Contents

CHAPTER 16

# A.C. BRIDGES

## Learning Objectives

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A wide variety of AC bridge circuits (such as wheatstone) may be used for the precision measurement of AC resistance, capacitance and inductance

## 16.1. A.C. Bridges

Resistances can be measured by direct-current Wheatstone bridge, shown in Fig. 16.1 (*a*) for which the condition of balance is that

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} \quad \text{or} \quad R_1 R_3 \quad R_2 R_4\ ^{*}$$

Inductances and capacitances can also be measured by a similar four-arm bridge, as shown in Fig. 16.1 (*b*); instead of using a source of direct current, alternating current is employed and galvanometer is replaced by a vibration galvanometer (for commercial frequencies or by telephone detector if frequencies are higher (500 to 2000 Hz)).

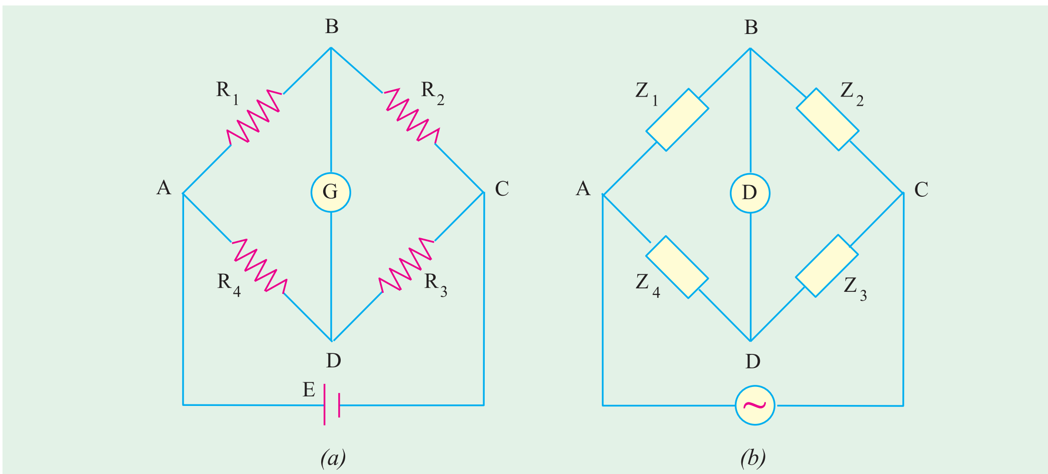



**Fig. 16.1**

The condition for balance is the same as before but instead of resistances, impedances are used *i.e.*

$$\mathbf{Z_1 / Z_2 = Z_4 / Z_3} \quad \text{or} \quad \mathbf{Z_1 Z_3 = Z_2 Z_4}$$

But there is one important difference *i.e.* not only should there be balance for the magnitudes of the impedances but also a phase balance. Writing the impedances in their polar form, the above condition becomes

$$Z_1\angle\phi_1 . Z_3\angle\phi_3 = Z_2\angle\phi_2 . Z_4\angle\phi_4 \text{ or } Z_1 Z_3 \angle\phi_1 + \phi_3 = Z_2 Z_4 \angle\phi_2 + \phi_4$$

Hence, we see that, in fact, there are two balance conditions which must be satisfied simultaneously in a four-arm a.c. impedance bridge.

(*i*) $Z_1 Z_3 = Z_2 Z_4$ ... for magnitude balance

(*ii*) $\phi_1 + \phi_3 = \phi_2 + \phi_4$ ... for phase angle balance

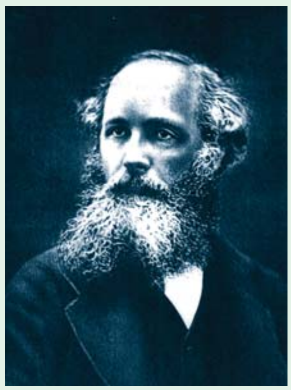
In this chapter, we will consider a few of the numerous bridge circuits used for the measurement of self-inductance, capacitance and mutual inductance, choosing as examples some bridges which are more common.

## 16.2. Maxwell's Inductance Bridge

The bridge circuit is used for medium inductances and can be arranged to yield results of considerable precision. As shown in Fig. 16.2, in the two arms, there are two pure resistances so

* Products of opposite arm resistances are equal.

that for balance relations, the phase balance depends on the remaining two arms. If a coil of an unknown impedance $Z_1$ is placed in one arm, then its positive phase angle $\phi_1$ can be compensated for in either of the following two ways:



James Clark Maxwell

(*i*) A known impedance with an equal positive phase angle may be used in either of the *adjacent* arms (so that $\phi_1 = {}_3$ or ${}_2 = \phi_4$ ), remaining two arms have zero phase angles (being pure resistances). Such a network is known as Maxwell's a.c. bridge or $L_1/L_4$ bridge.

(*ii*) Or an impedance with an equal *negative* phase angle (*i.e.* capacitance) may be used in *opposite* arm (so that $\phi_1 + \phi_3 = 0$). Such a network is known as Maxwell-Wien bridge (Fig. 16.5) or Maxwell's L/C bridge.

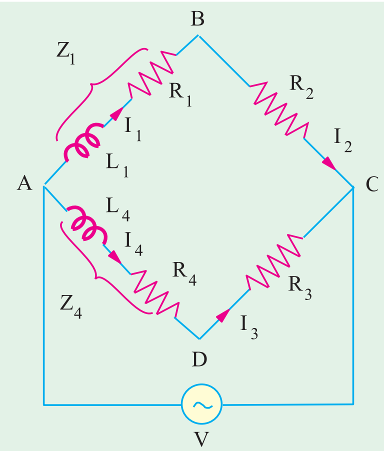



**Fig. 16.2**

Hence, we conclude that an inductive impedance may be measured in terms of another inductive impedance (of equal time constant) in either ***adjacent*** arm (Maxwell bridge) or the unknown inductive impedance may be measured in terms of a combination of resistance and capacitance (of equal time constant) in the ***opposite*** arm (Maxwell-Wien bridge). It is important, however, that in each case the time constants of the two impedances must be matched.

As shown in Fig. 16.2,

$Z_1 = R_1 + jX_1 = R_1 + j\omega L_1$ ... unknown; $\mathbf{Z_4} = R_4 + jX_4 = R_4 + j\omega L_4$ ...known

$R_2, R_3$ = known pure resistances; $D$ = detector

The inductance $L_4$ is a variable self-inductance of constant resistance, its inductance being of the same order as $L_1$. The bridge is balanced by varying $L_4$ and one of the resistances $R_2$ or $R_3$. Alternatively, $R_2$ and $R_3$ can be kept constant and the resistance of one of the other two arms can be varied by connecting an additional resistance in that arm (Ex. 16.1).

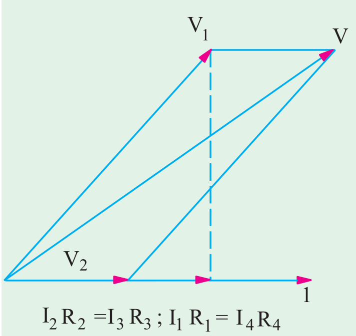



**Fig. 16.3**

The balance condition is that $\mathbf{Z_1Z_3 = Z_2Z_4}$

$\therefore \quad (R_1 + j\omega L_1)R_3 = (R_4 + j\omega L_4)R_2$

Equating the real and imaginary parts on both sides, we have

$R_1R_3 = R_2R_4$ or $R_1/R_4 = R_2/R_3$ *

(*i.e.* products of the resistances of opposite arms are equal).

and $\quad \omega L_1R_3 = \omega L_4R_2$ or $L_1 = L_4\dfrac{R_2}{R_3}$

We can also write that $L_1 \quad L_4 \cdot \dfrac{R_1}{R_4}$

* Or $\dfrac{L_1}{R_1} = \dfrac{L_4}{R_4}$ *i.e.*, the time constants of the two coils are matched.

Hence, the unknown self-inductance can be measured in terms of the known inductance $L_4$ and the two resistors. Resistive and reactive terms balance independently and the conditions are independent of frequency. This bridge is often used for measuring the iron losses of the transformers at audio frequency.

The balance condition is shown vectorially in Fig. 16.3. The currents $I_4$ and $I_3$ are in phase with $I_1$ and $I_2$. This is, obviously, brought about by adjusting the impedances of different branches, so that these currents lag behind the applied voltage $V$ by the same amount. At balance, the voltage drop $V_1$ across branch 1 is equal to that across branch 4 and $I_3 = I_4$. Similarly, voltage drop $V_2$ across branch 2 is equal to that across branch 3 and $I_1 = I_2$.

**Example 16.1.** *The arms of an a.c. Maxwell bridge are arranged as follows: AB and BC are non-reactive resistors of 100 Ω each, DA is a standard variable reactor $L_1$ of resistance 32.7 Ω and CD comprises a standard variable resistor R in series with a coil of unknown impedance. Balance was obtained with $L_1$ = 47.8 mH and R = 1.36 Ω. Find the resistance and inductance of the coil.*

**(Elect. Inst. & Meas. Nagpur Univ. 1993)**

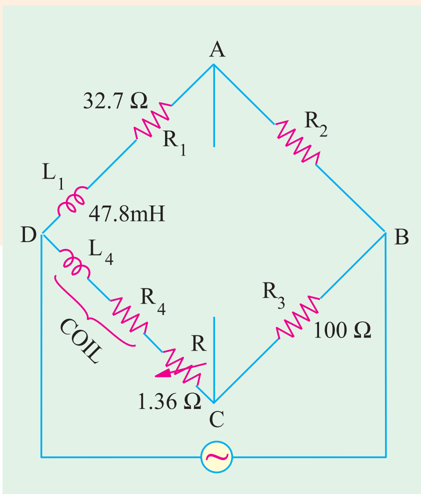



**Fig. 16.4**

**Solution.** The a.c. bridge is shown in Fig. 16.4.

Since the products of the resistances of opposite arms are equal

$\therefore\ 32.7 \times 100 = (1.36 + R_4)100$

$\therefore\ 32.7 = 1.36 + R_4$ or $R_4 = 32.7 - 1.36 =$ **31.34Ω**

Since $L_1 \times 100 = L_4 \times 100\ \therefore\ L_4 = L_1 =$ **47.8 mH**

or because time constants are the same, hence

$L_1/32.7 = L_4/(31.34 + 1.36)\ \therefore\ L_4 = 47.8$ mH

## 16.3. Maxwell-Wien Bridge or Maxwell's L/C Bridge

As referred to in Art. 16.2, the *positive* phase angle of an inductive impedance may be compensated by the *negative* phase angle of a capacitive impedance put in the *opposite* arm. The unknown inductance then becomes known in terms of this capacitance.

Let us first find the combined impedance of arm 1.

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{-jX_C} = \frac{1}{R_1} + \frac{j}{X_C} = \frac{1}{R_1} + j\omega C = \frac{1 + j\omega CR_1}{R_1}$$

$$\therefore\ \mathbf{Z}_1 \quad \frac{R_1}{1 \quad j \quad CR_1};\ \mathbf{Z}_2 \quad R_2$$

$$\mathbf{Z_3} = R_3 + j\omega L_3 \text{ and } Z_4 = R_4$$

Balance condition is $\mathbf{Z_1Z_3 = Z_2Z_4}$

or $$\frac{R_1(R_3 \quad j \quad L_3)}{1 \quad j \quad CR_1} \quad R_2R_4 \text{ or } R_1R_3 \quad j \quad L_3 R_1 \quad R_2R_4 \quad j \quad CR_1R_2R_4$$

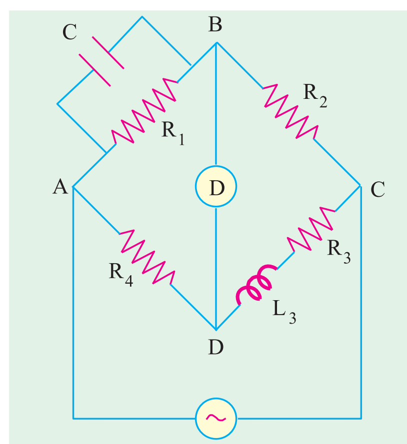



**Fig. 16.5**

Separating the real and imaginaries, we get

$$R_1R_3 \quad R_2R_4 \text{ and } L_3 R_1 \quad C R_1R_2R_4; R_3 \quad \frac{R_2R_4}{R_1} \text{ and } L_3 \quad C R_2R_4$$

**Example 16.2.** *The arms of an a.c. Maxwell bridge are arranged as follows: AB is a non-inductive resistance of 1,000 $\Omega$ in parallel with a capacitor of capacitance 0.5 $\mu F$, BC is a non-inductive resistance of 600 $\Omega$ CD is an inductive impedance (unknown) and DA is a non-inductive resistance of 400 $\Omega$. If balance is obtained under these conditions, find the value of the resistance and the inductance of the branch CD.*

**[Elect. & Electronic Meas, Madras Univ.]**

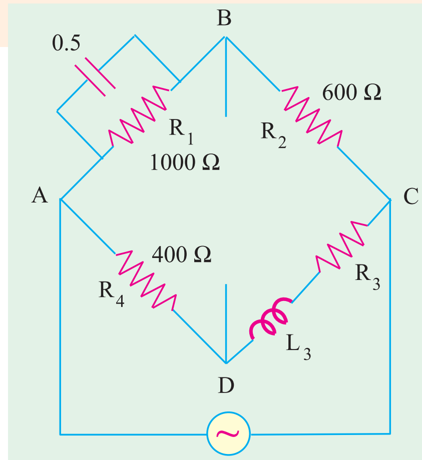


Fig. 16.6

**Solution.** The bridge is shown in Fig. 16.6. The conditions of balance have already been derived in Art. 16.3 above.

Since $R_1R_3 = R_2R_4 \quad \therefore \quad R_3 = R_2R_4 / R_1$

$$\therefore \qquad R_3 = \frac{600 \times 400}{1000} = \mathbf{240\ \Omega}$$

Also $\quad L_3 = CR_2R_4$

$$= 0.5 \times 10^{-6} \times 400 \times 600$$

$$= 12 \times 10^{-2} = 0.12 \text{ H}$$

## 16.4. Anderson Bridge

It is a very important and useful modification of the Maxwell-Wien bridge described in Art. 16.3. In this method, the unknown inductance is measured in terms of a known capacitance and resistance, as shown in Fig. 16.7.

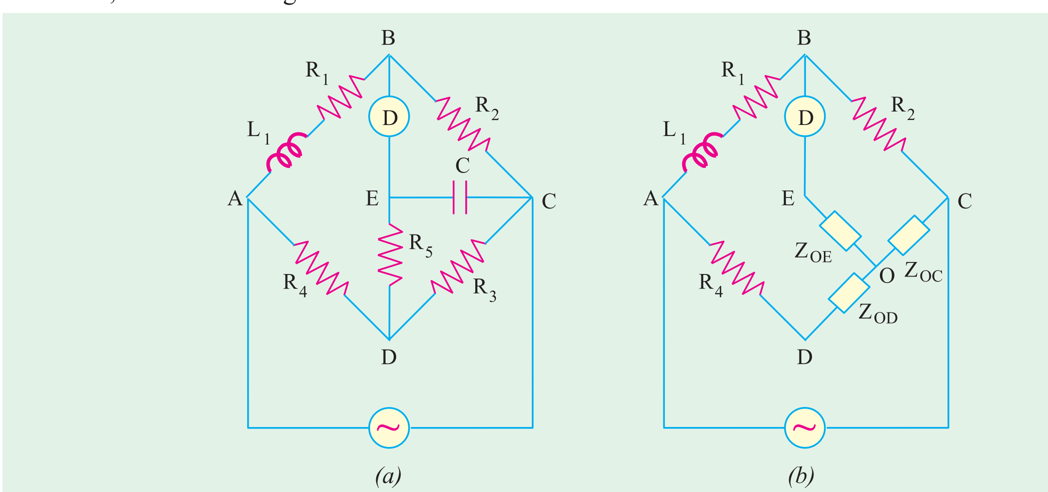


Fig. 16.7

The balance conditions for this bridge may be easily obtained by converting the mesh of impedances $C$, $R_5$ and $R_3$ to an equivalent star with star point $O$ by $\Delta / Y$ transformation. As seen from Fig. 16.7 ($b$).

$$\mathbf{Z}_{OD} \quad \frac{R_3R_5}{(R_3 \quad R_5 \quad 1/j \quad C)}; \quad \mathbf{Z}_{OC} \quad \frac{R_3/j \quad C}{(R_3 \quad R_5 \quad 1/j \quad C)} \quad \mathbf{Z}_3$$

With reference to Fig. 16.7 ($b$) it is seen that

$\mathbf{Z_1} = (R_1 + j\omega L_1)$ $\mathbf{Z_2} = R_2$; $\mathbf{Z_3} = \mathbf{Z_{OC}}$ and $\mathbf{Z_4} = R_4 + \mathbf{Z_{OD}}$

For balance $\mathbf{Z_1Z_3 = Z_2Z_4}$ $\therefore$ $(R_1 \quad j \quad L_1) \quad \mathbf{Z_{OC}} \quad R_2(R_4 \quad \mathbf{Z_{OD}})$

$$\therefore \ (R_1 \quad j \quad L_1)\frac{R_3 / j \quad C}{(R_3 \quad R_5 \quad 1/j \quad C)} \quad R_2 \quad R_4 \quad \frac{R_3R_5}{R_3 \quad R_5 \quad 1/j \quad C}$$

Further simplification leads to $R_2R_3R_4 \quad R_2R_4R_5 \quad j\frac{R_2R_4}{C} \quad R_2R_3R_5 \quad\quad j\frac{R_1R_3}{C} \quad \frac{R_3L_1}{C}$

$$\therefore \quad \frac{jR_2R_4}{C} \quad \frac{jR_1R_3}{C} \text{ or } R_1 \quad R_2R_4 / R_3$$

Also $\frac{R_3L_1}{C} \quad R_2R_3R_4 \quad R_2R_3R_5 \quad R_2R_4R_5 \quad \therefore \ L_1 = CR_2 \ R_4 + R_5 + \frac{R_4R_5}{R_3}$

This method is capable of precise measurements of inductances over a wide range of values from a few micro-henrys to several henrys and is one of the commonest and the best bridge methods.

**Example 16.3.** *An alternating current bridge is arranged as follows: The arms AB and BC consists of non-inductive resistances of 100-ohm each, the arms BE and CD of non-inductive variable resistances, the arm EC of a capacitor of 1 μF capacitance, the arm DA of an inductive resistance. The alternating current source is connected to A and C and the telephone receiver to E and D. A balance is obtained when resistances of arms CD and BE are 50 and 2,500 ohm respectively. Calculate the resistance and inductance of arm DA.*

*Draw the vector diagram showing voltage at every point of the network.*

**(Elect. Measurements, Pune Univ.)**

**Solution.** The circuit diagram and voltage vector diagram are shown in Fig. 16.8. As seen, $I_2$ is vector sum of $I_C$ and $I_3$. Voltage $V_2 = I_2 R_2 = I_C X_C$. Also, vector sum of $V_1$ and $V_2$ is $V$ as well as that of $V_3$ and $V_4$. $I_C$ is at right angles to $V_2$.

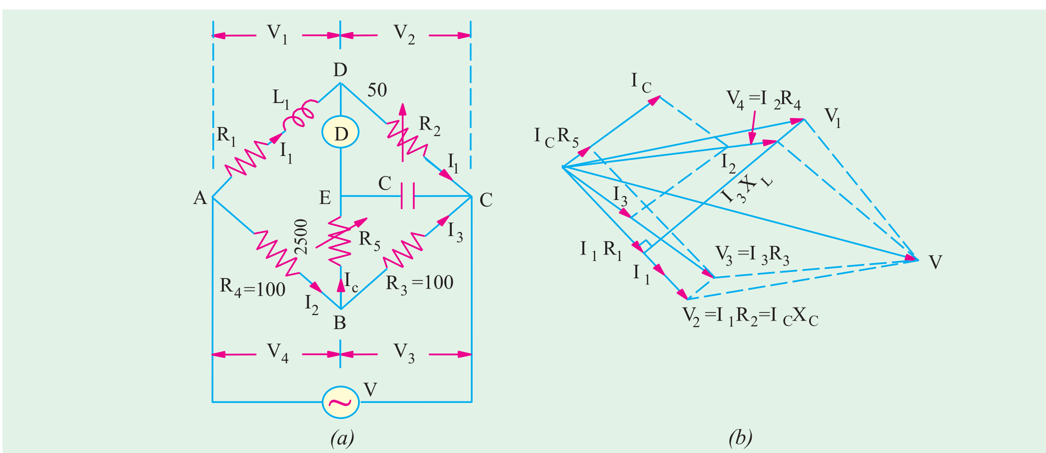



**Fig. 16.8**

Similarly, $V_3$ is the vector sum of $V_2$ and $I_CR_5$.

As shown in Fig. 16.8, $R_1 = R_2 . R_4/R_3 = 50 \times 100/100 = 50\ \Omega$

The inductance is given by $L \quad CR_2 (R_4 \quad R_5 \quad R_4R_5 / R_3)$

$$\therefore \ L \quad 1 \quad 10^{6} \quad 50(100 \quad 2500 \quad 100 \quad 2500/100)$$

$$= \mathbf{0.2505\ H}$$

**Example 16.4.** *Fig. 16.9 gives the connection of Anderson's bridge for measuring the inductance $L_1$ and resistance $R_1$ of a coil. Find $R_1$ and $L_1$ if balance is obtained when $R_3 = R_4 = 2000$ ohms, $R_2 = 1000$ ohms $R_5 = 200$ ohms and $C = 1\mu F$. Draw the vector diagram for the voltages and currents in the branches of the bridge at balance.*

**(Elect. Measurements, AMIE Sec. B Summer 1990)**

**Solution.** $R_1 = R_2\,R_4\,/\,R_3 = 1000 \times 2000\,/\,2000 =$ **1000 Ω**

$$L_1 \quad CR_2 \quad R_4 \quad R_5 \quad \frac{R_4R_5}{R_3}$$

$$= 1 \quad 10^{\;6} \quad 1000 \;\; 2000 \quad 200 \quad \frac{2000 \quad 200}{2000} \quad = \mathbf{2.4\ H}$$

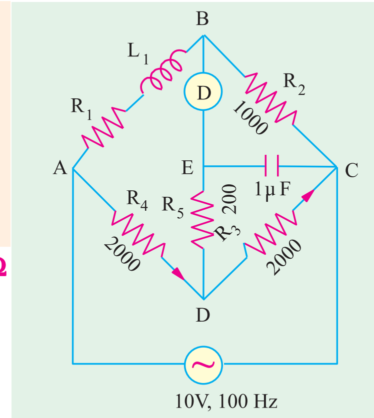



Fig. 16.9

## 16.5. Hay's Bridge

It is also a modification of the Maxwell-Wien bridge and is particularly useful if the phase angle of the inductive impedance $_m \quad \tan^{\;1}(\;\; L/R)$ is large. The network is shown in Fig. 16.10. It is seen that, in this case, a comparatively smaller series resistance $R_1$ is used instead of a parallel resistance (which has to be of a very large value).

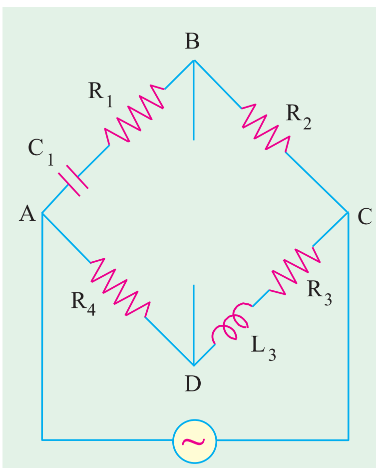



Fig. 16.10

Here $Z_1 \quad R_1 \quad \frac{j}{C_1}; Z_2 \quad R_2$

$Z_3 \quad R_3 \quad j \;\; L_3; Z_4 \quad R_4$

Balance condition is $\mathbf{Z_1Z_3 = Z_2Z_4}$

$$\text{or} \qquad R_1 \quad \frac{j}{C_1} \quad (R_3 \quad j \;\; L_3) \quad R_2R_4$$

Separating the reals and the imaginaries, we obtain

$$R_1R_3 \quad \frac{L_3}{C_1} \quad R_2R_4 \text{ and } L_3R_1 \quad \frac{R_3}{C_1} \quad 0$$

Solving these simultaneous equations, we get

$$L_3 \quad \frac{C_1R_2R_4}{1 \quad {}^2R_1^2C_1^2} \text{ and } R_3 \quad \frac{{}^2C_1^2R_1R_2R_4}{1 \quad {}^2R_1^2C_1^2}$$

The symmetry of expressions should help the readers to remember the results even when branch elements are exchanged, as in Ex. 16.5.

**Example 16.5.** *The four arms of a Hay's a.c. bridge are arranged as follows: AB is a coil of unknown impedance; BC is a non-reactive resistor of 1000 Ω; CD is a non-reactive resistor of 833 Ω in series with a standard capacitor of 0.38 μF; DA is a non-reactive resistor of 16,800 Ω. If the supply frequency is 50 Hz, determine the inductance and the resistance at the balance condition.*

**(Elect. Measu. A.M.I.E. Sec B, 1992)**

**Solution.** The bridge circuit is shown in Fig. 16.11.

$$277 \quad 50 \quad 314.22\,\text{rad/s}; \quad {}^2 \quad 314.2^2 \quad 98{,}721$$

$$R_1 \quad \frac{98,721 \quad (0.38 \quad 10^{\ 6})^2 \quad 833 \quad 16,800 \quad 1000}{1 \quad 98,721 \quad 833^2 \quad (0.38 \quad 10^{\ 6})^2} = 210\ \Omega$$

$$L_1 \quad \frac{16,800 \quad 1000 \quad 0.38 \quad 10^{\ 6}}{1 \quad 98,721 \quad 833^2 \quad (0.38 \quad 10^{\ 6})^2} \quad 6.38\,\text{H}$$

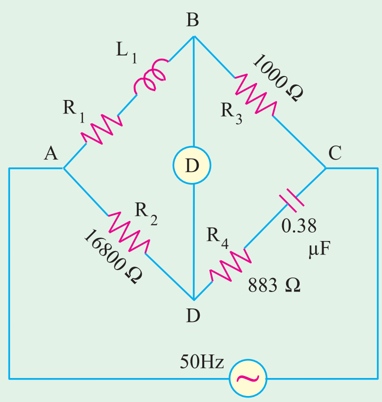



Fig. 16.11

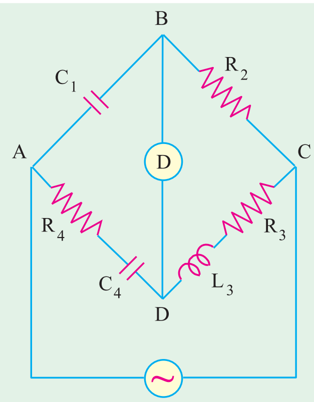



Fig. 16.12

## 16.6. The Owen Bridge

The arrangement of this bridge is shown in Fig. 16.12. In this method, also, the inductance is determined in terms of resistance and capacitance. This method has, however, the advantage of being useful over a very wide range of inductances with capacitors of reasonable dimensions.

Balance condition is $\mathbf{Z_1\, Z_3 = Z_2\, Z_4}$

Here $\mathbf{Z_1} = -\dfrac{j}{\omega C_1}$; $\qquad Z_2 \quad R_2;\ Z_3 \quad R_3 \quad j \quad L_3;\ \ Z_4 \quad R_4 \quad \dfrac{j}{C_4}$

$$\therefore \qquad \frac{j}{C_1}(R_3 \quad j \quad L_3) \quad R_2 \quad R_4 \quad \frac{j}{C_4}$$

Separating the reals and imaginaries, we get $R_3 = R_2 \dfrac{C_1}{C_4}$ and $L_3 = C_1 R_2 R_4$.

Since $\omega$ does not appear in the final balance equations, hence the bridge is unaffected by frequency variations and wave-form.

## 16.7. Heavisible-Campbell Equal Ratio Bridge

It is a mutual inductance bridge and is used for measuring self-inductance over a wide range in terms of mutual inductometer readings. The connections for Heaviside's bridge employing a standard variable mutual inductance are shown in Fig. 16.13. The primary of the mutual inductometer is inserted in the supply circuit and the secondary having self-inductance $L_2$ and resistance $R_2$ is put in arm 2 of the bridge. The unknown inductive impedance having self-inductance of $L_1$ and resistance $R_1$ is placed in arm 1. The other two arms have pure resistances of $R_3$ and $R_4$.

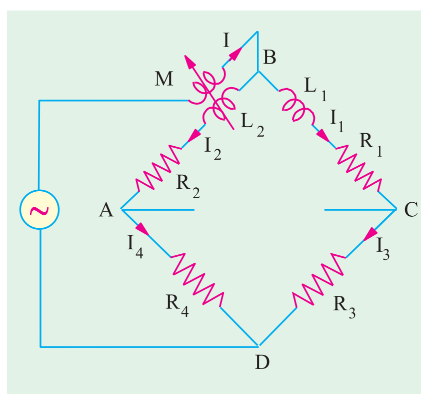



Fig. 16.13

Balance is obtained by varying mutual inductance $M$ and resistances $R_3$ and $R_4$.

For balance, $I_1 R_3 \quad I_2 R_4$ ... ***(i)***

$$I_1(R_1 \quad j \quad L_1) \quad I_2(R_2 \quad j \quad L_2) \quad j \quad MI$$

... ***(ii)***

Since $I = I_1 + I_2$, hence putting the value of $I$ in equation ***(ii)***, we get

$I_1[R_1 \quad j \quad (L_1 \quad M)] \quad I_2[R_2 \quad j \quad (L_2 \quad M)]$ ... ***(iii)***

Dividing equation ***(iii)*** by ***(i)***, we have $\dfrac{R_1 \quad j \ (L_1 \quad M)}{R_3} \quad \dfrac{R_2 \quad j \ (L_2 \quad M)}{R_4}$

$\therefore$ $\qquad R_3[R_2 \quad j \quad (L_2 \quad M)] \quad R_4[R_1 \quad j \quad (L_1 \quad M)]$

Equating the real and imaginaries, we have $R_2R_3 = R_1R_4$ ... ***(iv)***

Also, $R_3(L_2 + M) = R_4(L_1 - M)$. If $R_3 = R_4$, then $L_2 + M = (L_1 - M)$ $\therefore$ $L_1 - L_2 = 2M$ ... ***(v)***

This bridge, as modified by Campbell, is shown in Fig. 16.14. Here $R_3 = R_4$. A balancing coil or a test coil of self-inductance equal to the self-inductance $L_2$ of the secondary of the inductometer and of resistance slightly greater than $R_2$ is connected in series with the unknown inductive impedance ($R_1$ and $L_1$) in arm 1. A non-inductive resistance box along with a constant-inductance rheostat are also introduced in arm 2 as shown.

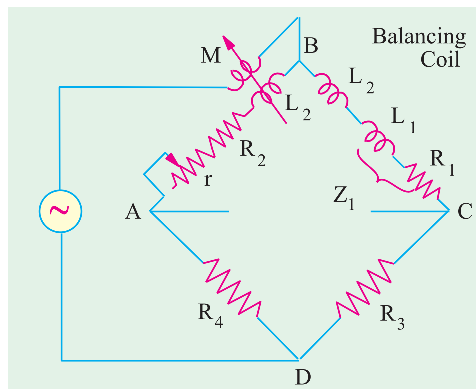



**Fig. 16.14**

Balance is obtained by varying $M$ and $r$. Two readings are taken; one when $Z_1$ is in circuit and second when $Z_1$ is removed or short-circuited across its terminals.

With unknown impedance $Z_1$ still in circuit, suppose for balance the values of mutual inductance and $r$ are $M_1$ and $r_1$. With $Z_1$ short-circuited, let these values be $M_2$ and $r_2$. Then

$$L_1 = 2(M_1 - M_2) \text{ and } R_1 = r_1 - r_2$$

By this method, the self-inductance and resistance of the leads are eliminated.

**Example 16.6.** *The inductance of a coil is measured by using the Heaviside-Campbell equal ratio bridge. With the test coil short-circuited, balance is obtained when adjustable non-reactive resistance is 12.63 Ω and mutual inductometer is set at 0.1 mH. When the test coil is in circuit, balance is obtained when the adjustable resistance is 25.9 Ω and mutual inductometer is set at 15.9 mH. What is the resistance and inductance of the coil?*

**Solution.** With reference to Art. 16.7 and Fig. 16.14, $r_1 = 25.9\ \Omega$, $M_1 = 15.9$ mH

With test coil short-circuited $r_2 = 12.63\ \Omega$; $M_2 = 0.1$ mH

$$L_1 = 2(M_1 - M_2) = 2(15.9 - 0.1) = \mathbf{31.6\ mH}$$

$$R_1 = -r_1 - r_2 = 25.9 - 12.63 = \mathbf{13.27\ \Omega}$$

## 16.8. Capacitance Bridges

We will consider only De Sauty bridge method of comparing two capacitances and Schering bridge used for the measurement of capacitance and dielectric loss.

## 16.9. De Sauty Bridge

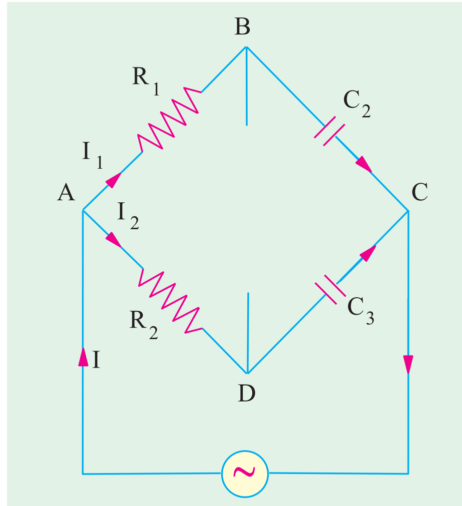



Fig. 16.15

With reference to Fig. 16.15, let

$C_2$ = capacitor whose capacitance is to be measured

$C_3$ = a standard capacitor

$R_1$, $R_2$ = non-inductive resistors

Balance is obtained by varying either $R_1$ or $R_2$. For balance, points $B$ and $D$ are at the same potential.

$$\therefore \quad I_1R_1 \quad I_2R_2 \text{ and } \frac{j}{C_2}.I_1 \quad \frac{j}{C_3}.I_2$$

Dividing one equation by the other, we get

$$\frac{R_1}{R_2} \quad \frac{C_2}{C_3};\ C_2 \quad C_3\frac{R_1}{R_2}$$

The bridge has maximum sensitivity when $C_2 = C_3$. The simplicity of this method is offset by the impossibility of obtaining a perfect balance if both the capacitors are not free from the dielectric loss. A perfect balance can only be obtained if air capacitors are used.

## 16.10. Schering Bridge

It is one of the very important and useful methods of measuring the capacitance and dielectric loss of a capacitor. In fact, it is a device for comparing an imperfect capacitor $C_2$ in terms of a loss-free standard capacitor $C_1$ [Fig. 16.16 (*a*)]. The imperfect capacitor is represented by its equivalent loss-free capacitor $C_2$ in series with a resistance $r$ [Fig. 16.16 (*b*)].

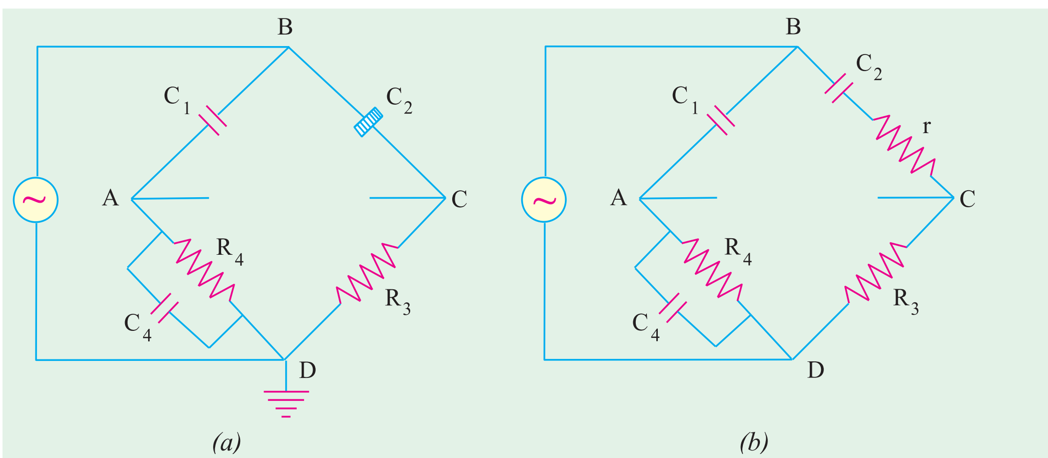



Fig. 16.16

For high voltage applications, the voltage is applied at the junctions shown in the figure. The junction between arms 3 and 4 is earthed. Since capacitor impedances at lower frequencies are much higher than resistances, most of the voltage will appear across capacitors. Grounding of the junction affords safety to the operator form the high-voltage hazards while making balancing adjustment in arms 3 and 4.

$$\text{Now} \quad Z_1 \quad \frac{j}{C_1};Z_2 \quad r \quad \frac{j}{C_2};Z_3 \quad R_3;Z_4 \quad \frac{1}{(1/R_4) \quad j \quad C_4} \quad \frac{R_4}{1 \quad j \quad C_4R_4}$$

For balance, $\mathbf{Z_1Z_3 = Z_2Z_4}$

or $\quad \dfrac{jR_3}{C_1} \quad r \quad \dfrac{j}{C_2} \quad \dfrac{R_4}{1 \quad j \quad C_4R_4}$ or $\dfrac{jR_3}{C_1}(1 \quad {}_aC_4R_4) \quad R_4 \quad r \quad \dfrac{j}{C_2}$

Separating the real and imaginaries, we have $C_2 = C_1(R_4 / R_3)$ and $r = R_3.(C_4 / C_1)$.

The quality of a capacitor is usually expressed in terms of its phase defect angle or dielectric loss angle which is defined as the angle by ***which current departs from exact quadrature from the applied voltage*** *i.e.* the complement of the phase angle. If $\phi$ is the actual phase angle and $\delta$ the defect angle, then $\quad 90 \quad$. For small values of $\delta$, tan $\delta$ = sin $\delta$ = cos $\phi$ (approximately). Tan $\delta$ is usually called the *dissipation factor* of the *R–C* circuit. For low power factors, therefore, dissipation factor is approximately equal to the power factor.

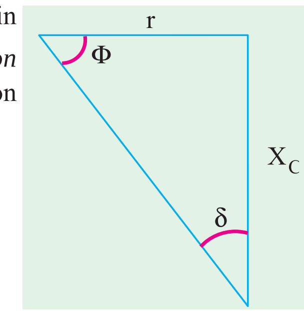



**Fig. 16.17**

As shown in Fig. 16.17,

Dissipation factor = power factor = tan $\delta$

$$= \frac{r}{X_C} = \frac{r}{1/\omega C_2} = \omega r C_2$$

Putting the value of $rC_2$ from above,

Dissipation factor = $\quad rC_2 \quad C_4R_4$ = power factor.

**Example 16.7.** *In a test on a bakelite sample at 20 kV, 50 Hz by a Schering bridge, having a standard capacitor of 106 pF, balance was obtained with a capacitance of 0.35 μF in parallel with a non-inductive resistance of 318 ohms, the non-inductive resistance in the remaining arm of the bridge being 130 ohms. Determine the capacitance, the p.f. and equivalent series resistance of the specimen. Derive any formula used. Indicate the precautions to be observed for avoiding errors.* **(Elect. Engg. Paper I, Indian Engg. Services 1991)**

**Solution.** Here $C_1$ = 106 pF, $C_4$ = 0.35 $\mu F$, $R_4$ = 318 $\Omega$, $R_3$ = 130 $\Omega$.

$$C_2 \quad C_1.(R_4 | R_3) \quad 106 \quad 318/130 = \mathbf{259.3\ pF}$$

$$r \quad R_3.(C_4/C_1) \quad 130 \quad 0.35 \quad 10^{\ 6}/106 \quad 10^{\ 12} = \mathbf{0.429}\ \text{M}\Omega$$

$$\text{p.f.} = \quad rC_2 \quad (2 \quad 50) \quad 0.429 \quad 10^{\ 6} \quad 259.3 \quad 10^{\ 12} = \mathbf{0.035}$$

**Example 16.8.** *A losy capacitor is tested with a Schering bridge circuit. Balance obtained with the capacitor under test in one arm, the succeeding arms being, a non-inductive resistor of 100 Ω, a non-reactive resistor of 309 Ω in parallel with a pure capacitor of 0.5 μF and a standard capacitor of 109 μμF. The supply frequency is 50 Hz. Calculate from the equation at balance the equivalent series capacitance and power factor (at 50 Hz) of the capacitor under test.* **(Measu. & Instru., Nagpur Univ. 1992)**

**Solution.** Here, we are given

$$C_1 \quad 109\,\text{pF};\ R_3 = 100 \quad ;\ C_4 = 0.5 \quad \text{F};\ R_4 = 309\ \Omega$$

Equivalent capacitance is $C_2 \quad 109 \quad 309/100 = \mathbf{336.8\ pF}$

$$p.f. = \quad C_4R_4 \quad 314 \quad 0.5 \quad 10^{\ 6} \quad 309 = \mathbf{0.0485}$$

## 16.11. Wien Series Bridge

It is a simple ratio bridge and is used for audio-frequency measurement of capacitors over a wide range. The bridge circuit is shown in Fig. 16.18.

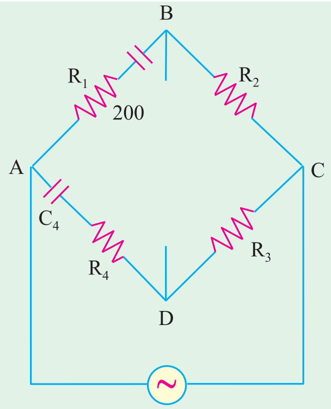



Fig. 16.18

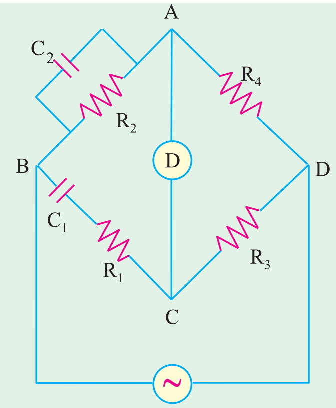



Fig. 16.19

The balance conditions may be obtained in the usual way. For balance

$$R_1 = R_2R_4/R_3 \text{ and } C_1 = C_4(R_3/R_2)$$

## 16.12. Wien Parallel Bridge

It is also a ratio bridge used mainly as the feedback network in the wide-range audio-frequency *R-C* oscillators. It may be used for measuring audio-frequencies although it is not as accurate as the modern digital frequency meters.

The bridge circuit is shown in Fig. 16.19. In the simple theory of this bridge, capacitors $C_1$ and $C_2$ are assumed to be loss-free and resistances $R_1$ and $R_2$ are separate resistors.

The usual relationship for balance gives

$$R_4\left(R_1 - \frac{j}{\omega C_1}\right) = R_3 \frac{R_2}{1 + j\omega C_2R_2} \quad \text{or} \quad R_4\left(R_1 - \frac{j}{\omega C_1}\right)(1 + j\omega C_2R_2) = R_2R_3$$

Separating the real and imaginary terms, we have

$$R_1R_4 + R_2R_4\frac{C_2}{C_1} = R_2R_3 \quad \text{or} \quad \frac{C_2}{C_1} = \frac{R_3}{R_4} - \frac{R_1}{R_2} \qquad \text{... }(i)$$

and
$$\omega C_2R_2R_4 - \frac{R_4}{\omega C_1} = 0 \quad \text{or} \quad \omega^2 = \frac{1}{R_1R_2C_1C_2} \qquad \text{... }(ii)$$

or
$$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \text{ Hz}$$

**Note.** Eq. (*ii*) may be used to find angular frequency $\omega$ of the source if terms are known. For such purposes, it is convenient to make $C_1 = 2C_2, R_3 = R_4$ and $R_2 = 2R_1$. In that case, the bridge has equal ratio arms so that Eq. (*i*) will always be satisfied. The bridge is balanced simultaneously by adjusting $R_2$ and $R_1$ (though maintaining $R_2 = 2R_1$). Then, as seen from Eq. (*ii*) above

$$\omega^2 = 1/(R_1.2R_1.2C_2.C_2) \quad \text{or} \quad \omega = 1/(2R_1C_2)$$

**Example 16.9.** *The arms of a four-arm bridge ABCD, supplied with a sinusoidal voltage, have the following values:*

*AB : 200 ohm resistance in parallel with 1 $\mu F$ capacitor; BC : 400 ohm resistance; CD : 1000 ohm resistance and DA : resistance R in series with a $2\mu F$ capacitor.*

*Determine* ***(i)*** *the value of R and* ***(ii)*** *the supply frequency at which the bridge will be balanced.* **(Elect. Meas. A.M.I.E. Sec. 1991)**

**Solution.** The bridge circuit is shown in Fig. 16.20.

(*i*) As discussed in Art. 16.12, for balance we have

$$\frac{C_2}{C_1} \quad \frac{R_3}{R_4} \quad \frac{R_1}{R_2} \text{ or } \frac{2}{1} \quad \frac{1000}{4000} \quad \frac{R_1}{200}$$

$\therefore\ R_1 = 200 \times 0.5 =$ **$100\ \Omega$**

(*ii*) The frequency at which bridge is balanced is given by

$$f \quad \frac{1}{2 \quad \sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

$$= \frac{10^6}{2 \quad \sqrt{100 \quad 200 \quad 1 \quad 2}} = \textbf{796 Hz}$$

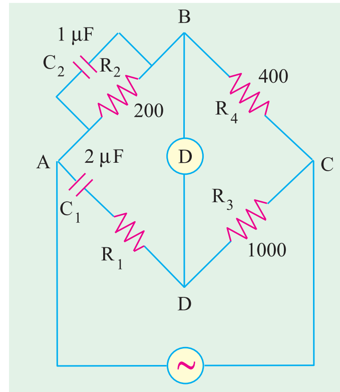



**Fig. 16.20**

## Tutorial Problems No. 16.1

**1.** In Anderson a.c. bridge, an impedance of inductance *L* and resistance *R* is connected between *A* and *B*. For balance following data is obtained. An ohmic resistance of $1000\,\Omega$ each in arms *AD* and *CD*, a non-inductive resistance of $500\,\Omega$ in *BC*, a pure resistance of $200\,\Omega$ between points *D* and *E* and a capacitor of $2\mu F$ between *C* and *E*. The supply is 10 volt (A.C.) at a frequency of 100 Hz and is connected across points *A* and *C*. Find *L* and *R*. **[1.4 H; 500 Ω]**

**2.** A balanced bridge has the following components connected between its five nodes, A, B, C, D and E:

| | |
|---|---|
| Between *A* and *B* : 1,000 ohm resistance; | Between *B* and *C* : 1,000 ohm resistance |
| Between *C* and *D* : an inductor; | Between *D* and *A* : 218 ohm resistance |
| Between *A* and *E* : 469 ohm resistance; | Between *E* and *B* : 10 μ F capacitance |
| Between *E* and *C* : a detector; | Between *B* and *D* : a power supply (a.c.) |

Derive the equations of balance and hence deduce the resistance and inductance of the inductor.

**[R = 218 Ω, L = 7.89 H] *(Elect. Theory and Meas. London Univ.)***

**3.** An a.c. bridge is arranged as follows: The arms *AB* and *BC* consist of non-inductive resistance of $100\,\Omega$, the arms, *BE* and *CD* of non-inductive variable resistances, the arm *EC* of a capacitor of $1\ \mu F$ capacitance, the arm *DA* of an inductive resistance. The a.c. source is connected to *A* and *C* and the telephone receiver to *E* and *D*. A balance is obtained when the resistances of the arms *CD* and *BE* are 50 Ω and 2500 Ω respectively.

Calculate the resistance and the inductance of the arm *DA*. What would be the effect of harmonics in the waveform of the alternating current source? **[50 Ω; 0.25 H]**

**4.** For the Anderson's bridge of Fig. 16.21, the values are underbalance conditions. Determine the values of unknown resistance *R* and inductance *L*. **[R = 500 Ω; L = 1.5 H]**

***(Elect. Meas & Inst. Madras Univ. Nov. 1978)***

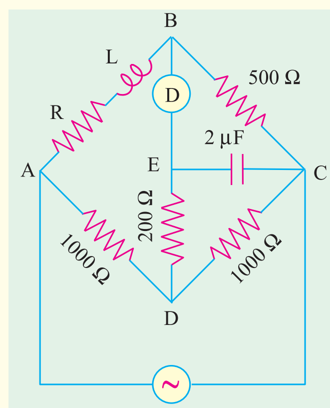



**Fig. 16.21**

**5.** An Anderson's bridge is arranged as under and balanced for the following values of the bridge components:

Branch *AB* – unknown coil of inductance *L* and resistance *R*

Branch *BC* – non-inductive resistance of 500 Ω

Branches *AD* & *CD* – non-inductive resistance of 100 Ω each

Branch *DE* – non-inductive resistance of 200 Ω

Branch *EB* – vibration galvanometer

Branch *EC* – 2.0 μF capacitance

Between *A* and *C* is 10 V, 100-Hz a.c. supply. Find the values of *R* and *L* of the unknown coil.

**[R = 500 Ω; L = 0.5 H] *(Elect. Meas & Meas. Inst., Gujarat Univ.)***

**6.** An a.c. Anderson bridge is arranged as follows:

(*i*) branches *BC* and *ED* are variable non-reactive resistors

(*ii*) branches *CD* and *DA* are non-reactive resistors of 200 ohm each

(*iii*) branch *CE* is a loss-free capacitor of $1\ \mu F$ capacitance.

The supply is connected across $A$ and $C$ and the detector across $B$ and $E$. Balance is obtained when the resistance of $BC$ is 400 ohm and that of $ED$ is 500 ohm. Calculate the resistance and inductance of $AB$.

Derive the relation used and draw the vector diagram for balanced condition of the bridge.

**[400 $\Omega$ ; 0.48 H] *(Elect. Measurements, Poona Univ.)***

**7.** In a balanced bridge network, $AB$ is a resistance of 500 ohm in series with an inductance of 0.18 henry, the non-inductive resistances $BC$ and $DA$ have values of 1000 ohm and arm $CD$ consists of a capacitance of $C$ in series with a resistance $R$. A potential difference of 5 volts at a frequency $5000/2\pi$ is the supply between the points $A$ and $C$. Find out the values of $R$ and $C$ and draw the vector diagram. **[472 $\Omega$ ; 0.235 $\mu$F] *(Elect. Measurements, Poona Univ.)***

**8.** A sample of bakelite was tested by the Schering bridge method at 25 kV, 50-Hz. Balance was obtained with a standard capacitor of 106 pF capacitance, a capacitor of capacitance 0.4 μF in parallel with a non-reactive resistor of 318 $\Omega$ and a non-reactive resistor of 120 $\Omega$. Determine the capacitance, the equivalent series resistance and the power factor of the specimen. Draw the phase or diagram for the balanced bridge. **[281 pF ; 0.452 M$\Omega$ ; 0.04] *(Elect. Measurements-II; Bangalore Univ.)***

**9.** The conditions at balance of a Schering bridge set up to measure the capacitance and loss angle of a paper dielectric capacitor are as follows:

$f$ = 500 Hz
$Z_1$ = a pure capacitance of 0.1 μF
$Z_2$ = a resistance of 500 $\Omega$ shunted by a capacitance of 0.0033 μF
$Z_3$ = pure resistance of 163 $\Omega$
$Z_4$ = the capacitor under test

Calculate the approximate values of the loss resistance of the capacitor assuming–
(*a*) series loss resistance (*b*) shunt loss resistance. **[5.37 $\Omega$, 197,000 $\Omega$ ] *(London Univ.)***

**10.** Name and draw the bridge used for measurements of Inductance. ***(Anna University, April 2002)***

**11.** A Wheat-stone bridge network has the following resistances :
$AB = 10\Omega$, $BC = 15\Omega$, $CD = 25\Omega$, $DA = 20\Omega$ and $BD = 10\Omega$

***(V.T.U., Belgaum Karnataka University, February 2002)***

## OBJECTIVE TESTS – 16

**1.** Maxwell-Wien bridge is used for measuring
(*a*) capacitance (*b*) dielectric loss
(*c*) inductance (*d*) phase angle

**2.** Maxwell's $L/C$ bridge is so called because
(*a*) it employs $L$ and $C$ in two arms
(*b*) ratio $L/C$ remains constant
(*c*) for balance, it uses two opposite impedances in opposite arms
(*d*) balance is obtained when $L = C$

**3.** ........ bridge is used for measuring an unknown inductance in terms of a known capacitance and resistance.
(*a*) Maxwell's $L/C$ (*b*) Hay's
(*c*) Owen (*d*) Anderson

**4.** Anderson bridge is a modification of ....... bridge.
(*a*) Owen (*b*) Hay's
(*c*) De Sauty (*d*) Maxwell-Wien

**5.** Hay's bridge is particularly useful for measuring
(*a*) inductive impedance with large phase angle
(*b*) mutual inductance
(*c*) self inductance
(*d*) capacitance and dielectric loss

**6.** The most useful ac bridge for comparing capacitances of two air capacitors is ......... bridge.
(*a*) Schering (*b*) De Sauty
(*c*) Wien series (*d*) Wien parallel

**7.** Heaviside-Campbell Equal Ratio bridge is used for measuring
(*a*) self-inductance in terms of mutual inductance
(*b*) capacitance in terms of inductance
(*c*) dielectric loss of an imperfect capacitor
(*d*) phase angle of a coil

## ANSWERS

**1.** *c* **2.** *c* **3.** *d* **4.** *d* **5.** *a* **6.** *b* **7.** *a*

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