in the unbalanced Y-connected load of Fig. 19.78, Z_a = (10 + j0), Z_b = (3 + j4) and Z_c = (0 – j10) and the load is put across a 3-phase, 200-V circuit with balanced voltages, find the three line currents and voltages across each branch impedance. Assume phase sequence of V_{ab}, V_{bc}, V_{ca}.*

Solution. Take V_{ab} along the axis of reference. The vector expressions for the three voltages are

$$V_{ab} = 200 + j0$$

$$V_{bc} \quad 200 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \qquad 100 \quad j173.2; \quad V_{ca} \quad 200 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \qquad 100 \quad j173.2$$

From equation **(9)** given above

$$\mathbf{I}_{oa} \quad \frac{(\ 100 \quad j173.2)(3 \quad j4) \quad (200 \quad j0)(0 \quad j10)}{(0 \quad j10)(10 \quad j0) \quad (10 \quad j0)(3 \quad j4) \quad (3 \quad j4)(0 \quad j10)}$$

$$= \frac{-992.8 + j2119.6}{70 - j90} = -20.02 + j4.54$$

$$\mathbf{I}_{ob} \quad \frac{(200 \quad j0)(0 \quad j10) \quad (\ 100 \quad j173.2)(10 \quad j0)}{(10 \quad j0)(3 \quad j4) \quad (3 \quad j4)(0 \quad j10) \quad (0 \quad j10)(10 \quad j0)}$$

$$= \frac{1000 - j268}{70 - j90} = 7.24 + j5.48$$

Now, $\mathbf{I}_{oc}$ may also be calculated in the same way or it can be found easily from equation (4) of Art. 19.33.

$$\mathbf{I}_{oc} = \mathbf{I}_{ao} + \mathbf{I}_{bo} = -\mathbf{I}_{oa} - \mathbf{I}_{ob} = 20.02 - j4.54 - 7.24 - j5.48 = 12.78 - j10.02$$

Now

$$\mathbf{V}_{oa} = \mathbf{I}_{oa}\mathbf{Z}_a = (-20.02 + j4.54)(10 + j0) = 200.2 + j45.4$$

$$\mathbf{V}_{ob} = \mathbf{I}_{ob}\mathbf{Z}_b = (7.24 + j5.48)(3 + j4) = -0.2 + j45.4$$

$$\mathbf{V}_{oc} = \mathbf{I}_{oc}\mathbf{Z}_c = (12.78 - j10.02)(0 - j10) = -100.2 - j127.8$$

As a check, we may combine $\mathbf{V}_{oa}$, $\mathbf{V}_{ob}$ and $\mathbf{V}_{oc}$ to get the line voltages which should be equal to the applied line voltages. In passing from *a* to *b* through the circuit internally, we find that we are in opposition to $\mathbf{V}_{oa}$ but in the same direction as the positive direction of $\mathbf{V}_{ob}$.

$$\mathbf{V}_{ab} \quad \mathbf{V}_{ao} \quad \mathbf{V}_{ob} \qquad \mathbf{V}_{oa} \quad \mathbf{V}_{ob} \qquad (\ 200.2 \quad j45.4) \quad (\ 0.2 \quad j45.4) \quad 200 \quad j0$$

$$\mathbf{V}_{bc} \quad \mathbf{V}_{bo} \quad \mathbf{V}_{oc} \qquad \mathbf{V}_{ob} \quad \mathbf{V}_{oc} \qquad (\ 0.2 \quad j45.4) \quad (\ 100.2 \quad j127.8) \qquad 100 \quad j173.2$$

$$\mathbf{V}_{ca} \quad \mathbf{V}_{co} \quad \mathbf{V}_{oa} \qquad \mathbf{V}_{oc} \quad \mathbf{V}_{oa} \qquad (\ 100.2 \quad j127.8) \quad (\ 200.2 \quad j45.4) \qquad 100 \quad j173.2$$

19.34. Delta/Star and Star/Delta Conversions

Let us consider the unbalanced Δ-connected load of Fig. 19.85 (*a*) and *Y*-connected load of Fig. 19.85 (*b*). If the two systems are to be equivalent, then the impedances between corresponding pairs of terminals must be the same.

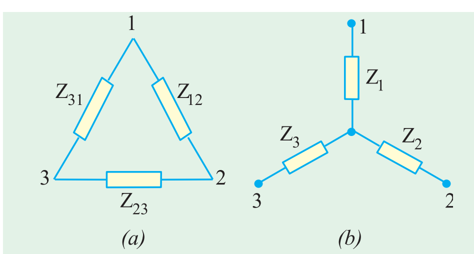


Fig. 19.85

(*i*) Delta/Star Conversion

For *Y*-load, total impedance between terminals 1 and 2 is = $\mathbf{Z}_1 + \mathbf{Z}_2$ (it should be noted that double subscript notation of $\mathbf{Z}_{01}$ and $\mathbf{Z}_{02}$ has been purposely avoided).

Considering terminals 1 and 2 of Δ-load, we find that there are two parallel paths having impedances of $\mathbf{Z}_{12}$ and $(\mathbf{Z}_{31} + \mathbf{Z}_{23})$. Hence, the equivalent impedance between terminals 1 and 2 is given by

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_{12}} + \frac{1}{\mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \text{or} \quad \mathbf{Z} = \frac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$$

Therefore, for equivalence between the two systems $\mathbf{Z}_1 + \mathbf{Z}_2 = \dfrac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$ ... **(1)**

Similarly $\mathbf{Z}_2+\mathbf{Z}_3=\dfrac{\mathbf{Z}_{23}(\mathbf{Z}_{31}+\mathbf{Z}_{12})}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}}$ ... **(2)** $\quad \mathbf{Z}_3+\mathbf{Z}_1=\dfrac{\mathbf{Z}_{31}(\mathbf{Z}_{12}+\mathbf{Z}_{23})}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}}$ ... **(3)**

Adding equation **(3)** to **(1)** and subtracting equation **(2)**, we get

$$2\mathbf{Z}_1=\frac{\mathbf{Z}_{12}(\mathbf{Z}_{23}+\mathbf{Z}_{31})+\mathbf{Z}_{31}(\mathbf{Z}_{12}+\mathbf{Z}_{23})-\mathbf{Z}_{23}(\mathbf{Z}_{31}+\mathbf{Z}_{12})}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}}=\frac{2\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}}$$

$$\therefore \quad \mathbf{Z}_1=\frac{\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}} \quad \text{... (4)}$$

The other two results may be written down by changing the subscripts cyclically

$$\therefore \quad \mathbf{Z}_2=\frac{\mathbf{Z}_{23}\mathbf{Z}_{12}}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}}; \quad \text{... (5)} \quad \mathbf{Z}_3=\frac{\mathbf{Z}_{31}\mathbf{Z}_{23}}{\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31}} \quad \text{... (6)}$$

The above expression can be easily obtained by remembering that (Art. 2.19)

$$\text{Start } \mathbf{Z}=\frac{\text{Product of } \Delta\,\mathbf{Z}\text{'s connected to the same terminals}}{\text{Sum of } \Delta\,\mathbf{Z}\text{'s}}$$

In should be noted that all **Z'** are to be expressed in their complex form.

***(iii)* Star/Delta Conversion**

The equations for this conversion can be obtained by rearranging equations **(4), (5)** and **(6)**, Rewriting these equations, we get

$$\mathbf{Z}_1(\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31})=\mathbf{Z}_{12}\mathbf{Z}_{31} \quad \text{... (7)}$$
$$\mathbf{Z}_2(\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31})=\mathbf{Z}_{23}\mathbf{Z}_{12} \quad \text{... (8)}$$
$$\mathbf{Z}_3(\mathbf{Z}_{12}+\mathbf{Z}_{23}+\mathbf{Z}_{31})=\mathbf{Z}_{31}\mathbf{Z}_{23} \quad \text{... (9)}$$

Dividing equation **(7)** by **(9)**, we get $\dfrac{\mathbf{Z}_1}{\mathbf{Z}_3}=\dfrac{\mathbf{Z}_{12}}{\mathbf{Z}_{23}} \qquad \therefore \ \mathbf{Z}_{23}=\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}$

Dividing equation **(8)** by **(9)**, we get $\dfrac{\mathbf{Z}_2}{\mathbf{Z}_3}=\dfrac{\mathbf{Z}_{12}}{\mathbf{Z}_{31}} \qquad \therefore \ \mathbf{Z}_{31}=\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2}$

Substituting these values in equation **(7)**, we have $\mathbf{Z}_1 \quad \mathbf{Z}_{12} \quad \mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_1} \quad \mathbf{Z}_{31} \quad \mathbf{Z}_{12}.\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2}$

or $\quad \mathbf{Z}_1\mathbf{Z}_{12}\ 1+\dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}+\dfrac{\mathbf{Z}_{31}}{\mathbf{Z}_{12}} = \mathbf{Z}_{12}\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2} \quad \therefore \ \mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1=\mathbf{Z}_{12}\times\mathbf{Z}_3$

$$\mathbf{Z}_{12}=\frac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \text{ or } \mathbf{Z}_{12}=\mathbf{Z}_1+\mathbf{Z}_2+\frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_3}$$

Similarly, $\quad \mathbf{Z}_{23}=\mathbf{Z}_2+\mathbf{Z}_3+\dfrac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}=\dfrac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$

$$\mathbf{Z}_{31}=\mathbf{Z}_3+\mathbf{Z}_1+\frac{\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}=\frac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

As in the previous case, it is to be noted that all impedances must be expressed in their complex form.

Another point for noting is that *the line currents of this equivalent delta are the currents in the phases of the Y-connected load.*

Example 19.73. *An unbalanced star-connected load has branch impedances of Z_1 = 10 $\angle 30°\ \Omega$, Z_2 = 10 $\angle -45°\ \Omega$, Z_3 = 20 $\angle 60°\ \Omega$ and is connected across a balanced 3-phase, 3-wire supply of 200 V. Find the line currents and the voltage across each impedance using Y/Δ conversion method.*

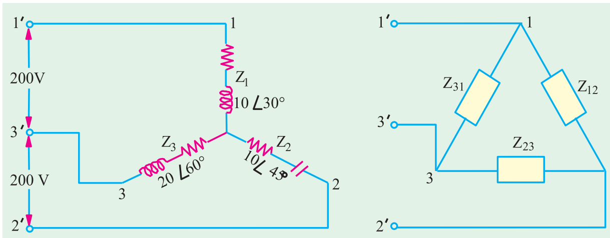


Fig. 19.86

Solution. The unbalanced *Y*-connected load and its equivalent Δ -connected load are shown in Fig. 19.86.

Now $\mathbf{Z_1Z_2} + \mathbf{Z_2Z_3} + \mathbf{Z_3Z_1}$ = (10 ∠ 30°) (10 ∠ – 45°) + (10 ∠ – 45°) (20 ∠ 60°) + (20 ∠ 60°) (10 ∠ 30°) = 100 ∠ – 15° + 200 ∠ 15° + 200 ∠ 90°

Converting these into their cartesian form, we get

= 100 [cos (– 15°) – *j* sin 15°] + 200 (cos 15° + *j* sin 15°) + 200 (cos 90° + *j* sin 90°)

= 96.6 – *j*25.9 + 193.2 + *j*51.8 + 0 + *j*200 = 289.8 + *j*225.9 = 368 ∠ 38°

$$\mathbf{Z}_{12} \quad \frac{\mathbf{Z}_1\mathbf{Z}_2 \quad \mathbf{Z}_2\mathbf{Z}_3 \quad \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \quad \frac{368 \quad 38}{20 \quad 60} \quad 18.4 \quad 22 \quad 17.0 \quad j6.9$$

$$\mathbf{Z}_{23} \quad \frac{368 \quad 38}{10 \quad\quad 30} \quad 36.8 \quad 8 \quad 36.4 \quad j5.1$$

$$\mathbf{Z}_{31} \quad \frac{368 \quad 38}{10 \quad\quad 45} \quad 36.8 \quad 83 \quad 4.49 \quad j36.5$$

Assuming clockwise phase sequence of voltages $\mathbf{V}_{12}$, $\mathbf{Z}_{23}$ and $\mathbf{V}_{31}$, we have

$\mathbf{V}_{12}$ = 200 0 , $\mathbf{V}_{23}$ 200 120 , $\mathbf{V}_{31}$ 200 120

$$\mathbf{I}_{12} \quad \frac{\mathbf{V}_{12}}{\mathbf{Z}_{12}} \quad \frac{200 \quad 0}{18.4 \quad\quad 22} \quad 10.86 \quad 22 \quad 10.07 \quad j4.06$$

$$\mathbf{I}_{23} \quad \frac{\mathbf{V}_{23}}{\mathbf{V}_{23}} \quad \frac{200 \quad\quad 120}{36.8 \quad 8} \quad 5.44 \quad 128 \quad 3.35 \quad j4.29$$

$$\mathbf{I}_{31} \quad \frac{\mathbf{V}_{31}}{\mathbf{Z}_{31}} \quad \frac{200 \quad 120}{36.8 \quad 83} \quad 5.44 \quad 37 \quad 4.34 \quad j3.2$$

Line current = $\mathbf{I}'_{11} = \mathbf{I}_{12} + \mathbf{I}_{13} = \mathbf{I}_{12} - \mathbf{I}_{31}$

$= (10.07 + j4.06) - (4.34 + j3.2) = 5.73 + j0.86 = 5.76\angle 8°32'$

$\mathbf{I}_{22}$ $\mathbf{I}_{23}$ $\mathbf{I}_{12}$ (3.35 *j*4.29) (10.07 *j*4.06) 13.42 *j*8.35 15.79 148 6

$\mathbf{I}_{33}$ $\mathbf{I}_{31}$ $\mathbf{I}_{23}$ (4.34 *j*3.2) (3.35 *j*4.29) 7.69 *j*7.49 10.73 44 16

These are currents in the *phases of the Y-connected unbalanced load*. Let us find voltage drop across each star-connected branch impedance.

Voltage drop across $\mathbf{Z}_1 = \mathbf{V}_{10} = \mathbf{I}_{11}\mathbf{Z}_1$ 5.76 8 32 .10 30 57.6 38 32

Voltage drop across $\mathbf{Z}_2 = \mathbf{V}_{20} = \mathbf{I}_{22}\,\mathbf{Z}_2$ 15.79 148 6.10 45 157.9 193 6

Voltage drop across $\mathbf{Z}_3 = \mathbf{V}_{30} = \mathbf{I}_{33}\,\mathbf{Z}_3$ 10.73 44 16 .20 60 214.6*104 16

Example 19.74. *A 300-V (line) 3-phase supply feeds! star-connected load consisting of non-inductive resistors of 15, 6 and 10 Ω connected to the R, Y and B lines respectively. The phase sequence is RYB. Calculate the voltage across each resistor.*

Solution. The *Y*-connected unbalanced load and its equivalent Δ -connected load are shown in Fig. 19.87. Using Y/Δ conversion method we have

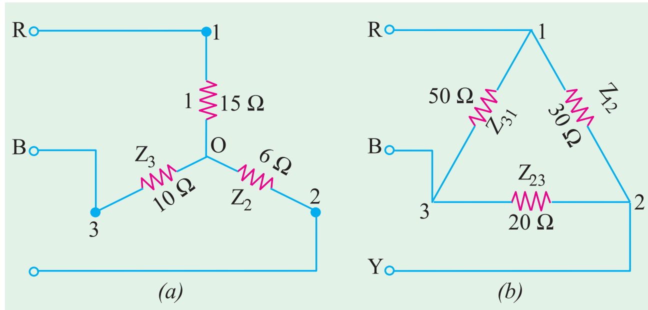


Fig. 19.87

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$= \frac{90+60+150}{10} = 30\,\Omega$$

$$\mathbf{Z}_{23}\quad 300/15\quad 20$$

$$\mathbf{Z}_{31}\quad 300/6\quad 50$$

Phase current $I_{RY} = V_{RY}/Z_{12} = 300/30 = 10\text{ A}$

Similarly $I_{YB} = V_{YB}/Z_{23} = 300/20 = 15\text{ A}$

$I_{BR} = V_{BR}/Z_{31} = 300/50 = 6\text{A}$

Each current is in phase with its own voltage because the load is purely resistive.

The line currents for the delta connection are obtained by compounding these phase currents in pairs, either trigonometrically or by phasor algebra. Using phasor algebra and choosing $\mathbf{V}_{RY}$ as the reference axis, we get

$$\mathrm{I}_{RY}\quad 10\quad j0;\ \mathrm{I}_{YB}\quad 15(\ \ \frac{1}{2}\quad j\sqrt{3}/2)\quad 7.5\quad j13.0;\mathrm{I}_{BR}\quad 6(\ \ \frac{1}{2}\quad j\sqrt{3}/2)\quad 3.0\quad j5.2$$

Line currents for delta-connection [Fig. 19.66 (*b*)] are

$\mathbf{I}_R\quad \mathbf{I}_{RY}\quad \mathbf{I}_{RB}\quad \mathbf{I}_{RY}\quad \mathbf{I}_{BR}\quad (10\quad j0)\quad (\ \ 3\quad j5.2)\quad 13\quad j5.2$ or 14 A in magnitude

$\mathbf{I}_Y\quad \mathbf{I}_{YR}\quad \mathbf{I}_{YB}\quad \mathbf{I}_{YB}\quad \mathbf{I}_{RY}\quad (\ \ 7.5\quad j13.0)\quad (10\quad j0)\quad 17.5\quad j13$ or 21.8 A in magnitude

$\mathbf{I}_B\quad \mathbf{I}_{BR}\quad \mathbf{I}_{BY}\quad \mathbf{I}_{BR}\quad \mathbf{I}_{YB}\quad (\ \ 3.0\quad j5.2)\quad (\ \ 7.5\quad j13.0)\quad 4.5\quad j18.2\quad 18.7$ A in magnitude

These line currents for Δ -connection are the phase currents for *Y*-connection. Voltage drop across each limb of *Y*-connected load is

$\mathbf{V}_{RN}\quad \mathbf{I}_R\mathbf{Z}_1\quad (13\quad j5.2)(15\quad j0)\quad 195\quad 78$ volt or 210 V

$\mathbf{V}_{YN} = \mathbf{I}_Y\mathbf{Z}_2 = (-17.5 - j13.0)(6 + j0) = -105 - j78$ volt or 131 V

$\mathbf{V}_{BN} = \mathbf{I}_B\mathbf{Z}_3 = (4.5 + j18.2)(10 + j0) = 45 + j182$ volt or 187 V

As a check, it may be verified that the difference of phase voltages taken in pairs should give the three line voltages. Going through the circuit internally, we have

$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = (195 - j78) - (105 - j78) = 300\ \angle 0°$

$\mathbf{V}_{YB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = (-105 - j78) - (45 + j182) = -150 - j260 = 300\ \angle -120°$

$\mathbf{V}_{BR} = \mathbf{V}_{BN} - \mathbf{V}_{RN} = (45 + j182) - (195 - j78) = -150 + j260 = 300\ \angle 120°$

This question could have been solved by direct geometrical methods as shown in Ex. 19.52.

Example 19.75 *A Y-connected load is supplied from a 400-V, 3-phase, 3-wire symmetrical system RYB. The branch circuit impedances are*

$$Z_R = 10\sqrt{3} + j10;\ Z_Y = 20 + j20\sqrt{3};\ Z_B = 0 - j10$$

Determine the current in each branch. Phase sequence is RYB.

(Network Analysis, Nagpur Univ. 1993)

Solution. The circuit is shown in Fig. 19.88. The problem will be solved by using all the four possible ways in which 3-wire unbalanced *Y* connected load can be handled.

Now, $\mathbf{Z}_R \quad 20 \quad 30 \quad (17.32 \quad j10)$

$\mathbf{Z}_Y \quad 40 \quad 60 \quad (20 \quad j34.64)$

$\mathbf{Z}_B = 10\angle -90° = -j10$

Also, let $\mathbf{V}_{RY} \quad 400 \quad 0 \quad 400 \quad j0$

$\mathbf{V}_{RB} = 400\angle -120° = -200 - j346$

$\mathbf{V}_R = 400\angle 120° = -200 + j346$

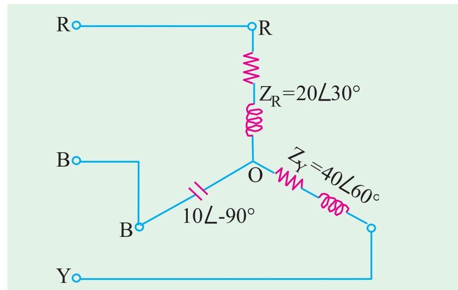


Fig. 19.88

(*a*) By applying Kirchhoff's Laws

With reference to Art. 19.33, it is seen that

$$I_{RO} = I_R = \frac{V_{RY}Z_B - V_{BR}Z_Y}{Z_RZ_Y + Z_YZ_B + Z_BZ_R}; I_{YO} = \mathrm{I}_Y = \frac{V_{YB}Z_R - V_{RY}Z_B}{Z_RZ_Y + Z_YZ_B + Z_BZ_R};$$

$$I_{BO} = I_B = \frac{V_{BR}Z_Y - V_{YB}Z_R}{Z_RZ_Y + Z_YZ_B + Z_BZ_R}$$

Now, $\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R$

$= 20\angle 30°.40\angle 60° + 40\angle 60°.10\angle -90° + 10\angle -90°.20\angle 30°$

$= 800\angle 90° + 400\angle -30° + 200\angle -60° = 446 + j426 = 617\angle 43.7°$

$\mathbf{V}_{RY}\mathbf{Z}_B \quad \mathbf{V}_{BR}\mathbf{Z}_Y \quad 400 \quad 10 \quad 90 \quad 400 \quad 120 .40 \quad 60$

$= 16{,}000 - j4000 = 16{,}490\angle -14°3'$

$\therefore \quad \mathbf{I}_R \quad \frac{16{,}490 \quad 14 \quad 3}{617 \quad 43.7} \quad 26.73 \quad 57 \quad 45$

$\mathbf{V}_{YB}\mathbf{Z}_R \quad \mathbf{V}_{RY}\mathbf{Z}_B \quad 400 \quad 120 .20 \quad 30 \quad 400.10 \quad 90 \quad j4000 \quad 4000 \quad 90$

$\mathbf{I}_Y \quad \frac{4000 \quad 90}{617 \quad 43.7} \quad 6.48 \quad 133.7$

$\mathbf{V}_{BR}\mathbf{Z}_Y \quad \mathbf{V}_{YB}\mathbf{Z}_R \quad 400 \quad 120 .40 \quad 60 \quad 400 \quad 120 \; 20. \quad 30$

$= -16{,}000 + j8{,}000 = 17{,}890\angle 153°26'$

$\therefore \quad \mathbf{I}_B \quad \frac{17{,}890 \quad 153 \quad 26}{617 \quad 43.7} \quad 29 \quad 109 \quad 45$

(*b*) By Star/Delta Conversion (Fig. 19.89)

The given star may be converted into the equivalent delta with the help of equations given in Art. 19.34.

$$\mathbf{Z}_{RY} \quad \frac{\mathbf{Z}_R\mathbf{Z}_Y \quad \mathbf{Z}_Y\mathbf{Z}_B \quad \mathbf{Z}_B\mathbf{Z}_R}{\mathbf{Z}_B} \quad \frac{617 \quad 43.7}{10 \quad 90} \quad 61.73 \quad 133.7$$

$$\mathbf{Z}_{YB} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_R} \quad \frac{617 \quad 43.7}{20 \quad 30} \quad 30.87 \quad 13.7$$

$$\mathbf{Z}_{BR} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_Y} \quad \frac{617 \quad 43.7}{40 \quad 60} \quad 15.43 \quad 16.3$$

$$\mathbf{I}_{RY} \quad \frac{\mathbf{V}_{RY}}{\mathbf{Z}_{RY}} \quad \frac{400}{61.73 \quad 133.7} \quad 6.48 \quad 133.7 \quad (\; 4.47 \quad j4.68)$$

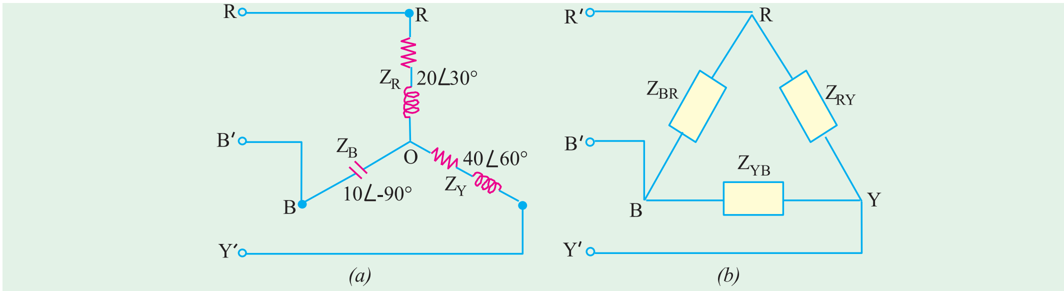


Fig. 19.89

$$\mathbf{I}_{YB} \quad \frac{\mathbf{V}_{YB}}{\mathbf{Z}_{YB}} \quad \frac{400 \qquad 120}{30.87 \quad 13.7} \quad 12.95 \qquad 133.7 \qquad (\;\; 8.95 \qquad j9.35)$$

$$\mathbf{I}_{BR} \quad \frac{\mathbf{V}_{BR}}{\mathbf{Z}_{BR}} \quad \frac{400 \quad 120}{15.43 \qquad 16.3} \quad 25.9 \quad 136.3 \qquad (\;\; 18.7 \qquad j17.9)$$

$$\mathbf{I}_{RR} \quad \mathbf{I}_{RY} \quad \mathbf{I}_{BR} \quad 14.23 \quad j22.58 \quad 26.7 \qquad 57 \;\; 48$$

$$\mathbf{I}_{YY} \quad \mathbf{I}_{YB} \quad \mathbf{I}_{RY} \qquad 4.48 \quad j4.67 \quad 6.47 \qquad 134 \;\; 6$$

$$\mathbf{I}_{YB} \quad \mathbf{I}_{BR} \quad \mathbf{I}_{YB} \quad 9.85 \quad j27.25 \quad 29 \quad 109 \;\; 48$$

$$\mathbf{I} = (0 + j0) \qquad \text{–as a check}$$

As explained in Art. 19.34, these line currents of the equivalent delta represent the phase currents of the star-connected load of Fig. 19.89 (*a*).

Note. Minor differences are due to accumulated errors.

(*c*) By Using Maxwell's Loop Current Method

Let the loop or mesh currents be as shown in Fig. 19.90. It may be noted that

$$\mathbf{I}_R = \mathbf{I}_1; \mathbf{I}_Y = \mathbf{I}_2 - \mathbf{I}_1 \text{ and } \mathbf{I}_B = -\mathbf{I}_2$$

Considering the drops across *R* and *Y*-arms, we get

$$\mathbf{I}_1\mathbf{Z}_R + \mathbf{Z}_Y(\mathbf{I}_1 - \mathbf{I}_2) = \mathbf{V}_{RY}$$

or $$\mathbf{I}_1(\mathbf{Z}_R + \mathbf{Z}_Y) - \mathbf{I}_2\mathbf{Z}_Y = \mathbf{V}_{RY} \qquad \ldots (i)$$

Similarly, considering the legs *Y* and *B*, we have

$$\mathbf{Z}_Y(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{Z}_B\mathbf{I}_2 = V_{YB}$$

or $$-\mathbf{I}_1\mathbf{Z}_Y + \mathbf{I}_2(\mathbf{Z}_B + \mathbf{Z}_Y) = \mathbf{V}_{YB} \qquad \ldots (ii)$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$, we get

$$\mathbf{I}_1 = \frac{\mathbf{V}_{RY}(\mathbf{Z}_Y + \mathbf{Z}_B) + \mathbf{Z}_{YB}\mathbf{V}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y)(\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2};$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{YB}(\mathbf{Z}_R + \mathbf{Z}_Y) + \mathbf{V}_{RY}\mathbf{Z}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y)(\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2}$$

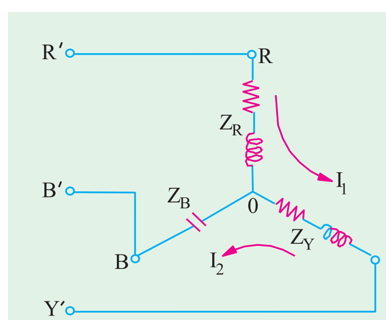


Fig. 19.90

$$\mathbf{I}_1 \quad \frac{400(20 \quad j24.64) \quad 400 \qquad 120 \;.40 \quad 60}{(37.32 \quad j44.64)(20 \quad j24.64) \quad 1600 \quad 120}$$

$$= \frac{16{,}000 - j4{,}000}{448 + j427} = \frac{16{,}490\angle -14°3'}{617\angle 43.7°}$$

$$= 26\angle -57°45' = (13.9 - j22)$$

$$\mathbf{I}_2 \quad \frac{(\;\;200 \quad j346)(37.32 \quad j44.64) \quad 400(20 \quad j34.64)}{484 \quad j427}$$

$$= \frac{16{,}000 - j8{,}000}{448 + j427} = \frac{17{,}890\angle -26°34'}{617\angle 43.7°} = 28.4\angle -70°16'$$

$$= 28.4\angle -70°16' = (9.55 - j26.7)$$

$\therefore$

$$\mathbf{I}_R \quad \mathbf{I}_1 \quad 26 \quad 57\;45$$

$$\mathbf{I}_Y \quad \mathbf{I}_2 \quad \mathbf{I}_1 \quad (9.55 \quad j26.7) \quad (13.9 \quad j22) \quad 4.35 \quad j4.7 \quad 6.5 \quad 134$$

$$\mathbf{I}_B \quad \mathbf{I}_2 \quad 28.4 \quad 70\;16 \quad 28.4 \quad 109\;44$$

(*d*) By Using Millman's Theorem*

According to this theorem, the voltage of the load star point O' with respect to the star point or neutral O of the generator or supply (normally zero potential) is given by

$$\mathbf{V}_{O'O} = \frac{\mathbf{V}_{RO}\mathbf{Y}_R + \mathbf{V}_{YO}\mathbf{Y}_Y + \mathbf{V}_{BO}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

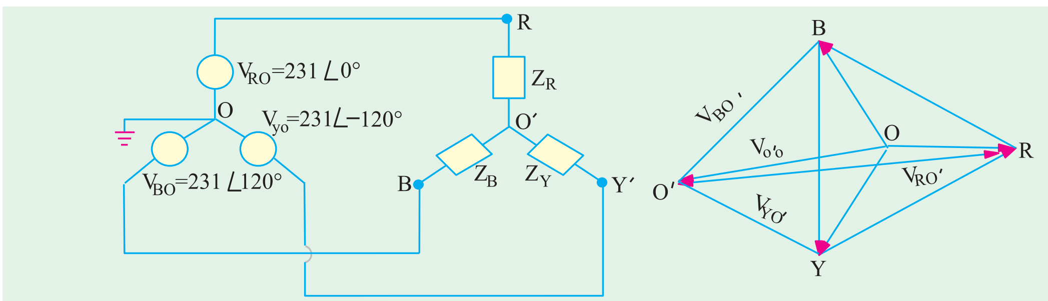


Fig. 19.91

where $\mathbf{V}_{RO}$, $\mathbf{V}_{YO}$ and $\mathbf{V}_{BO}$ are the phase voltages of the generator or 3-phase supply.

As seen from Fig. 19.91, voltage across each phase of the load is

$$V_{RO} \quad V_{RO} \quad V_{O\,O} V_{YO} \quad V_{YO} \quad V_{OO}\; V_{BO} \quad V_{BO} \quad V_{OO}$$

Obviously, $\mathbf{I}_{RO} \quad (\mathbf{V}_{RO} \quad \mathbf{V}_{OO}\;)\mathbf{Y}_R ; \mathbf{I}_{YO} \quad (\mathbf{V}_{YO} \quad \mathbf{V}_{O\,O})\mathbf{Y}_Y$ and

$$\mathbf{I}_{BO}' = (\mathbf{V}_{BO} - \mathbf{V}_{OO})\mathbf{Y}_B$$

Here $\mathbf{Y}_R \quad \frac{1}{20 \quad 30} \quad 0.05 \quad 30 \quad (0.0433 \quad j0.025)$

$$\mathbf{Y}_Y \quad \frac{1}{40 \quad 60} \quad 0.025 \quad 60 \quad (0.0125 \quad j0.0217)$$

$$\mathbf{Y}_B \quad \frac{1}{10 \quad 90} \quad 0.1\;\;90 \quad 0 \quad j0.1$$

---

* Incidentally, it may be noted that the p.d. between load neutral and supply neutral is given by

$$V_{OO}' = \frac{V_{RO}' + V_{YO}' + Y_{BO}'}{3}$$

$\therefore \quad \mathbf{Y}_R \quad \mathbf{Y}_Y \quad \mathbf{Y}_B \quad 0.0558 \quad j0.0533 \quad 0.077 \quad 43.7$

Let $\quad \mathbf{V}_{RO} \quad \frac{400}{\sqrt{3}} \quad 0 \quad (231 \quad j0)$

$\mathbf{V}_{BO} \quad 231 \quad 120 \quad 115.5 \quad j200$

$\mathbf{V}_{BO} \quad 231 \quad 120 \quad 115.5 \quad j200$

$$\mathbf{V}_{OO} \quad \frac{231.0.05 \quad 30 \quad 231 \quad 120 \; .0.025 \quad 60 \quad 231 \quad 120 \; .01 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{-15.8 - j17.32}{0.077\angle 43.7°} = \frac{23.5\angle -132.4\angle}{0.077\angle 43.7} = 305\angle -176.1° = (304.5 - j20.8)$$

$\mathbf{V}_{RO} \quad \mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 231 \quad (\; 304.5 \quad j20.8) \quad 535.5 \quad j20.8 \quad 536 \quad 2.2$

$\mathbf{V}_{YO} \quad (\; 115.5 \quad j200) \quad (\; 304.5 \quad j20.8) \quad 189 \quad j179 \quad 260 \quad 43 \; 27$

$\mathbf{V}_{BO} \quad (\; 115.5 \quad j200) \quad (\; 304.5 \quad j20.8) \quad 189 \quad j221 \quad 291 \quad 49 \; 27$

$\therefore \quad \mathbf{I}_{RO} \quad 536 \quad 2.2 \quad 0.05 \quad 30 \quad 26.5 \quad 27.8$

$\mathbf{I}_{YO} \quad 260 \quad 43 \; 27 \quad 0.025 \quad 60 \quad 6.5 \quad 103 \; 27$

$\mathbf{I}_{BO} \quad 291 \quad 49 \; 27 \quad 0.1 \quad 90 \quad 29.1 \quad 139 \; 27$

Note. As seen from above, $\mathbf{V}'_{RO} = \mathbf{V}'_{RO} - \mathbf{V}'_{OO}$

Substituting the value of $\mathbf{V}'_{OO}$, we have

$$\mathbf{V_{RO}} = \mathbf{V_{RO}} - \frac{\mathbf{V_{RO}Y_R} + \mathbf{V_{YO}Y_Y} + \mathbf{V_{BO}Y_B}}{\mathbf{V_R} + \mathbf{V_Y} + \mathbf{V_B}}$$

$$= \frac{(\mathbf{V}_{RO} - \mathbf{V}_{YO})\mathbf{Y}_Y + (\mathbf{Y}_{RO} - \mathbf{V}_{BO})\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

$$= \frac{\mathbf{V}_{RY}\mathbf{Y}_Y + \mathbf{V}_{RB}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

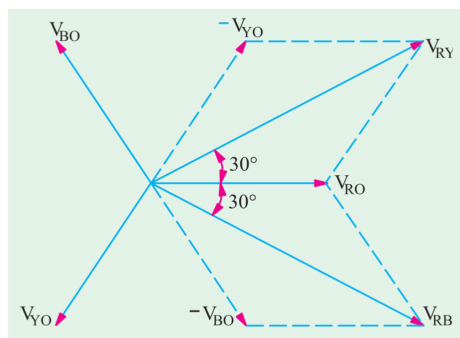


Fig. 19.92

Since $\mathbf{V}_{RO}$ is taken as the reference vector, then as seen from Fig. 19.92.

$\mathbf{V}_{RY} \quad 400 \quad 30$ and $\mathbf{V}_{RB} \quad 400 \quad 30$

$$\therefore \mathbf{V}_{RO} \quad \frac{400 \quad 30 \quad 0.025 \quad 60 \quad 400 \quad 30 \quad 0.1 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{28.6 + j29.64}{0.077\angle 43.7°} = \frac{41\angle 46°}{0.077\angle 43.7°} = 532.5\angle 2.3°$$

$\mathbf{I}_{RO} \quad \mathbf{V}_{RO}\mathbf{Y}_R \quad 532.5 \quad 2.3 \quad 0.05 \quad 30 \quad 26.6 \quad 27.7$

Similarly, $\mathbf{V}_{YO}$ and $\mathbf{V}_{BO}$ may be found and $\mathbf{I}_Y$ and $\mathbf{I}_B$ calculated therefrom.

Example 19.76. *Three impedances, Z_R, Z_Y and Z_B are connected in star across a 440-V, 3-phase supply. If the voltage of star-point relative to the supply neutral is 200 $\angle$ 150° volt and Y and B line currents are 10 $\angle$ – 90° A and 20 $\angle$ 90° A respectively, all with respect to the voltage between the supply neutral and the R line, calculate the values of Z_R, Z_Y and Z_B.*

(Elect Circuit; Nagpur Univ. 1991)

Solution. Let O and O' be the supply and load neutrals respectively. Also, let,

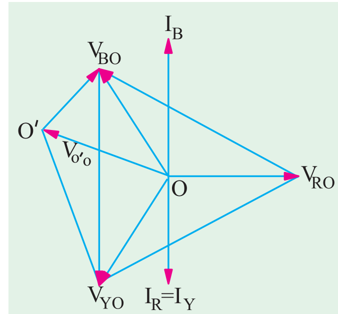


Fig. 19.93

$$\mathbf{V}_{RO} \quad \frac{440}{\sqrt{3}} \quad 0 \quad 254 \quad 0 \quad 254 \quad j0$$

$$\mathbf{Y}_{YO} \quad 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{V}_{BO} \quad 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{I}_Y = 10\angle -90° = -j10; \; \mathbf{I}_B = 20\angle 90° = j20$$

$$\mathbf{I}_R = -(\mathbf{I}_Y + \mathbf{I}_B) = -j10$$

Also, $\mathbf{V}_{OO} \quad 200 \quad 150 \quad 173 \quad j100$

$$\mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 254 \quad (\; 173 \quad j100) \quad 427 \quad j100 \quad 438.5 \quad 13.2$$

$$\mathbf{V}_{YO} \quad \mathbf{V}_{OO} \quad (\; 127 \quad j220) \quad (\; 173 \quad j100) \quad 46 \quad j320 \quad 323 \quad 81.6$$

$$\mathbf{V}_{BO} \quad \mathbf{V}_{OO} \quad (\; 127 \quad j220) \quad (\; 173 \quad j100) \quad 46 \quad j120 \quad 128.6 \quad 69$$

As seen from Art. 19.32.

$$\mathbf{Z}_R \quad \frac{\mathbf{V}_{RO} \quad \mathbf{V}_{00}}{\mathbf{I}_R} \quad \frac{438.5 \quad 13.2}{10 \quad 90} \quad 43.85 \quad 76.8$$

$$\mathbf{Z}_Y \quad \frac{\mathbf{V}_{YO} \quad \mathbf{V}_{OO}}{\mathbf{I}_Y} \quad \frac{323}{10 \quad 90} \quad 32.3 \quad 8.4$$

$$\mathbf{Z}_B \quad \frac{\mathbf{V}_{BO} \quad \mathbf{V}_{OO}}{\mathrm{I}_B} \quad \frac{128.6 \quad 69}{20 \quad 90} \quad 6.43 \quad 21$$

19.35. Unbalanced Star-connected Non-inductive Load

Such a case can be easily solved by direct geometrical method. If the supply system is symmetrical, the line voltage vectors can be drawn in the form of an equilateral triangle *RYB* (Fig. 19.94). As the load is an unbalanced one, its neutral point will not, obviously, coincide with the centre of the gravity or centroid of the triangle. Let it lie at any other point like *N*. If point *N* represents the potential of the neutral point if the unbalanced load, then vectors drawn from *N* to points, *R*, *Y* and *B* represent the voltages across the branches of the load. These voltages can be represented in their rectangular co-ordinates with respect to the rectangular axis drawn through *N*. It is seen that taking co-ordinates of *N* as (0, 0), the co-ordinates of point *R* are [(*V*/2 – *x*), – *y*]

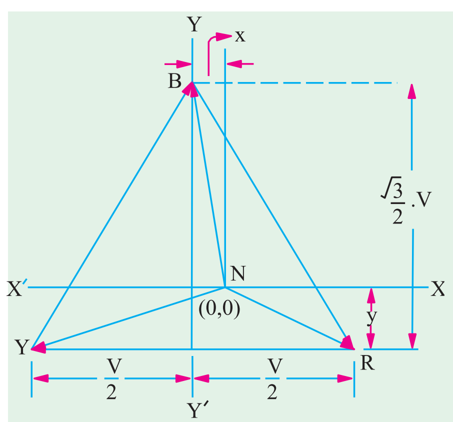


Fig. 19.94

of point *Y* are $[-(V/2+x),-y]$

and of point *B* are $[-x,(\sqrt{3}V/2-y)]$

$$\mathrm{V}_{RN} \quad \frac{V}{2} \quad x \quad jy; \; \mathrm{V}_{YN} \quad \frac{V}{2} \quad x \quad iy$$

$$\mathrm{V}_{BN} \quad x \quad j \quad \frac{\sqrt{3}V}{2} \quad y$$

Let R_1, R_2 and R_3 be the respective branch impedances, Y_1, Y_2 and Y_3 the respective admittances and $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$ the respective currents in them.

Then $\qquad \mathbf{I}_R = \mathbf{V}_{RN} / \mathbf{R}_1 = \mathbf{V}_{RN}\mathbf{Y}_1$

Similarly, $\qquad \mathbf{I}_Y = \mathbf{V}_{YN}\mathbf{Y}_2$ and $\mathbf{I}_B = \mathbf{V}_{BN}\mathbf{Y}_3$. Since $\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$

$\therefore \qquad \mathbf{V}_{RN}\mathbf{Y}_1 + \mathbf{V}_{YN}\mathbf{Y}_2 + \mathbf{V}_{BN}\mathbf{Y}_3 = 0$

$$\text{or } Y_1 \quad \frac{V}{2} \quad x \quad jy \quad Y_2 \quad \frac{V}{2} \quad x \quad jy \quad Y_3 \quad x \quad j\frac{\sqrt{3}V}{2} \quad y \quad 0$$

$$\text{or} \quad x(Y_1 \quad Y_2 \quad Y_3) \quad \frac{V}{2}(Y_1 \quad Y_2) \quad j \quad Y_3\frac{\sqrt{3}V}{2} \quad y(Y_1 \quad Y_2 \quad Y_3) \quad 0$$

$$\therefore \qquad -x(Y_1+Y_2+Y_3)+\frac{V}{2}(Y_1-Y_2)=0 \qquad \therefore \; x = \frac{V(Y_1-Y_2)}{2Y_1+Y_2+Y_3}$$

$$\text{Also} \qquad Y_3\frac{\sqrt{3}V}{2} - y(Y_1+Y_2+Y_3)=0 \qquad \therefore \; y = \frac{\sqrt{3}V}{2}\,\frac{Y_3}{(Y_1+Y_2+Y_3)}$$

Knowing the values of x, the values of $\mathbf{V}_{RN}$, $\mathbf{V}_{YN}$ and $\mathbf{V}_{BN}$ and hence, of $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$ can be found as illustrated by Ex. 19.68.

Example 19.77. *Three non-inductive resistances of 5, 10 and 15 Ω are connected in star and supplied from a 230-V symmetrical 3-phase system. Calculate the line currents (magnitudes).*
(Principles of Elect. Engg. Jadavpur Univ.)

Solution.

(*a*) Star/Delta Conversion Method

The Y-connected unbalanced load and its equivalent Δ-connected load are shown in Fig. 19.95 (*a*) and (*b*) respectively. Using Y/Δ conversion, we have

$$\mathbf{Z}_{12} \quad \frac{\mathbf{Z}_1\mathbf{Z}_2 \quad \mathbf{Z}_2\mathbf{Z}_3 \quad \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \quad \frac{50 \quad 150 \quad 75}{15} \quad \frac{55}{3}$$

$$Z_{23} \quad 275/5 \quad 55 \quad \text{and } Z_{31} \quad 275/10 \quad 27.5$$

Phase current $\mathbf{I}_{RY} = \mathbf{V}_{RY}/\mathbf{Z}_{12}$ = 230/(55/3) = 12.56 A

Similarly, $\mathbf{I}_{YB} = \mathbf{V}_{YB}/\mathbf{Z}_{23}$ = 230/55 = 4.18 A; $\mathbf{I}_{BR} = \mathbf{V}_{BR}/\mathbf{Z}_{31}$ = 230/27.5 = 8.36 A

The line currents for Δ-connection are obtained by compounding the above phase currents trigonometrically or vectorially. Choosing vector addition and taking $\mathbf{V}_{RY}$ as the reference vector, we get;

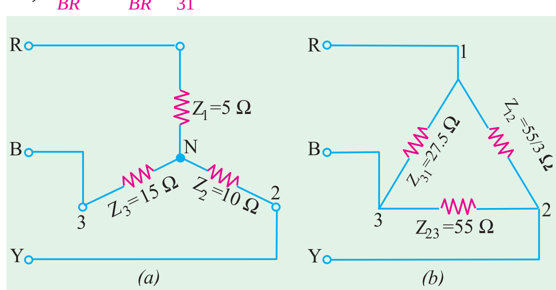


Fig. 19.95

$\mathbf{I}_{RY}$ = (12.56 + j0)

$$\mathbf{I}_{YB} = 4.18 \quad \frac{1}{2} \; j\frac{\sqrt{3}}{2} \quad = -2.09 - j3.62$$

$$\mathbf{I}_{BR} = 8.36 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad = -4.18 + j7.24$$

Hence, line currents for Δ-connection of Fig. 19.95 (*b*) are

$$\mathbf{I}_R = \mathbf{I}_{RY} + \mathbf{I}_{RB} = \mathbf{I}_{RY} - \mathbf{I}_{BR}$$
$$= (12.56 + j0) - (-4.18 + j7.24) = 16.74 - j7.24 \text{ or } 18.25 \text{ A – in magnitude}$$

$$\mathbf{I}_Y = \mathbf{I}_{YR} + \mathbf{I}_{YB} = \mathbf{I}_{YB} - \mathbf{I}_{RY}$$
$$= (-2.09 - j3.62) - (12.56 + j0) = -14.65 - j3.62 \text{ or } 15.08 \text{ A – in magnitude}$$

$$\mathbf{I}_B = \mathbf{I}_{BR} + \mathbf{I}_{BY} = \mathbf{I}_{BR} - \mathbf{I}_{YB}$$
$$= (-4.18 + j7.24) - (-2.09 - j3.62) = -2.09 + j10.86 \text{ or } 11.06 \text{ A –in magnitude}$$

(*b*) Geometrical Method

Here, $R_1 \quad 5 \quad , R_2 \quad 10 \quad$; and $R_3 \quad 15 \quad ; Y_1 \quad 1/5\text{S};\ Y_2 \quad 1/10\text{S};\ Y_3 \quad 1/15\text{S}$

As found above in Art. 19.35 $x = \frac{V}{2}(Y_1 - Y_2)/(Y_1 + Y_2 + Y_3)$

$$= \frac{230}{2} \quad \frac{1}{5} \quad \frac{1}{10} \Big/ \frac{1}{5} \quad \frac{1}{10} \quad \frac{1}{15} \qquad 31.4$$

$$y \qquad \frac{\sqrt{3}V}{2}.Y_3 \Big/ (Y_1 \quad Y_2 \quad Y_3) \quad (\sqrt{3} \quad 115 \quad 1/15)/(11/30) \quad 36.2$$

$$\mathbf{V}_{RN} \qquad \frac{V}{2} \quad x \quad jy \quad (115 \quad 31.4) \quad j36.2 \quad 83.6 \quad j36.2$$

$$\mathbf{V}_{YN} \qquad \frac{V}{2} \quad x \quad jy \qquad 146.4 \quad j36.2$$

$$\mathbf{V}_{BN} \qquad x \quad j \frac{\sqrt{3}V}{2} \quad y \qquad 31.4 \quad j163$$

$$\mathbf{I}_R \quad \mathbf{V}_{RN}\mathbf{Y}_1 \quad (83.6 \quad j36.2) \quad 1/5 \quad 16.72 \quad j7.24$$

$$\mathbf{I}_Y \quad \mathbf{V}_{YN}\mathbf{Y}_2 \quad (\ 146.4 \quad j36.2) \quad 1/10 \qquad 14.64 \quad j3.62$$

$$\mathbf{I}_B \quad \mathbf{V}_{BN}\mathbf{Y}_3 \quad (\ 31.4 \quad j163) \quad 1/15 \qquad 2.1 \quad j10.9$$

These are the same currents as found before.

(*c*) Solution by Millman's Theorem

$\mathbf{Y}_R \quad 1/5 \quad 0\ ;\ \mathbf{Y}_Y \quad 1/10 \quad 0\ ;\ \mathbf{Y}_B \quad 1/15 \quad 0$ and $\mathbf{Y}_R \quad \mathbf{Y}_Y \quad \mathbf{Y}_B \quad 11/30 \quad 0$ Siemens

Let the supply voltages be represented (Fig. 19.96) by

$\mathbf{V}_{RO} \quad 230/\sqrt{3} \quad 0 \qquad 133 \quad 0\ ;\ \mathbf{V}_{YO} \quad 133 \qquad 120\ ;\ \mathbf{V}_{BO} \quad 133 \quad 120$

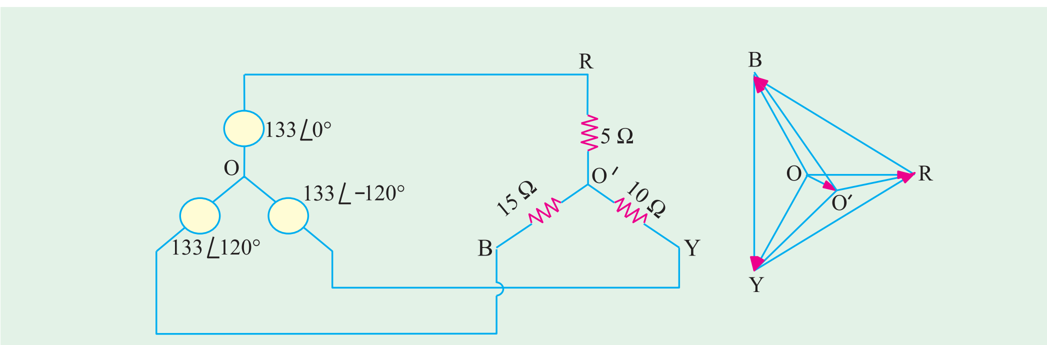


Fig. 19.96

The p.d. between load and supply neutral is

$$\mathbf{V}_{OO} \qquad \frac{133/5 \quad (133/10) \qquad 120 \qquad (133/15) \quad 120}{30/11 \quad 0}$$

$$= 42.3 - j10.4 = 43.6\angle -13.8°$$

$$\mathbf{V}_{RO} \quad 133 \quad (42.3 \quad j10.4) \quad 90.7 \quad j10.4$$

$$\mathbf{V}_{YO} \quad 133 \quad 120 \quad (42.3j \quad 10.4)$$
$$= (-66.5 - j115) - (42.3 - j10.4) = -108.8 - j104.6$$

$$\mathbf{V}_{BO} \quad 133 \;\; 120 \quad V_{OO} \quad (\;66.5 \quad j115) \quad (42.3 \quad j10.4) \quad 108.8 \quad j125.4$$

$$\mathbf{I}_R \quad \mathbf{V}_{RO}\,\mathbf{Y}_R \quad 1/5.(90.7 \quad j10.4) \quad 18.1 \quad j2.1 \text{ or } 18.22 \text{ A in magnitude}$$

$$\mathbf{I}_Y \quad 10.88 \quad j10.5 \text{ or } 15.1 \text{ A in magnitude}$$

$$I_B = -7.5 + j8.4 \text{ or } 11.7 \text{ A in magnitude}$$

Example 19.78. *The unbalanced circuit of Fig. 19.97 (a) is connected across a symmetrical 3-phase supply of 400-V. Calculate the currents and phase voltages. Phase sequence is RYB.*

Solution. The line voltages are represented by the sides of an equilateral triangle *ABC* in Fig. 19.97 (*b*). Since phase impedances are unequal, phase voltages are unequal and are represented by lengths, *NA*, *NB* and *NC* where *N* is the neutral point which is shifted from its usual position. *CM* and *ND* are drawn perpendicular to horizontal side *AB*. Let co-ordinates of point *N* be (0, 0). Obviously, *AM* = *BM* = 200 V, *CM* = $\sqrt{3}$ × 200 *V*, *CM* = $\sqrt{3}$ × 200 = 346 V. Let *DM* = *x* volts and *ND* = *y* volts.

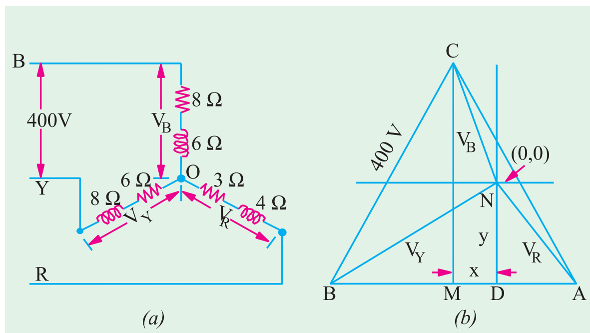


Fig. 19.97

Then, with reference to point *N*, the vector expressions for phase voltages are

$$\mathbf{V}_R \quad (200 \quad x) \quad jy, \quad \mathbf{V}_Y \quad (200 \quad x) \quad jy; \; \mathbf{V}_B \quad x \quad j(346 \quad y)$$

$$\mathbf{I}_R \quad \frac{\mathbf{V}_R}{\mathbf{Z}_R} \quad \frac{(200 \quad x) \quad jy}{3 \quad j4} \quad \frac{3 \quad j4}{3 \quad j4} \quad (24 \quad 0.12x \quad 0.16y) \quad j(\;32 \quad 0.16x \quad 0.12y)$$

$$\mathbf{I}_Y \quad \frac{\mathbf{V}_Y}{\mathbf{Z}_Y} \quad \frac{(200 \quad x) \quad jy}{6 \quad j8} \quad \frac{6 \quad j8}{6 \quad j8}$$
$$= (-12 - 0.06x - 0.08y) + j(16 + 0.08x - 0.06y)$$

$$\mathbf{I}_B \quad \frac{\mathbf{V}_B}{\mathbf{Z}_B} \quad \frac{x \quad j(346 \quad y)}{8 \quad j6} \quad \frac{8 \quad j6}{8 \quad j6}$$
$$= (20.76 - 0.08x - 0.06y) + j(27.68 + 0.06x - 0.08y)$$

Now, $\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$

$\therefore \quad (32.76 - 0.26x - 0.3y) + j(11.68 + 0.3x - 0.26y) = 0$

Obviously, the real component as well as the *j*-component must be zero.

$\therefore \quad 32.76 - 0.26 \times -0.3y = 0$ and $11.68 + 0.3x - 0.26y = 0$

Solving these equations for *x* and *y*, we have *x* = 31.9 V and *y* = 81.6 V

$$\mathbf{V}_R \quad (200 \quad 31.9) \quad j81.6 \quad 168 \quad j81.6 \quad 186.7 \quad 25.9$$
$$\mathbf{V}_Y \quad (200 \quad 31.9) \quad j81.6 \quad 231.9 \quad j81.6 \quad 245.8 \;\; 199.4$$
$$\mathbf{V}_B \quad 31.9 \quad j(346 \quad 81.6) \quad 31.9 \quad j264.4 \quad 266.3 \;\; 83.1$$

Substituting these values of *x* and *y* in the expressions for currents, we get

$\mathbf{I}_R \quad (24 \quad 0.12 \quad 31.9 \quad 0.16 \quad 81.6) \quad j(\; 32 \quad 0.16 \quad 31.9 \quad 0.12 \quad 81.6)$
$= 7.12 - j36.7$

Similarly $\mathbf{I}_Y \quad 20.44 \quad j13.65; I_B \quad 13.3 \quad j23.06$

$\Sigma I = (0.+j0)$ – as a check

Example 19.79. *A $3-\phi$, 4-wire, 400-V symmetrical system supplies a Y-connected load having following branch impedances:*

$$Z_R = 100\Omega, Z_Y = j10\,\Omega \text{ and } Z_B = -j10\Omega$$

Compute the values of load phase voltages and currents and neutral current. Phase sequence is RYB.

How will these values change in the event of an open in the neutral wire?

Solution. (*a*) When Neutral Wire is Intact. [Fig 19.98 (*a*)]. As discussed in Art. 19.30, the *load* phase voltages would be the same as *supply* phase voltages despite imbalance in the load. The three load phase voltages are:

$$\mathbf{V}_R \quad 231 \quad 0 \,, \mathrm{V}_Y \quad 231 \quad 120 \text{ and } \mathrm{V}_B \quad 231 \quad 120$$

$$\mathbf{I}_R \quad 231 \quad 0 \;/100 \quad 0 \quad 2.31 \quad 0 \quad 2.31 \quad j0$$

$$\mathbf{I}_Y \quad 231 \quad 120 \;/10 \quad 90 \quad 2.31 \quad 210 \quad 20 \quad j11.5$$

$$\mathbf{I}_B \quad 231 \quad 120 \;/10 \quad 90 \quad 23.1 \quad 210 \quad 20 \quad j11.5$$

$$\mathbf{I}_N \quad (\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad (2.31 \quad 20 \quad j11.5 \quad 20 \quad j11.5) \quad 37.7\,\mathrm{A}$$

(*b*) When Neutral is Open [Fig.19.98 (*b*)]

In this case, the load phase voltages will be no longer equal. The node pair voltage method will be used to solve the question. Let the supply phase voltages be given by

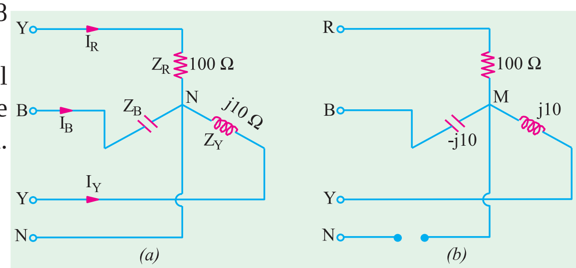


Fig. 19.98

$\mathbf{E}_R \quad 231 \quad 0 \,, \mathbf{E}_Y \quad 231 \quad 120$
$= -115.5 - j200$

$\mathbf{E}_B = 231\angle 120° = -115\cdot 5 + j200$

$\mathbf{Y}_R \quad 1/100 \quad 0.01; \; \mathbf{Y}_Y \quad 1/\, j10 \quad j0.1 \text{ and } \mathbf{Y}_B \quad 1/ \quad j10 \quad j0.1$

$$\mathbf{V}_{NN} = \frac{231\times 0.01+(-j0.1)(-115.5-j200)+j0.1(-115.5+j200)}{0.01+(-j0.1)+j0.1} = -3769+j0$$

The load phase voltages are given by

$$\boldsymbol{V}_R \quad \boldsymbol{E}_R \quad \boldsymbol{V}_{NN} \quad (231 \quad j0) \quad (\; 3769 \quad j0) = 4000 \text{ V}$$

$$\mathbf{V}_Y \quad \mathbf{E}_Y \quad \mathbf{V}_{NN} \quad 115.5 \quad j200 \quad (\; 3769 \quad j0) \quad (3653.5 \quad j200)$$

$$\mathbf{V}_B \quad \mathbf{E}_B \quad \mathbf{V}_{NN} \quad 115.5 \quad j200 \quad (\; 3769 \quad j0) \quad (3653.5 \quad j0)$$

$$\mathbf{I}_R \quad \mathbf{V}_R\,\mathbf{Y}_R \quad 4000 \quad 0.01 \quad 40\,\mathrm{A}$$

$$\mathbf{I}_Y \quad (\; j0.1)(3653.5 \quad j200) \quad (20 \quad j3653.5)$$

$$\mathbf{I}_B \quad (j0.1)(3653.5 \quad j200) \quad 20 \quad j3653.5$$

Obviously, the neutral current will just not exist.

Note. As hinted in Art. 19.30 (*i*), the load phase voltages and currents become abnormally high.

Example 19.80. *For the circuit shown in Fig. 19.99 find the readings on the two wattmeters W_a and W_c.*

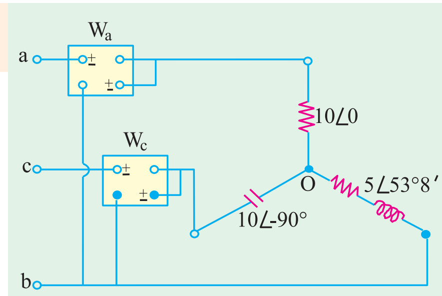


Fig. 19.99

Solution. The three line currents for this problem have already been determined in Example 19.43.

$$\mathbf{I}_{ao} = 20.02 - j4.54$$

$$\mathbf{I}_{bo} = -7.24 - j5.48$$

$$\mathbf{I}_{co} \quad 12.78 \quad j10.12$$

The line voltages are given by

$$\mathbf{V}_{ab} \quad 200 \quad j0$$

$$\mathbf{V}_{bc} = -100 - j\,173.2$$

$$\mathbf{V}_{ca} = -100 + j\,173.2$$

Wattmeter W_a carries a current of using $\mathbf{I}_{ao}$ = 20.02 – j 4.54 and has voltage $\mathbf{V}_{ab}$ impressed across its pressure coil. Power can be found by using current conjugate.

$\mathbf{P}_{VA}$ = (200 + j 0) (20.02 + j 4.54) = (200) (20.02) + j (200)(4.54)

Actual power = 200 × 20.02 = 4004 W ∴ W_a = **4004 W**

The other wattmeter W_c carries current of $\mathbf{I}_{co}$ = – 12.78 + j 10.02 and has a voltage $\mathbf{V}_{cb}$ = – $\mathbf{V}_{bc}$ = 100 + j 173.2 impressed across it. By the same method, wattmeter reading is

$$W_c = (100 \times -12.78) + (173.2 \times 10.02) = -1278 + 1735.5 = \mathbf{457.5\ W}$$

Example 19.81. *Three resistors 10, 20 and 20 Ω are connected in star to the terminals A, B and C of a $3-\phi$, 3 wire supply through two single-phase wattmeters for measurement of total power with current coils in lines A and C and pressure coils between A and B and C and B. Calculate **(i)** the line currents **(ii)** the readings of each wattmeter.*
The line voltage is 400-V. **(Electrical Engineering-I, Bombay Univ.)**

Solution. Let $\mathbf{V}_{AB}$ 400 0 ; $\mathbf{V}_{BC} = 400\angle -120°$ and $\mathbf{V}_{CA}$ 400 120

As shown in Fig. 19.100, current through wattmeter W_1 is $\mathbf{I}_{AO}$ or $\mathbf{I}_A$ and that through W_2 is $\mathbf{I}_{CO}$ or $\mathbf{I}_C$ and the voltages are $\mathbf{V}_{AB}$ and $\mathbf{V}_{CB}$ respectively. Obviously,

$$\mathbf{Z}_A = 10\angle 0°; \mathbf{Z}_B = 20\angle 0°, \mathbf{Z}_C = 20\angle 0°$$

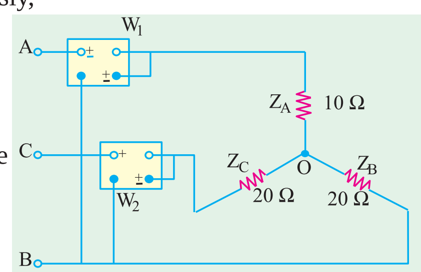


Fig. 19.100

The currents $\mathbf{I}_A$ and $\mathbf{I}_C$ may be found by applying either Kirchhoff's laws (Art. 19.33) or Maxwell's Mesh Method. Both methods will be used for illustration.

(a) From Eq. (10), (11) and (12) of Art. 19.33, we have

$$\mathbf{I}_A = \frac{400 \times 20 - 20(-200 + j346)}{(10 \times 20) + (20 \times 20) + (20 \times 10)}$$

$$= \frac{12{,}000 - j6{,}920}{800} = 15 - j8.65\,\text{A}$$

$$\mathbf{I}_C \quad \frac{20(\quad 200 \quad j346) \quad 10(\quad 200 \quad j346)}{800} \quad \frac{2000 \quad j10{,}380}{800} \quad 2.5 \quad j13$$

(b) From Eq. **(i)** and **(ii)** of solved example 17.48 (c) we get

$$\mathbf{I}_A \quad \mathbf{I}_1 \quad \frac{400 \quad 40 \quad 20(\quad 200 \quad j346)}{30 \quad 40 \quad 20^2} \quad 15 \quad j8.65\,\text{A}$$

$$\mathbf{I}_C \quad \mathbf{I}_2 \quad \frac{30 \quad (\quad 200 \quad j346) \quad 400 \quad 20}{800} \quad 25 \quad j13$$

As seen, wattmeter W_1 carries current $\mathbf{I}_A$ and has a voltage $\mathbf{V}_{AB}$ impressed across its pressure coil. Power may be found by using voltage conjugate.

$$\mathbf{P}_{VA} \quad (400 \quad j0)(15 \quad j8.65) \quad 6000 \quad j3{,}460$$

∴ reading of W_1 = 6000 W = **6 kW**

Similarly, W_2 carries $\mathbf{I}_C$ and has voltage $\mathbf{V}_{CB}$ impressed across its pressure coil.

Now, $\mathbf{V}_{CB} \quad \mathbf{V}_{BC} \quad (200 \quad j346)$. Using voltage conjugate, we get

$\mathbf{P}_{VA}$ (200 j346)(2.5 j13)

Real power = (200 × – 2.5) + (13 × 346) = 4000 W

∴ reading of W_2 = **4kW;** Total power = 10 kW

Example 19.82. *Three impedances Z_A, Z_B and Z_C are connected in delta to a 200-V, 3-phase three-wire symmetrical system RYB.*

Z_A = 10 ∠ 60° between lines R and Y ; Z_B = 10 ∠ 0° between lines Y and B

Z_C = 10 ∠ 60° between lines B and R

The total power in the circuit is measured by means of two wattmeters with their current coils in lines R and B and their corresponding pressure coils across R and Y and B and Y respectively. Calculate the reading on each wattmeter and the total power supplied. Phase sequence RYB.

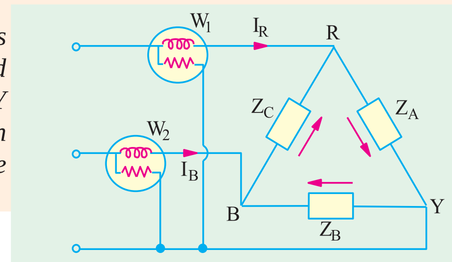


Fig. 19.101

Solution. The wattmeter connections are shown in Fig. 19.101.

$\mathbf{V}_{RY}$ 200 0 200 j0

$\mathbf{V}_{YB}$ 200 120 100 j173.2

$\mathbf{V}_{BR}$ 200 120 100 j173.2

$$\mathbf{I}_{BR} \quad \frac{200 \quad 0}{10 \quad 60} \quad 20 \quad 60 \quad 10 \quad j17.32$$

$$\mathbf{I}_{YB} \quad \frac{200 \quad 120}{10 \quad 0} \quad 20 \quad 120$$

$$= -10 - j17.32; \mathbf{I}_{BR} = \frac{200\angle 120°}{10\angle 60°} = 20\angle 60° = 10 + j17.32$$

As seen, current through W_1 is $\mathbf{I}_R$ and voltage across its pressure coil is $\mathbf{V}_{RY}$.

$\mathbf{I}_R$ $\mathbf{I}_{RY}$ $\mathbf{I}_{BR}$ j34.64 A

Using voltage conjugate, we have

$\mathbf{P}_{VA}$ (200 j0)(j34.64) 0 j6,928

Hence, W_1 reads zero.

Current through W_2 is $\mathbf{I}_B$ and voltage across its pressure coil is $\mathbf{V}_{BY}$

$\mathbf{I}_B$ $\mathbf{I}_{BR}$ $\mathbf{I}_{YB}$ 20 j34.64; $\mathbf{V}_{BY}$ $\mathbf{V}_{YB}$ 100 j173.2

Again using voltage conjugate, we get

$\mathbf{P}_{YA}$ (100 j173.2)(20 j34.64)

= 8000 + j0

∴ reading of W_2 = **8000 W**

19.36. Phase Sequence Indicators

In unbalanced 3-wire star-connected loads, phase voltages change considerably if the phase sequence of the supply is reversed. One or the other load phase voltage becomes dangerously large which may result in damage to the equipment. Some phase voltage becomes too

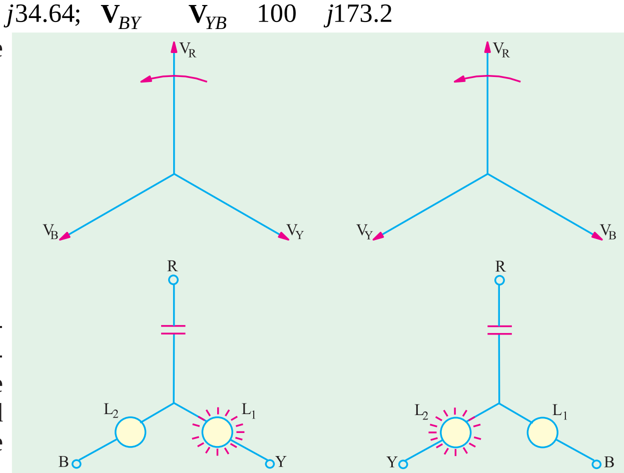


Fig. 19.102

small which is equally deterimental to some types of electrical equipment. Since phase voltage depends on phase sequence, this fact has been made the basis of several types of phase sequence indicators.* A simple phase sequence indicator may be made by connecting two suitable incandescent lamps and a capacitor in a *Y*-connection as shown in Fig. 19.102. It will be found that for phase sequence *RYB*, lamp L_1 will glow because its phase voltage will be large whereas L_2 will not glow because of low voltage across it.

When, phase sequence is *RBY*, opposite conditions develop so that this time L_2 glows but not L_1.

Another method of determining the phase sequence is by means of a small 3-phase motor. Once direction of rotation with a known sequence is found, the motor may be used thereafter for determining an unknown sequence.

Tutorial Problem No. 19.3

1. Three impedances $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ are mesh-connected to a symmetrical 3-phase, 400-V, 50-Hz supply of phase sequence $R \to Y \to B$.

$\mathbf{Z}_1$ = (10 + *j*0) ohm– between *R and* Y *lines*
$\mathbf{Z}_2$ = (5 + *j*6) ohm – between *Y* and *B* lines
$\mathbf{Z}_3$ = (5 – *j*5) ohm – between *B* and *R* lines

Calculate the phase and line currents and total power consumed.

[40 A, 40 A, 56.6 A ; 95.7 A, 78.4 A, 35.2 A; 44.8 kW]

2. A symmetrical $3\text{-}\phi$, 380-V supply feeds a mesh-connected load as follows :

Load *A* : 19 kVA at p.f. 0.5 lag ; Load *B* : 20 kVA at p.f. 0.8 lag : Load *C* : 10 kVA at p.f. 0.9 load Determine the line currents and their phase angles for *RYB* sequence.

[74.6 $\angle -51°$ A, 98.6 $\angle 172.7°$ A ; 68.3 $\angle 41.8°$ A]

3. Determine the line currents in an unbalanced *Y* connected load supplied from a symmetrical $3\text{-}\phi$, 440-V, 3-wire system. The branch impedances of the load are : $\mathbf{Z}_1 = 5 \angle 30°$ ohm, $\mathbf{Z}_2 = 10 \angle 45°$ ohm and $\mathbf{Z}_3 = 10 \angle 45°$ ohm and $\mathbf{Z}_4 = 10 \angle 60°$ ohm. The sequence is *RYB*. **[35.7 A, 32.8 A; 27.7 A]**

4. A $3\text{-}\phi$, *Y*-connected alternator supplies an unbalanced load consisting of three impedances (10 + *j*20), (10 – *j*20) and 10 Ω respectively, connected in star. There is no neutral connection. Calculate the voltage between the star point of the alternator and that of the load. The phase voltage of the alternator is 230 V. **[–245.2 V]**

5. Non-reactive resistors of 10, 20 and 25 Ω are star-connected to the *R*, *Y* are *B* phases of a 400-V, symmetrical system. Determine the current and power in each resistor and the voltage between star point and neutral. Phase sequence, *RYB*. **[16.5 A, 2.72 kW ; 13.1 A, 3.43 kW; 11.2 A, 3.14 kW ; 68 V]**

6. Determine the line current in an unbalanced, star-connected load supplied from a symmetrical 3-phase, 440-V system. The branch impedance of the load are $\mathbf{Z}_R = 5 \angle 30°\ \Omega$, $Z_Y = 10 \angle 45°\ \Omega$ and $Z_B = 10 \angle 60°\ \Omega$. The phase sequence is *RYB*. **[35.7 A, 32.8 A, 27.7 A]**

7. Three non-reactive resistors of 3, 4 and 5-Ω respectively are star-connected to a 3-phase, 400-V symmetrical system, phase sequence *RYB*. Find (*a*) the current in each resistor (*b*) the power dissipated in each resistor (*c*) the phase angles between the currents and the corresponding line voltages (*d*) the star-point potential. Draw to scale the complete vector diagram.

[(*a*) 66.5 A, 59.5 A, 51.8 A (*b*) 13.2, 14.15, 13.4 kW (*c*) 26°24′, 38°10′, 25°20 (*d*) 34 V]

8. An unbalanced *Y*-connected load is supplied from a 400-V, $3\text{-}\phi$, 3-wire symmetrical system. The branch circuit impedances and their connection are (2 + *j*2) Ω, *R* to *N* ; (3 – *j*3) Ω, *Y* to *N* and (4 + *j*1) Ω, *B* to *N* of the load. Calculate (*i*) the value of the voltage between lines *Y* and *N* and (*ii*) the phase of this voltage relative to the voltage between line *R* and *Y*. Phase sequence *RYB*.

[(*i*) (–216–j 135.2) or 225.5 V (*ii*) 2° or –178°]

9. A star-connection of resistors $R_a = 10\ \Omega$; $R_b = 20\ \Omega$ is made to the terminals *A*, *B* and *C* respectively of a symmetrical 400-V, ϕ supply of phase sequence $A \to B \to C$. Find the branch voltages and currents and star-point voltage to neutral.

* It may, however, be noted that phase sequence of *currents* in an unbalanced load is not necessarily the same as the *voltage* phase sequence. Unless indicated otherwise, voltage phase sequence is implied.

[V_A = 148.5 + *j*28.6 ; I_A = 14.85 + j2.86 ; V_B = – 198 – j171.4 ; I_B = – 9.9 – j8.57
V_C = – 198 + j228.6 ; I_C = –4.95 + j5.71. V_N = 82.5 – j28.6 (to be subtracted from supply voltage)]

10. Three non-reactive resistance of 5, 10 and 5 ohm are star-connected across the three lines of a 230-V 3-phase, 3-wire supply. Calculate the line currents.

[(18.1 + J21.1) A ; (– 10.9–j 10.45) A ; (– 7.3 + j8.4) A]

11. A 3-ϕ, 400-V symmetrical supply feeds a star-connected load consisting of non-reactive resistors of 3, 4 and 5 Ω connected to the *R*, *Y* and *B* lines respectively. The phase sequence is *RYB*. Calculate (*i*) the load star point potential (*ii*) current in each resistor and power dissipated in each resistor.

[(*i*) 34.5 V (*ii*) 66.4 A, 59.7 A, 51.8 A (*iii*) 13.22 kW, 14.21 kW, 13.42 kW]

12. A 20-Ω resistor is connected between lines *R* and *Y*, a 50-Ω resistor between lines *Y* and *B* and a 10-Ω resistor between lines *B* and *R* of a 415-V, 3-phase supply. Calculate the current in each line and the reading on each of the two wattmeters connected to measure the total power, the respective current coils of which a connected in lines *R* and *Y*. **[(25.9 – j9); (– 24.9 – j7.2); (– 1.04 + j16.2); 8.6 kW; 7.75 kW]**

13. A three-phase supply, giving sinusoidal voltage of 400 V at 50 Hz is connected to three terminals marked *R*, *Y* and *B*. Between *R* and *Y* is connected a resistance of 100 Ω, between *Y* and *B* an inductance of 318 mH and negligible resistance and between *B* and *R* a capacitor of 31.8 μF. Determine (*i*) the current flowing in each line and (*ii*) the total power supplied. Determine (*iii*) the resistance of each phase of a balanced star-connected, non-reactive load, which will take the same total power when connected across the same supply. **[(*i*) 7.73 A, 7.73 A, (*ii*) 1,600 W (*iii*) 100 Ω (*London Univ.*)]**

14. An unbalanced, star-connected load is fed from a symmetrical 3-phase system. The phase voltages across two of the arms of the load are $V_B = 295\ \angle 97°30'$ and $V_R = 206\ \angle -25°$. Calculate the voltage between the star-point of the load and the supply neutral. **[52.2 $\angle$–49.54']**

15. A symmetrical 440-V, 3-phase system supplies a star-connected load with the following branch impedances: $Z_R = 100\ \Omega$, $Z_Y = j\,5\ \Omega$, $Z_B = -j5\ \Omega$. Calculate the voltage drop across each branch and the potential of the neutral point to earth. The phase sequence is *RYB*. Draw the vector diagram.

$[8800\ \angle -30°,\ 8415\angle -31.5°,\ 8420 - \angle -28.5°,\ 8545\angle 150°]$

16. Three star-connected impedances, Z_1 = (20 + *j*37.7) Ω per phase are in parallel with three delta-connected impedances, Z_2 = (30 – *j*159.3) Ω per phase. The line voltage is 398 V. Find the line current, power factor, power and reactive volt-amperes taken by the combination.

[3.37 $\angle$10.4°; 0.984 lag; 2295 lag; 2295 W; 420 VAR.]

17. A 3-phase, 440-V, delta-connected system has the loads: branch *RY*, 20 KW at power factor. 1.0: branch *YB*, 30 kVA at power factor 0.8 lagging; branch *BR*, 20 kVA at power factor 0.6 leading. Find the line currents and readings on watt-meters whose current coils are in phases *R* and *B*.

[90.5 $\angle$176.5°; 111.4 $\angle$14°; 36.7 $\angle$– 119; °39.8 kW; 16.1 kW]

18. A 415 V, 50 Hz, 3-phase supply of phase sequence *RYB* is connected to a delta connected load in which branch *RY* consists of $R_1 = 100\ \Omega$, branch Y_B consists of $R_2 = 20\ \Omega$ in series with $X_2 = 60\ \Omega$ and branch *BR* consists of a capacitor *C* = 30 μF. Take V_{RY} as the reference and calculate the line currents. Draw the complete phasor diagrams. ***(Elect. Machines, A.M.I.E. Sec. B, 1989)***

[I_R = 7.78 $\angle$14.54°, I_Y = 10.66 $\angle$172.92°, I_B = 4.46 $\angle$– 47°]

19. Three resistances of 5, 10 and 15 Ω are connected in delta across a 3-phase supply. Find the values of the three resistors, which if connected in star across the same supply, would take the same line currents.

If this star-connected load is supplied from a 4-wire, 3-phase system with 260 V between lines, calculate the current in the neutral. **[2.5 Ω, 1.67 Ω, 5 Ω; 52 A] (*London Univ.*)**

20. Show that the power consumed by three identical phase loads connected in delta is equal to three times the power consumed when the phase loads are connected in star.

(Nagpur University, Summer 2002)

21. Prove, that the power consumed in balanced three- phase Delta-connected load is three times the power consumed in starconnected load. ***(Nagpur University, Winter 2002)***

22. A three-phase 230 volts systems supplies a total load of 2000 watts at a line current of 6 Amp when three identical impedances are in star-connection across the line terminals of the systems. Determine the resistive and reactive components of each impedance. *(Nagpur University, Winter 2002)*

23. Three simila coils each of impedance z = (8 + j10) ohms, are connected in star and supplied from 3-phase 400V, 50 Hz supply. Find the line current, power factor, power and total volt amperes. *(Nagpur University, Summer 2003)*

24. Three similar each having a resistance of 20 ohm and an inductance of 0.05 H are connected in star to a 3-phase 50 Hz supply with 400 V between lines. Calculate power factor, total power absorb and line current. If the same coil are reconnected in delta across the same supply what will be the power factor, total power absorbed and line current? *(Pune University 2003) (Nagpur University, Winter 2003)*

25. A 3 ϕ star connected load when supplied from 440 V, 50 Hz source takes a line current of 12 amp lagging w.r.t. line voltage by 70^0.Calculate : (*i*) limpedance parameters (*ii*) Power factor and its nature (*iii*) Draw phasor diagram indicating all voltages and currents. *(Nagpur University, Summer 2004)*

26. Derive the relationship between line current and phase current for Delta connected 3 phase load when supplied from 3 phase balanced supply. *(Nagpur University, Summer 2004)*

27. Derive the relationship between line voltage, phase voltage, line current and phase current in a 3 phase star connected and delta connected circuit. *(Gujrat University, June/July 2003)*

28. show that power input to a 3 phase circuit can be measured by two wattmeters connected properly in the circuit. Draw vector diagram. *(Gujrat University, June/July 2003)*

29. A balanced 3 phase star connected load of 100 kW takes a leading current of 100 A when connected across a 3 phase, 1100 V, 50 Hz supply. Calculate the circuit constants of the load per phase. *(Mumbai University 2003) (Gujrat University, June/July 2003)*

30. Establish relationship between line and phase voltages and currents in a balanced 3-phase star connec tion. Draw complete phasor diagram for voltages and currents. *(R.G.P.V. Bhopal University, June 2004)*

31. A delta connected load has the following impedances :
$Z_{RY} = j\ 10\ \Omega$, $Z_{YB} = 10 \angle 0^o\ \Omega$ and $Z_{BR} = -j\ 10\ \Omega$. If the load is connected across 100 volt balanced 3-phase supply, obtain the line currents. *(R.G.P.V. Bhopal University, June 2004)*

32. Two wattmeters ω_1 and ω_2 are used to measure power in a 3 phase balanced circuit. Mention the conditions under which (*i*) $\omega_1 = \omega_2$ (*ii*) $\omega_2 = 0$ (*iii*) $\omega_1 = 2\omega_2$. *(V.T.U. Belgaum Karnataka University, February 2002)*

33. Three coils each of impedance 20 | $60^0 \Omega$ are connected across a 400V, 3 phase supply. Find the reading of each of the two wattmeters connected to measure the power when the coils are connected in (*i*) star (*ii*) Delta. *(V.T.U. Belgaum Karnataka University, February 2002)*

34. The power input to a 3 phase circuit was measured by two wattmeter method and the readings were 3400 and - 1200 watts respectively. Calculate the total power and powerfactor. *(V.T.U. Belgaum Karnataka University, July/August 2002)*

35. With the help of connection diagram and vector diagram, obtain expressions for the two wattmeter readings used to measure power in a 3 phase the DC generator is running. *(V.T.U. Belgaum Karnataka University, July/August 2002)*

36. Obtain the relationship between line and phase values of current in a three phase, balanced, delta connected system. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

37. Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

38. Each of the two wattmeters connected to measure the input to a three phase circuit, reads 20kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three phase power remaining unchanged in the altered condition? *(V.T.U. Belgaum Karnataka University, January/February 2003)*

39. Two wattmeters connected to measure power in a 3 phase circuit read 5KW and 1KW, the latter reading being obtained after reversing current coil connections. Calculate power factor of the load and the total power consumed. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

39. Derive the relationship between phase and line values of voltages in a connected load. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

40. Three coils each of impedance $20\angle 60^0 \Omega$ are connected in delta across a 400, 3 phase, 50Hz, 50Hz Acsupply. Calculate line current and total power. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

41. What are the advantages of a three phase system over a single phase system? *(V.T.U. Belgaum Karnataka University, July/August 2003)*

42. With a neat circuit diagram and a vector diagram prove that two wattmeters are sufficient to measure total power in a 3 phase system. *(V.T.U. Belgaum Karnataka University, July/August 2003)*

43. A balanced star connected load of (8 + *jb*)Ω is connected to a 3 phase, 230 V supply. Find the line current, power factor, power, reactive voltamperes and total voltamperes. ***(V.T.U. Belgaum Karnataka University, July/August 2003)***

44. Two watt meters are used to measure the power delivered to a balance 3 phase load of power factor 0.281. One watt meter reads 5.2kW. Determine the reading of the second watt meter. What is the line current if the line voltage is 415 wolt? ***(V.T.U. Belgaum Karnataka University, January/February 2004)***

45. Write the equations for wattmeter reading W_1 and W_2 in 3 phase power measurement and therefrom for power factor. ***(Anna University, October/November 2002)***

46. Show that the wattmeters will read equal in two wattmeter method under unity power factor loading condition. ***(Anna University, November/December 2003)***

47. A star connected 3-phase load has a resistance of 6Ω and an inductive reactance of 8Ω in each brance. Line voltage is 220 volts. Write the phasor expressions for voltage across each branch, line voltages and line currents. Calculate the total power. ***(Anna University, November/December 2003)***

48. Two wattmeters connected to measure the total power in a 3-phase balanced circuit. One measures 4,800 W, while the other reads backwards. On reversing the latter it is found to read 400 W. What is the total power and power and power factor? Draw the connection diagram and phasor diagram of the circuit. ***(Mumbai University 2003) (RGPV Bhopal 2001)***

49. A star-network in which N is star point made up as follows :

AN = 70Ω, CN = 90 Ω. Find an equivalent delta network. If the above star-delta network are superimposed, what would be measured resistance between A and C? ***(Pune University, 2003) (RGPV Bhopal 2001)***

50. Explain with diagram the measurement of 3-phase power by two-wattmeter method. ***(RGPV Bhopal 2002)***

51. Show that the power taken by a 3-phase circuit can measured by two wattmeters connected properly in the circuit. ***(RGPV Bhopal)***

52. With the aid of star-delta connection diagram, state the basic equation from which star-delta conversionequation canbe derived. ***(Pune University, 2003) (RGPV Bhopal 2001)***

53. Star-delta connections in a 3-phase supply and their inter-relationship. ***(RGPV Bhopal 2001)***

54. Measurement of power in three-phase circuit in a balanced condition. ***(RGPV Bhopal 2001)***

55. Measurement of reactive power in three-phase circuit. ***(RGPV Bhopal 2001)***

56. Differentiate between balanced and unbalancedthree-phase supply and balanced and unbalanced three-phase load. ***(RGPV Bhopal June 2002)***

57. A 3-phase 3 wire supply feeds a load consisting of three equal resistors. By how much is the load reduced if one of the resistors be removed ? ***(RGPV Bhopal June 2002)***

58. Establish relationship between line and phase voltages and currents in a balanced delta connection. Draw complete phasor diagram of voltages and currents. ***(RGPV Bhopal December 2003)***

OBJECTIVE TESTS – 19

1. The minimum number of wattmeter (s) required to measure 3-phase, 3-wire balanced or unbalanced power is

(*a*) 1
(*b*) 2
(*c*) 3
(*d*) 4

(GATE 2001)

2. A wattmeter reads 400 W when its current coil is connected in the R phase and its pressure coil is connected between this phase and the neutral of a symmetrical 3-phase system supplying a balanced star connected 0.8 p.f. inductive load. The phase sequence is RYB. What will be the reading of this wattmeter if its pressure coil alone is reconnected between the B and Y phases, all other connections remaining as before?

(*a*) 400.0 (*b*) 519.6
(*c*) 300.0 (*d*) 692.8

(GATE 2003)

3. Total instantaneous power supplied by a 3-phase ac supply to a balanced R-L load is

(*a*) zero
(*b*) constant
(*c*) pulsating with zero average
(*d*) pulsating with non-zero average

(GATE 2004)

4. A balanced 3-phase, 3-wire supply feeds balanced star connected resistors. If one of the resistors is disconnected, then the percentage reduction in the load will be

(*a*) $33\frac{1}{3}$ (*b*) 50
(*c*) $66\frac{2}{3}$ (*d*) 75 ***(GATE)***

Top