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CHAPTER 13

SERIES A.C. CIRCUITS

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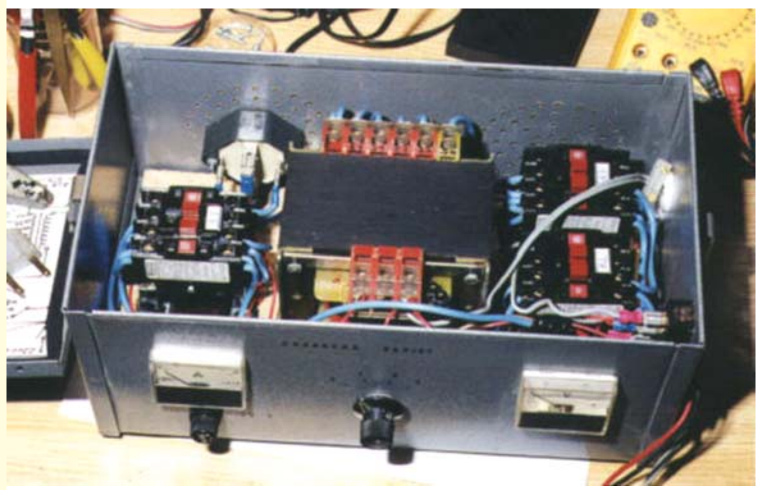

This chapter discusses series AC circuits, and how they function

13.1. A.C. Through Resistance and Inductance

A pure resistance R and a pure inductive coil of inductance L are shown connected in series in Fig. 13.1.

Let V = r.m.s. value of the applied voltage, I = r.m.s. value of the resultant current

$V_R = IR$ –voltage drop across R (in phase with I), $V_L = I \,.\, X_L$ –voltage drop across coil (ahead of I by 90°)

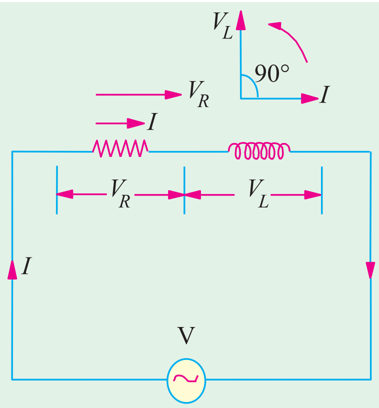

Fig. 13.1

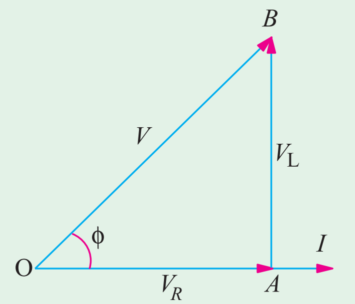

Fig. 13.2

These voltage drops are shown in voltage triangle OAB in Fig. 13.2. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. The applied voltage V is the vector sum of the two *i.e.* OB.

$$\therefore \quad V = \sqrt{(V_R^2 \quad V_L^2)} \quad \sqrt{[(IR)^2 \quad (I \,.\, X_L)^2]} \quad I\sqrt{R^2 \quad X_L^2}\,, \quad \frac{V}{\sqrt{(R^2 \quad X_L^2)}} \quad I$$

The quantity $\sqrt{(R^2 + X_L^2)}$ is known as the ***impedance*** (Z) of the circuit. As seen from the impedance triangle ABC (Fig. 13.3) $Z^2 = R^2 + X_L^2$.

i.e. ***(Impedance)^2 = (resistance)^2 + (reactance)^2***

From Fig. 13.2, it is clear that the applied voltage V leads the current I by an angle ϕ such that

$$\tan\phi = \frac{V_L}{V_R} = \frac{I \,.\, X_L}{I \,.\, R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{reactance}} \quad \therefore \ \phi = \tan^{-1}\frac{X_L}{R}$$

The same fact is illustrated graphically in Fig. 13.4.

In other words, current I lags behind the applied voltage V by an angle ϕ.

Hence, if applied voltage is given by $v = V_m \sin \omega t$, then current equation is

$$i = I_m \sin(\omega t - \phi) \text{ where } I_m = V_m / Z$$

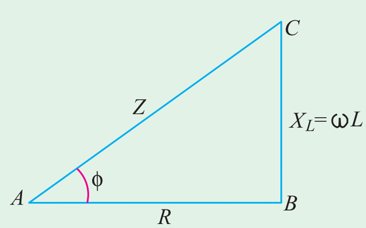

Fig. 13.3

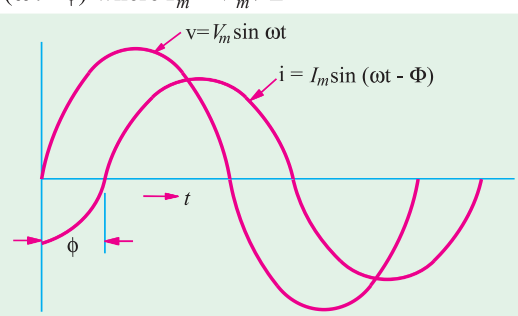

Fig. 13.4

In Fig. 13.5, I has been resolved into its two mutually perpendicular components, $I \cos \phi$ along the applied voltage V and $I \sin \phi$ in quadrature (*i.e.* perpendicular) with V.

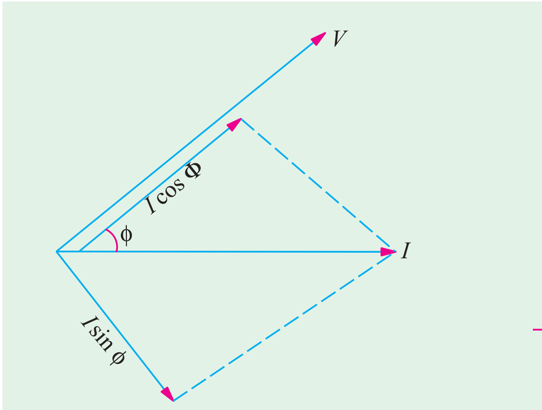

Fig. 13.5

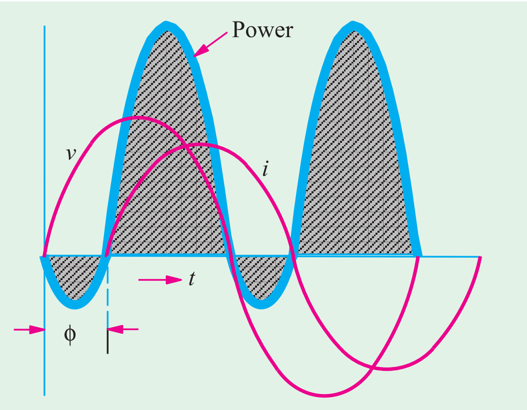

Fig. 13.6

The mean power consumed by the circuit is given by the product of V and ***that component of the current I which is in phase with V.***

So $P = V \times I \cos \phi$ = r.m.s. voltage × r.m.s. current × cos ϕ

The term 'cos ϕ' is called the power factor of the circuit.

Remember that in an a.c. circuit, the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (VA) and not true power in watts. True power (W) = volt-amperes (VA) × power factor.

or **Watts = VA × cos ϕ***

It should be noted that power consumed is due to ohmic resistance only because pure inductance does not consume any power.

Now $P = VI \cos \phi = VI \times (R/Z) = (V/Z) \times I.\ R = I^2 R$ ($\because \cos \phi = R/Z$) or $P = I^2 R$ watt

Graphical representation of the power consumed is shown in Fig. 14.6.

Let us calculate power in terms of instantaneous values.

Instantaneous power is $= v\ i = V_m \sin \omega t \times I_m \sin\ (\omega t - \phi) = V_m I_m \sin \omega t \sin (\omega t - \phi)$

$$= \frac{1}{2} V_m\ I_m [\cos \phi - \cos (2\omega t - \Phi)]$$

Obviously, this power consists of two parts (Fig. 13.7).

(*i*) a constant part $\frac{1}{2} V_m I_m \cos \phi$ which contributes to real power.

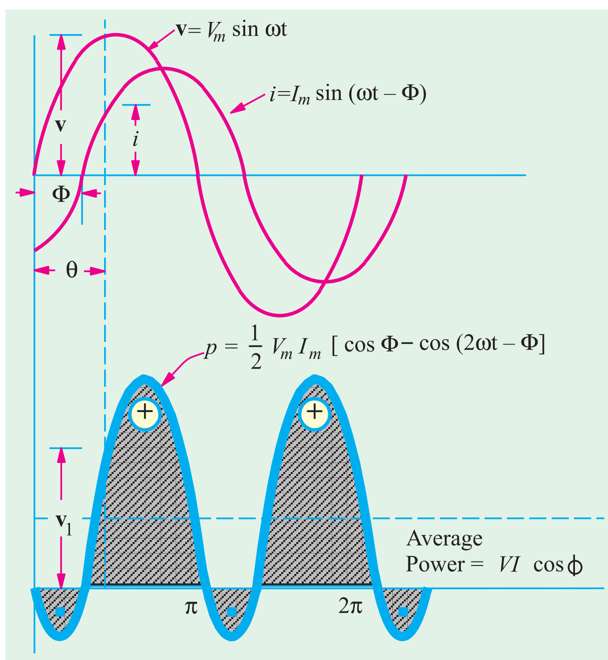

Fig. 13.7

* While dealing with large supplies of electric power, it is convenient to use kilowatt as the unit kW = kVA × cos ϕ

(ii) a pulsating component $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed $= \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi = VI\cos\phi$, where V and I represent the r.m.s values.

Symbolic Notation. $Z = R + jX_L$

Impedance vector has numerical value of $\sqrt{(R^2 + X_L^2)}$.

Its phase angle with the reference axis is $\phi = \tan^{-1}(X_L/R)$

It may also be expressed in the polar form as $\mathbf{Z} = \mathbf{Z} \angle \phi°$

(i) Assuming $\mathbf{V} = V \angle 0°$; $\mathbf{I} = \frac{V}{Z} = \frac{V \angle 0°}{\mathbf{Z} \angle \phi°} = \frac{V}{Z} \angle -\phi°$ (Fig. 13.8)

It shows that current vector is lagging behind the voltage vector by $\phi°$. The numerical value of current is V/Z.

(ii) However, if we assumed that

$\mathbf{I} = I \angle 0$, then

$\mathbf{V} = \mathbf{IZ} = I \angle 0° \times Z \angle \phi°$
$= IZ \angle \phi°$

It shows that voltage vector is $\phi°$ ahead of current vector in ccw direction as shown in Fig. 13.9.

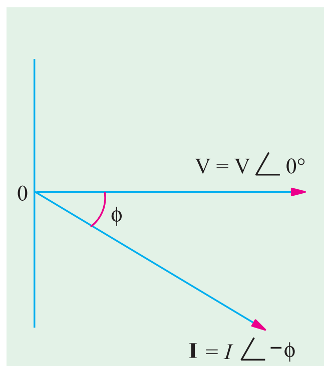

Fig. 13.8

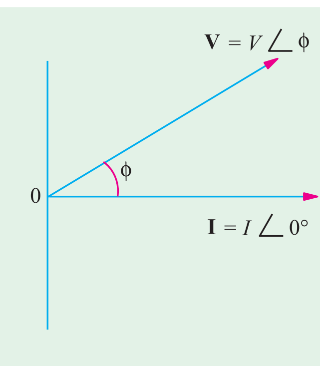

Fig. 13.9

13.2. Power Factor

It may be defined as

(i) cosine of the angle of lead or lag

(ii) the ratio $\frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$ (...Fig. 13.3) *(iii)* the ratio $\frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt} - \text{amperes}} = \frac{W}{VA}$

13.3. Active and Reactive Components of Circuit Current I

Active component is that which is in phase with the applied voltage V *i.e.* $I\cos\phi$. It is also known as 'wattful' component.

Reactive component is that which in quadrature with V *i.e.* $I\sin\phi$. It is also known as 'wattless' or 'idle' component.

It should be noted that the product of volts and amperes in an a.c. circuit gives voltamperes (VA). Out of this, the actual power is $VA\cos\phi = W$ and reactive power is $VA\sin\phi$. Expressing the values in kVA, we find that it has two rectangular components :

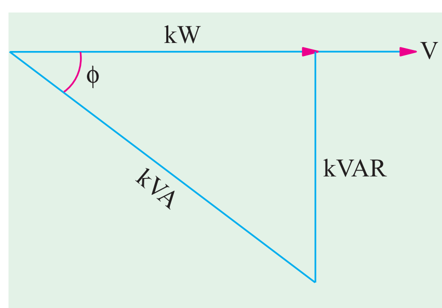

Fig. 13.10

(i) active component which is obtained by multiplying kVA by cos φ and this gives power in kW.

(ii) the reactive component known as reactive kVA and is obtained by multiplying kVA by sin ϕ. It is written as kVAR (kilovar). The following relations can be easily deduced.

$$kVA = \sqrt{kW^2 + kVAR^2}\ ;\ kW = kVA \cos\phi \text{ and } kVAR = kVA \sin\phi$$

These relationships can be easily understood by referring to the kVA triangle of Fig. 13.10 where it should be noted that lagging kVAR has been taken as negative.

For example, suppose a circuit draws a current of 1000 A at a voltage of 20,000 V and has a power factor of 0.8. Then

input = 1,000 × 20,000/1000 = 20,000 kVA; cos ϕ = 0.8; sin ϕ = 0.6

Hence kW = 20,000 × 0.8 = 16,000 ; kVAR = 20,000 × 0.6 = 12,000

Obviously, $\sqrt{16000^2 + 12000^2} = 20,000$ *i.e.* $kVA = \sqrt{kW^2 + kVAR^2}$

13.4. Active, Reactive and Apparent Power

Let a series *R-L* circuit draw a current of *I* when an alternating voltage of r.m.s. value *V* is applied to it. Suppose that current lags behind the applied voltage by ϕ. The three powers drawn by the circuit are as under :

(i) apparent power (S)

It is given by the product of r.m.s. values of applied voltage and circuit current.

$\therefore$ $S = VI = (IZ) . I = I^2 Z$ volt-amperes (*VA*)

(ii) active power (P or W)

It is the power which is actually dissipated in the circuit resistance. $P = I^2R = VI \cos\phi$ watts

(iii) reactive power (Q)

It is the power developed in the inductive reactance of the circuit.

$Q = I^2X_L = I^2 . Z \sin\phi = I . (IZ). \sin\phi = VI \sin\phi$ volt-amperes-reactive (VAR)

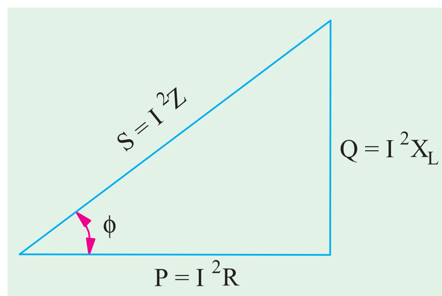

Fig. 13.11

These three powers are shown in the power triangle of Fig. 13.11 from where it can be seen that

$$S^2 = P^2 + Q^2 \text{ or } S = \sqrt{P^2 + Q^2}$$

13.5. Q-factor of a Coil

Reciprocal of power factor is called the *Q*-factor of a coil or its figure of merit. It is also known as quality factor of the coil.

$$Q \text{ factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R}$$

If *R* is small as compared to reactance, then *Q*-factor = $Z/R = \omega L/R$

Also, $$Q = 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$ —in the coil

Example 13.1. *In a series circuit containing pure resistance and a pure inductance, the current and the voltage are expressed as :*

$$i(t) = 5 \sin(314\,t + 2\pi/3) \text{ and } v(t) = 15 \sin(314\,t + 5\pi/6)$$

(a) *What is the impedance of the circuit ?* **(b)** *What is the value of the resistance ?* **(c)** *What is the inductance in henrys ?* **(d)** *What is the average power drawn by the circuit ?* **(e)** *What is the power factor ?* **[Elect. Technology, Indore Univ.]**

Solution. Phase angle of current = 2 π/3 = 2 180°/3 = 120° and phase angle of voltage = 5 π/6 = 5 × 180°/6 = 150°. Also, $Z = V_m/I_m = 3\ \Omega$.

Hence, current *lags* behind voltage by 30°. It means that it is an *R-L* circuit. Also 314 = 2π *f* or *f* = 50 Hz. Now, *R*/*Z* = cos 30° = 0.866; *R* = 2.6 Ω ; X_L/Z = sin 30° = 0.5

$\therefore \quad X_L = 1.5\ \Omega \qquad 314\ L = 1.5, \qquad L = 4.78$ mH

(a) $Z =$ **3 Ω** (b) $R =$ **2.6 Ω** **(c)** $L =$ **4.78 mH**

(d) $P = I^2 R = (5/\sqrt{2})^2 \times 2.6 =$ **32.5 W** **(e)** p.f. = cos 30° = **0.866 (lag).**

Example 13.2. *In a circuit the equations for instantaneous voltage and current are givn by v = 141.4 sin* $t \; \frac{2}{3}$ *volt and i = 7.07 sin* $t \; \frac{}{2}$ *amp, where ω= 314 rad/sec.*

(i) *Sketch a neat phasor diagram for the circuit.* **(ii)** *Use polar notation to calculate impedance with phase angle.* **(iii)** *Calculate average power & power factor.* **(iv)** *Calculate the instantaneous power at the instant t = 0.* **(F.Y. Engg. Pune Univ.)**

Solution. **(i)** From the voltage equation, it is seen that the voltage lags behind the reference quantity by 2π/3 radian or 2 × 180/3 = 120°. Similarly, current lags behind the reference quantity by π/2 radian or 180/2 = 90°. Between themselves, voltage lags behind the current by (120 −90) = 30° as shwon in Fig. 13.12 (b).

(ii) $V = V_m/\sqrt{2} = 141.4\sqrt{2} = 100$ V; $I = I_m/\sqrt{2} = 7.07/\sqrt{2} =$ **5 A.**

$\therefore \quad V = 100 \angle -120°$ and $I = 5 \angle -90° \quad \therefore Z = \dfrac{100 \angle -120°}{5 \angle -90°} = 20 \angle -30°$

(iii) Average power = $VI \cos\phi = 100 \times 5 \times \cos 30° =$ **433 W**

(iv) At $t = 0$; $v = 141.4 \sin (0 - 120°) = -122.45$ V; $i = 7.07 \sin (0 - 90°) = -7.07$ A.

$\therefore$ instantaneous power at $t = 0$ is given by $vi = (-122.45) \times (-7.07) =$ **865.7 W.**

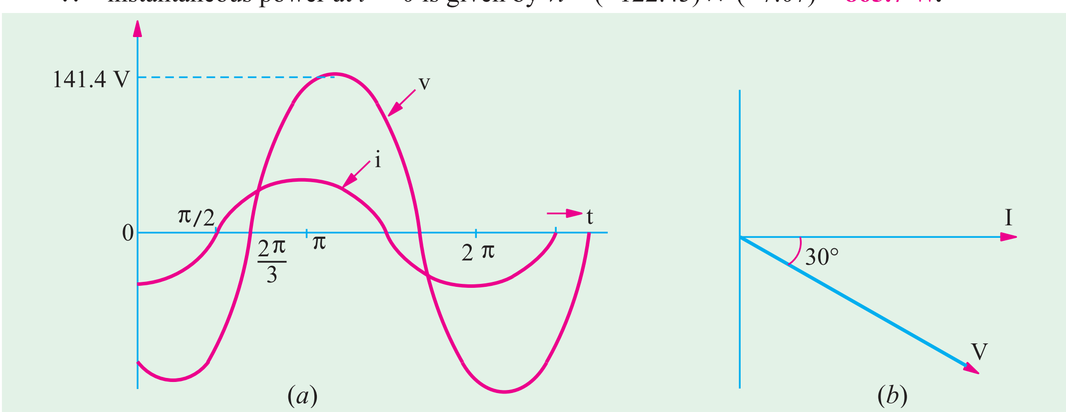

Fig. 13.12

Example 13.3. *The potential difference measured across a coil is 4.5 V, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9 A at 25 Hz, the potential difference is 24 V. Find the current, the power and the power factor when it is supplied by 50 V, 50 Hz supply.* **(F.Y. Pune Univ.)**

Solution. Let R be the d.c. resistance and L be the inductance of the coil.

$\therefore \quad R = V/I = 4.5/9 = 0.5\ \Omega;$

With a.c. current of 25 Hz, $Z = V/I = 24/9 = 2.66\ \Omega$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2} = 2.62\ \Omega.$ Now $X_L = 2\pi \times 25 \times L$; $L = 0.0167\ \Omega$

At 50 Hz

$X_L = 2.62 \times 2 = 5.24\ \Omega;\ Z = \sqrt{0.5^2 + 5.24^2} = 5.26\ \Omega$

Current $I = 50/5.26 =$ **9.5 A ;** Power $= I^2 R = 9.5^2 \times 0.5 =$ **45 W.**

Example 13.4. *In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 mA while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit ?* **(Network Analysis A.M.I.E. Sec. B, S 1990)**

Solution. (*i*) $Z = \sqrt{R^2 + (2\pi \times 50\, L)^2} = \sqrt{R^2 + 98696\, L^2}$; $V = IZ$ or $10 = 700 \times 10^{-3}\sqrt{(R^2 + 98696\, L^2)}$

$$\sqrt{(R^2 + 98696\, L^2)} = 10/700 \times 10^{-3} = 100/7 \text{ or } R^2 + 98696\, L^2 = 10000/49 \qquad ...(i)$$

(*ii*) In the second case $Z = \sqrt{R^2 + (2\pi \times 75L)^2} = \sqrt{(R^2 + 222066\, L^2)}$

$\therefore 10 = 500 \times 10^{-3}\sqrt{(R^2 + 222066\, L^2)}$ *i.e.* $\sqrt{(R^2 + 222066\, L^2)} = 20$ or $R^2 + 222066L^2 = 400$ (*ii*)

Subtracting Eq. (*i*) from (*ii*), we get

$222066\, L^2 - 98696\, L^2 = 400 - (10000/49)$ or $123370\, L^2 = 196$ or $L = 0.0398$ H = **40 mH.**

Substituting this value of L in Eq. (*ii*), we get, $R^2 + 222066\,(0.398)^2 = 400$ $\therefore$ **R = 6.9 Ω**

Example 13.5. *A series circuit consists of a resistance of 6 Ω and an inductive reactance of 8 Ω. A potential difference of 141.4 V (r.m.s.) is applied to it. At a certain instant the applied voltage is + 100 V, and is increasing. Calculate at this current,* **(i)** *the current* **(ii)** *the voltage drop across the resistance and* **(iii)** *voltage drop across inductive reactance.* **(F.E. Pune Univ.)**

Solution. $Z = R + jX = 6 + j8 = 10 \angle 53.1°$

It shows that current lags behind the applied voltage by 53.1°. Let V be taken as the reference quantity. Then $v = (141.4 \times \sqrt{2})\ \sin \omega t = 200 \sin \omega t$; $i = (V_m/Z \sin \omega t) - 30° = 20 \sin(\omega t - 53.1°)$.

(*i*) When the voltage is + 100 Vand increasing; $100 = 200 \sin \omega t$; $\sin \omega t = 0.5$; $\omega t = 30°$

At this instant, the current is given by $i = 20 \sin(30° - 53.1°) = -20 \sin 23.1° =$ **−7.847 A.**

(*ii*) drop across resistor = $iR = -7.847 \times 6 = -47$ V.

(*iii*) Let us first find the equation of the voltage drop V_L across the inductive reactance. Maximum value of the voltage drop = $I_m X_L = 20 \times 8 = 160$ V. It leads the current by 90°. Since current itself lags the applied voltage by 53.1°, the reactive voltage drop across the applied voltage by (90° – 53.1°) = 36.9°. Hence, the equation of this voltage drop at the instant when $\omega t = 30°$ is

$$V_L = 160 \sin(30° + 36.9°) = 160 \sin 66.9° = \mathbf{147.2\ V.}$$

Example 13.6. *A 60 Hz sinusoidal voltage v = 141 sin ωt is applied to a series R-L circuit. The values of the resistance and the inductance are 3 Ω and 0.0106 H respectively.*

(i) *Compute the r.m.s. value of the current in the circuit and its phase angle with respect to the voltage.*

(ii) *Write the expression for the instantaneous current in the circuit.*

(iii) *Compute the r.m.s. value and the phase of the voltages appearing across the resistance and the inductance.*

(iv) *Find the average power dissipated by the circuit.*

(v) *Calculate the p.f. of the circuit.* **(F.E. Pune Univ.)**

Solution. $V_m = 141$ V; $V = 141/\sqrt{2} = 100\,V$ $\therefore$ $V = 100 + j0$

$$X_L = 2\pi \times 60 \times 0.0106 = 4\,\Omega.\ Z = 3 + j\,4 = 5 \angle 53.1°$$

(*i*) $I = V/Z = 100 \angle 0°/5 \angle 53.1° = 20 \angle -53.1°$

Since angle is minus, the current lags behind the voltage by 53.1

(*ii*) $I_m = \sqrt{2} \times 20 = 28.28$; $\therefore$ $i = 28.28 \sin(\omega t - 53.1°)$

(*iii*) VR = IR = $20 \angle -53.1° \times 3 = 60 \angle -53.1°$ volt.

$V_L = jIX_L = 1 \angle 90° \times 20 \angle -53.1° \times 4 = 80 \angle 36.9°$

(*iv*) $P = VI \cos\phi = 100 \times 20 \times \cos 53.1° =$ **1200 W.**

(*v*) p.f. = $\cos\phi = \cos 53.1° =$ **0.6.**

Example 13.7. *In a given R-L circuit, R = 3.5 Ω and L = 0.1 H. Find* **(i)** *the current through the circuit and* **(ii)** *power factor if a 50-Hz voltage V = 220 ∠ 30° is applied across the circuit.*

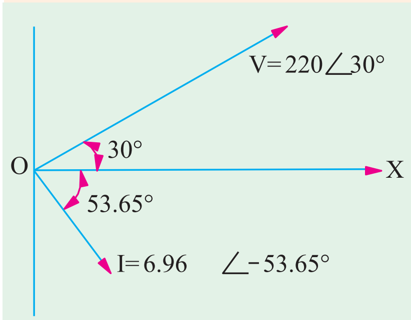

Fig. 13.13

Solution. The vector diagram is shown in Fig. 13.13.

$$X_L = 2\,\pi\, fL = 2\pi \times 50 \times 0.1 = 31.42\,\Omega$$

$$Z = \sqrt{(R^2 \quad X_L^2)} \quad \sqrt{3.5^2 \quad 31.42^2} \quad 31.6$$

$$\therefore\ \mathbf{Z} = 31.6 \angle \tan^{-1}(31.42/3.5) = 31.6 \angle 83.65°$$

$$\text{(i)}\ \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \quad \frac{220 \quad 30}{31.6 \quad 83.65} \quad 6.96 \qquad 53.65$$

(ii) Phase angle between voltage and current is

= 53.65° + 30° = 83.65° with current lagging.

p.f. = cos 83.65° = **0.11 (lag).**

Example 13.8. *In an alternating circuit, the impressed voltage is given by V = (100 –j50) volts and the current in the circuit is I = (3 – j4) A. Determine the real and reactive power in the circuit.* **(Electrical Engg., Calcutta Univ. 1991)**

Solution. Power will be found by the conjugate method. Using current conjugate, we have

$$P_{VA} = (100 - j\,50)\,(3 + j\,4) = 300 + j\,400 \;\; -j\,150 + 200 = 500 + j\,250$$

Hence, real power is **500 W** and reactive power of VAR is **250.** Since the second term in the above expression is positive, the reactive volt-amperes of 250 are inductive.*

Example 13.9. *In the circuit of Fig. 14.14, applied voltage V is given by (0 + j10) and the current is (0.8 + j 0.6) A. Determine the values of R and X and also indicate if X is inductive or capacitive.* **(Elect. Technology, Nagpur Univ. 1991)**

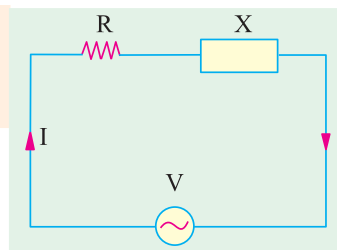

Fig. 13.14

Solution. **V** = 0 + j10 = ∠90°; **I** = 0.8 + j0.6 = 1∠36.9°

As seen, V leads the reference quantity by 90° whereas I leads by 36.9°. In other words, I *lags* behind the applied voltage by (90° – 36.9°) = 53.1°

Hence, the circuit of Fig. 13.14 is an R-L circuit.

Now, **Z = V/I** = 10 ∠90°/1 ∠36.9° = 10 ∠53.1° = 6 + j8

Hence, R = **6 Ω** and X_L = **8 Ω**

Example 13.10. *A two-element series circuit is connected across an a.c. source e =* $200\sqrt{2}$ *sin (ωt + 20°) V. The current in the circuit then is found to be i =* $10\sqrt{2}$ *cos (314 t –25°) A. Determine the parameters of the circuit.* **(Electromechanic Allahabad Univ. 1991)**

Solution. The current can be written as $i = 10\sqrt{2}\,\sin(314\,t - 25° + 90°) = 10\sqrt{2}\,\sin(314\,t + 65°)$. It is seen that applied voltage leads by 20° and current leads by 65° with regards to the reference quantity, their mutual phase difference is = 65° –(20°) = 45°. Hence, p.f. = cos 45° = **$1/\sqrt{2}$ (lead).**

Now, $V_m = 200\sqrt{2}$ and $I_m = 10\sqrt{2}$ $\quad\therefore\ Z = V_m/I_m = 200\sqrt{2}/10\sqrt{2} = 20\,\Omega$

$R = Z\cos\phi = 20/\sqrt{2}\,\Omega$ = **14.1 Ω** ; $X_c = Z\sin\phi = 20/\sqrt{2}$ = **14.1 Ω**

Now, f = 314/2π = 50 Hz. Also, $X_c = 1/2\pi fC$ ∴ $C = 1/2\pi \times 50 \times 14.1$ = **226 μF**

Hence, the given circuit is an R-C circuit.

---

* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs are positive.

Example 13.11. *Transform the following currents to the time domain :* **(*i*)** $6 - j\,8$ **(*ii*)** $-6 + j8$ **(*iii*)** $-j5$.

Solution. **(*i*)** Now, $(6 - j8)$ when expressed in the polar form is $\sqrt{6^2 + 8^2}\ \angle -\tan^{-1} 8/6 = 10\ \angle -53.1°$. The time domain representation of this current is $i\,(t) = 10 \sin (\omega t - 53.1°)$

(*ii*) $-6 + j8 = \sqrt{6^2 + 8^2}\ \angle \tan^{-1} 8/-6 = 10 \angle 126.9°$

$\therefore \quad i\,(t) = 10 \sin (\omega t + 126.9°)$

(*iii*) $-j\,5 = 10 \angle -90° \ \therefore i\,(t) = 10 \sin (\omega t - 90°)$

Example 13.12. *A choke coil takes a current of 2 A lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also, determine the power consumed when it is connected across 100-V, 25-Hz supply.*

(Elect. Engg. & Electronics, Bangalore Univ.)

Choke coil

Solution. **(*i*)** $Z_{coil} = 200/2 = 100\ \Omega$; $R = Z \cos \phi =- 100 \cos 60° = 50\ \Omega$

$X_L = Z \sin \phi = 100 \sin 60° = 86.6\ \Omega\ X_L = 2\,\pi\,fL = 86.6\ \therefore L = 86.6/2\pi \times 50 = 0.275$ H

(*ii*) Now, the coil will have different impedance because the supply frequency is different but its resistance would remain the same *i.e.* 50 Ω. Since the frequency has been halved, the inductive reactance of the coil is also halved *i.e.* it becomes 86.6/2 = 43.3 Ω.

$$Z_{coil} = \sqrt{50^2 + 43.3^2} = 66.1\ \Omega$$

$$I = 100/66.1 = 1.5 \text{ A, p.f.} = \cos \phi = 50/66.1 = 0.75$$

Power consumed by the coil $= VI \cos \phi = 100 \times 1.5 \times 0.75 =$ **112.5 W**

Example 13.13. *An inductive circuit draws 10 A and 1 kW from a 200-V, 50 Hz a.c. supply. Determine :*

(*i*) *the impedance in cartesian from (a + jb)* **(*ii*)** *the impedance in polar from Z ∠ θ* **(*iii*)** *the power factor* **(*iv*)** *the reactive power* **(*v*)** *the apparent power.*

Solution. $Z = 200/10 = 20\ \Omega$; $P = I^2R$ or $1000 = 10^2 \times R$; $R = 10\ \Omega$; $X_L = \sqrt{20^2 - 10^2} = 17.32\Omega$

(*i*) Z $= 10 + j17.32$ **(*ii*)** $|Z| = \sqrt{(10^2 + 17.32^2)} = 20\ \Omega$; $\tan \phi = 17.32/10 = 1.732$; $\phi = \tan^{-1}(1.732)$ $= 60°\ \therefore$ **Z = 20 ∠ 60°.** **(*iii*)** p.f. $= \cos \phi = \cos 60° =$ **0.5 lag** **(*iv*)** reactive power $= VI \sin \phi$ $= 200 \times 10 \times 0.866 =$ **1732 VAR** (*v*) apparent power $= VI = 200 \times 10 =$ **2000 VA.**

Example 13.14. *When a voltage of 100 V at 50 Hz is applied to a choking coil A, the current taken is 8 A and the power is 120 W. When applied to a coil B, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series ?* **(Elements of Elect. Engg., Bangalore Univ.)**

Solution.

$$Z_1 = 100/8 = 12.5\ \Omega\ ;\ P = I^2 . R_1 \text{ or } 120 = 8^2 \times R_1\ ;\ R_1 = 15/8\ \Omega$$

$$X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{12.5^2 - (15/8)^2} = 12.36\ \Omega$$

$$Z_2 = 100/10 = 10\ \Omega\ ;\ 500 = 10^2 \times R_2 \quad \text{or} \quad R_2 = 5\ \Omega$$

$$X_2 = \sqrt{10^2 - 5^2} = 8.66\ \Omega$$

With Joined in Series

$$R = R_1 + R_2 = (15/8) + 5 = 55/9\ \Omega;\ X = 12.36 + 8.66 = 21.02\ \Omega$$

$$Z = \sqrt{(55/8)^2 + (21.02)^2} = 22.1\ \Omega,\ I = 100/22.1 = 4.52\ \text{A},\ P = I^2R = 4.52^2 \times 55/8 = \mathbf{140\ W}$$

Example 13.15. *A coil takes a current of 6 A when connected to a 24-V d.c. supply. To obtain the same current with a 50-Hz a.c. supply, the voltage required was 30 V.*

Calculate ***(i)*** *the inductance of the coil* ***(ii)*** *the power factor of the coil.*

(F.Y. Engg. Pune Univ.)

Solution. It should be kept in mind the coil offers only resistance to direct voltage whereas it offers impedance to an alternating voltage.

$\therefore \quad R = 24/6 = 4\ \Omega;\ Z = 30/6 = 5\ \Omega$

(*i*) $\therefore \quad X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 4^2} = 3\ \Omega$ **(*ii*)** p.f. $= \cos\phi = R/Z = 4/5 =$ **0.8 (lag)**

Example 13.16. *A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150 μF are connected in series and are fed by a 230 V, 50 Hz supply. Find X_L, X_C, Z, Y, p.f., active power and reactive power.* **(Elect. Science-I, Allahabad Univ. 1992)**

Solution. $X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.8\Omega;\ X_C = 1/2\pi fC$

$= 10^{-6}\, 2\pi \times 50 \times 150 = 21.2\ \Omega;\ X = (X_L - X_C) = 41.6\ \Omega;$

$Z = \sqrt{R^2 + X^2} = \sqrt{20^2 + 41.6^2} = 46.2\ \Omega;\ I = V/Z = 230/46.2 = 4.98\ A$

Also, $Z = R + jX = 20 + j\,41.6 = 46.2\ \angle\ 64.3°$ ohm

$\therefore \quad Y = 1/Z = 1/46.2\ \angle\ 64.3° = 0.0216\ \angle -64.3°$ siemens

p.f. $= \cos 64.3° = 0.4336$ (lag)

Active power $= VI\cos\phi = 230 \times 4.98 \times 0.4336 = 497$ W

Reactive power $= VI\sin\phi = 230 \times 4.98 \times \sin 64.3° = 1031$ VAR

Example 13.17. *A 120-V, 60-W lamp is to be operated on 220-V, 50-Hz supply mains. Calculate what value of* ***(a)*** *non-inductive resistance* ***(b)*** *pure inductance would be required in order that lamp is run on correct voltage. Which method is preferable and why ?*

Solution. Rated current of the bulb = 60/120 = 0.5 A

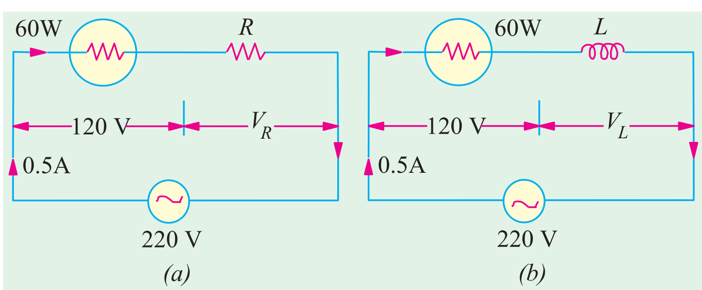

Fig. 13.15

(*a*) Resistor has been shown connected in series with the lamp in Fig. 13.15 (*a*).

P.D. across R is $V_R = 220 - 1200 = 100$ V

It is in phase with the applied voltage, $\therefore R = 100/0.5 =$ **200 Ω**

(*b*) P.D. across bulb = 120 V

P.D. across L is

$$V_L = \sqrt{(220^2 - 120^2)} = 184.4\ \text{V}$$

(Remember that V_L is in quadrature with V_R –the voltage across the bulb).

Now, $V_L = 0.5 \times X_L$ or $184.4 = 0.5 \times L \times 2\pi \times 50 \quad \therefore\ L = 184.4/0.5 \times 3.14 =$ **1.17 H**

Method **(*b*)** is preferable to **(*a*)** because in method **(*b*),** there is no loss of power. Ohmic resistance of 200 Ω itself dissipates large power (*i.e.* 100 × 0.5 = **50 W**).

Example 13.18. *A non-inductive resistor takes 8 A at 100 V. Calculate the inductance of a choke coil of negligible resistance to be connected in series in order that this load may be supplied from 220-V, 50-Hz mains. What will be the phase angle between the supply voltage and current ?*

(Elements of Elect. Engg.-I, Bangalore Univ.)

Solution. It is a case of pure resistance in series with pure inductance as shown in Fig. 13.16 (*a*).

Here $V_R = 100$ V,

$V_L = \sqrt{(220^2 - 100^2)} = 196$ V

Now, $V_L = I \cdot X_L$

or $196 = 8 \times 2\pi \times 50 \times L =$ **0.078 H**

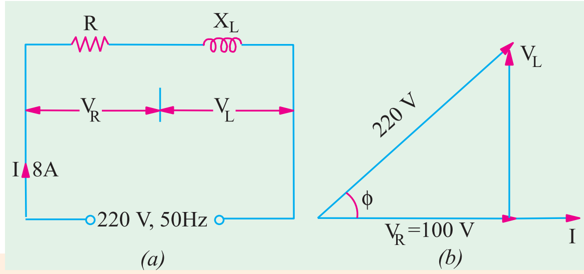

Fig. 13.16

Example 13.19. *A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250-V, 50-Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate*

(a) *impedance, reactance and resistance of the coil* ***(b)*** *the power absorbed by the coil and* ***(c)*** *the total power. Draw the vector diagram.* **(Elect. Engg., Madras Univ.)**

Solution. As seen from the vector diagram of Fig. 13.17 (*b*).

$BC^2 + CD^2 = 200^2$ ...***(i)*** $(125 + BC)^2 + CD^2 = 250^2$ ...***(ii)***

Subtracting Eq. ***(i)*** from ***(ii)***, we get, $(125 + BC)^2 - BC^2 = 250^2 - 200^2$

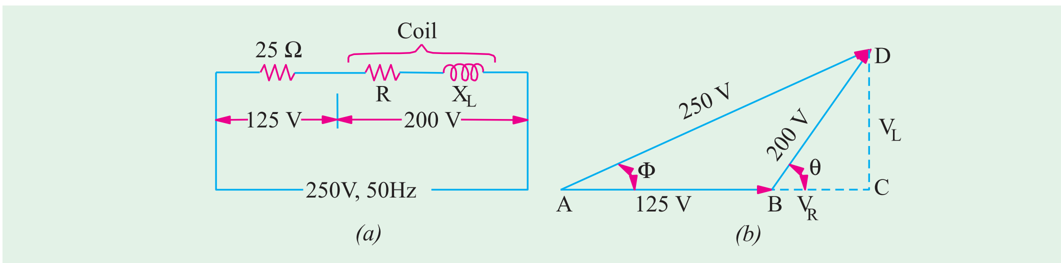

Fig. 13.17

$\therefore \quad BC = 27.5$ V; $CD = \sqrt{200^2 - 27.5^2} = 198.1$ V

(i) Coil impedance $= 200/5 =$ **40 Ω**

$V_R = IR = BC$ or $5R = 27.5 \quad \therefore \; P = 27.5/5 =$ **5.5 Ω**

Also $V_L = I \cdot X_L = CD = 198.1 \quad \therefore \; X_L = 198.1/5 =$ **39.62 Ω**

or $X_L = \sqrt{40^2 - 5.5^2} =$ **39.62 Ω**

(ii) Power absorbed by the coil is $= I^2 R = 5^2 \times 5.5 = 137.5$ W

Also $P = 200 \times 5 \times 27.5/200 =$ **137.5 W**

(iii) Total power $= VI \cos\phi = 250 \times 5 \times AC/AD = 250 \times 5 \times 152.5/250 =$ **762.5 W**

The power may also be calculated by using $I^2 R$ formula.

Series resistance $= 125/5 = 25\ \Omega$

Total *circuit* resistance $= 25 + 5.5 = 30.5\ \Omega$

$\therefore$ Total power $= 5^2 \times 30.5 =$ **762.5 W**

Example 13.20. *Two coils A and B are connected in series across a 240-V, 50-Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.* **(Elect. Technology Hyderabad Univ. 1991)**

Solution. The kVA triangle is shown in Fig. 13.18 (*b*) and the circuit in Fig. 13.18(*a*). The circuit kVA is given by, kVA $= \sqrt{(3^2 + 2^2)} = 3.606$ or $VA = 3{,}606$ voltmeters

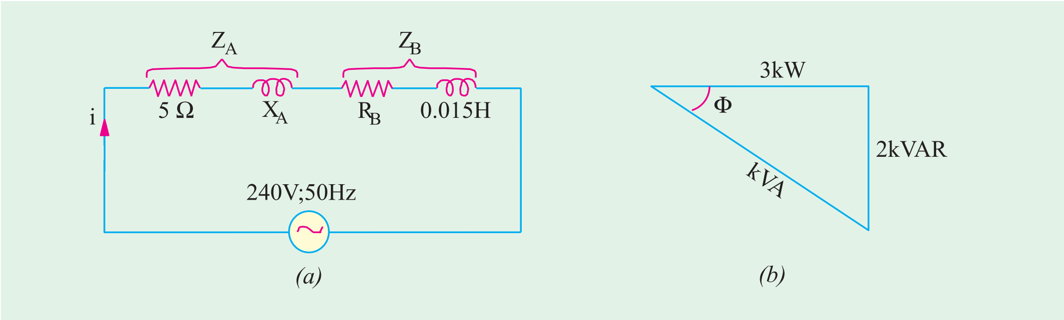

Fig. 13.18

Circuit current = 3,606/240 = 15.03 A $\quad \therefore\ 15.03^2 (R_A + R_B) = 3{,}000$

$\therefore\ R_A + R_B = 3{,}000/15.03^2 = 13.3\ \Omega \quad \therefore\ R_B = 13.3 - 5 =$ **8.3 Ω**

Now, impedance of the whole circuit is given by $Z = 240/15.03 = 15.97\ \Omega$

$$\therefore \quad X_A + X_B = \sqrt{(Z^2 - (R_A + R_B)^2} = \sqrt{15.97^2 - 13.3^3} = 8.84\ \Omega$$

$$\text{Now} \quad X_B = 2\pi \times 50 \times 0.015 = 4.713\ \Omega \quad \therefore\ X_A = 8.843 - 4.713 = 4.13\ \Omega$$

$$\text{or} \quad 2\pi \times 50 \times L_A = 4.13 \quad \therefore\ L_A = \mathbf{0.0132\ H\ (approx)}$$

$$\text{Now} \quad Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.13^2} = 6.585\ \Omega$$

$$\text{P.D. across coil } A = I \,.\, Z_A = 15.03 \times 6.485 = \mathbf{97.5\ V};\ Z_b = \sqrt{8.3^2 + 4.713^2} = 9.545\ \Omega$$

$$\therefore \quad \text{p.d. across coil } B = I \,.\, Z_B = 15.03 \times 9.545 = \mathbf{143.5\ V}$$

Example 13.21. *An e.m.f. e_0 = 141.4 sin (377 t + 30°) is impressed on the impedance coil having a resistance of 4 Ω and an inductive reactance of 1.25 Ω, measured at 25 Hz. What is the equation of the current ? Sketch the waves for i, e_R, e_L and e_0.*

Solution. The frequency of the applied voltage is $f = 377/2\pi$ = 60 Hz

Since coil reactance is 1.25 Ω at 25 Hz, its value at 60 Hz = 1.25 × 60/25 = 3 Ω

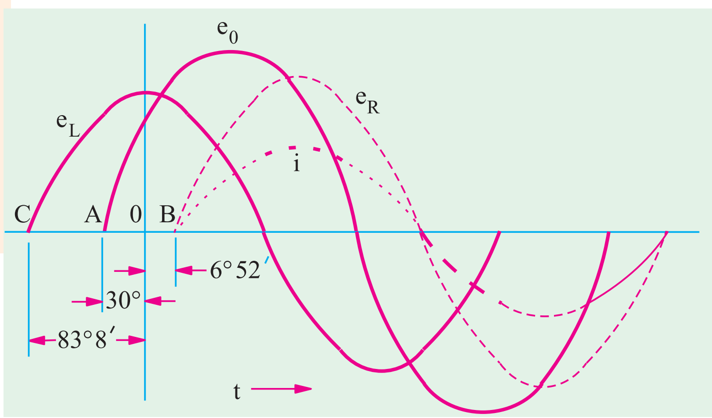

Fig. 13.19

Coil impedance, $Z = \sqrt{4^2 + 3^2}$, $= 5\ \Omega$; $\phi = \tan^{-1}(3/4) = 36°52'$

It means that circuit current lags behind the applied voltage by 36°52′ . Hence, equation of the circuit current is

$i = (141.4/5) \sin (377\,t + 30° - 36°52') = 28.3 \sin (377\,t - 6°52')$

Since, resistance drop is in phase with current, its equation is $e_R = iR = 113.2 \sin (377\,t - 6°52')$

The inductive voltage drop ***leads the current*** by 90°, hence its equation is

$$e_L = iX_L = 3 \times 28.3 \sin (377\,t - 6°52' + 90°) = 54.9 \sin (377\,t + 83°8')$$

The waves for i, e_R, e_L and e_0 have been drawn in Fig. 13.19.

Example 13.22. *A single phase, 7.46 kW motor is supplied from a 400-V, 50-Hz a.c. mains. If its efficiency is 85% and power factor 0.8 lagging, calculate* **(a)** *the kVA input* **(b)** *the reactive components of input current and* **(c)** *kVAR.*

Solution. Efficiency $= \dfrac{\text{output in watts}}{\text{input in watts}}$ $\quad \therefore \; 0.85 = \dfrac{7.46 \quad 1000}{VI \quad \cos} \quad \dfrac{7,460}{VI \quad 0.8}$

$\therefore \qquad VI = \dfrac{7460}{0.85 \times 0.8} = 10,970$ voltamperes

(*a*) $\therefore$ Input = 10,970/1000 = **10.97 kVA**

(*b*) Input current $I = \dfrac{\text{voltamperes}}{\text{volts}} = \dfrac{10,970}{400} = 27.43$ A

Active component of current = $I \cos \phi$ = **27.43** × 0.8 = **21.94 A**

Reactive component of current = $I \sin \phi$ = 27.43 × 0.6 = **16.46 A** ($\because \sin \phi = 0.6$)

(Reactive component = $\sqrt{27.43^2 \quad 21.94^2} \quad 16.46$ A

(*c*) kVAR = kVA sin φ = 10.97 × 0.6 = **6.58** (or kVAR = $VI \sin \phi \times 10^{-3} = 400 \times 16.46 \times 10^{-3}$ = 6.58)

Example 13.23. *Draw the phasor diagram for each of the following combinations :*

(*i*) *R and L in series and combination in parallel with C.*

(*ii*) *R, L and C in series with $X_C > X_L$ when ac voltage source is connected to it.*

[Nagpur University—Summer 2000]

Solution. (*i*)

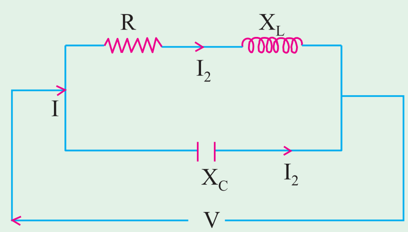

Fig. 13.20 (*a*) Circuit

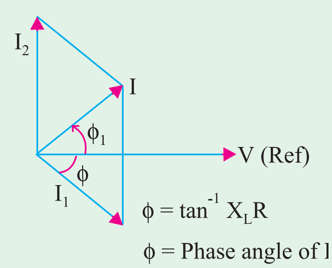

Fig. 13.20 (*b*) Phasor diagram

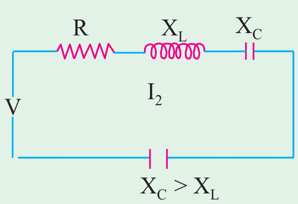

Fig. 13.20 (*c*) Circuit

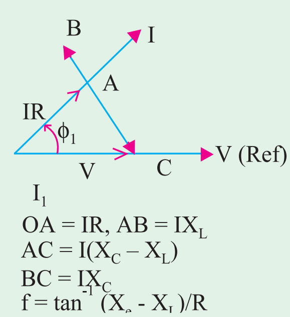

Fig . 13.20 (*d*) Phasor diagram

Example 13.24. *A voltage v (t) = 141.4 sin (314 t + 10°) is applied to a circuit and the steady current given by i(t) = 14.14 sin (314 t –20°) is found to flow through it.*

Determine

(i) *The p.f. of the circuit* *(ii)* *The power delivered to the circuit*

(iii) *Draw the phasor diagram.* **[Nagpur University Summer 2000]**

Solution. $v(t) = 141.4 \sin(314\,t + 10°)$

This expression indicates a sinusoidally varying alternating voltage at a frequency

$\omega = 314$ rad/sec, $f = 50$ Hz

V = RMS voltage (Peak voltage)/$\sqrt{2}$ = 100 volts

The expression for the current gives the following data :

I = RMS value = $14.14/\sqrt{2} = 10$ amp

frequency = 50 Hz, naturally.

Phase shift between I and $V = 30°$, I lags behind V.

(i) Power factor of the circuit = cos 30° = 0.866 lag

(ii) $P = VI \cos\phi = 100 \times 10 \times 0.866 = 866$ watts

(iii) Phasor diagram as drawn below, in Fig. 13.21 (*a*).

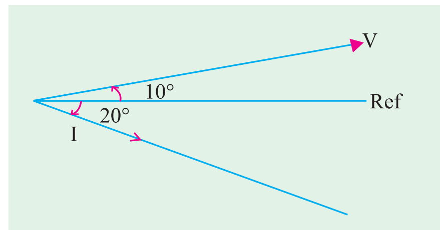

Fig. 13.21 (*a*) Phasor diagram

Example 13.25. *A coil of 0.8 p.f. is connected in series with 110 micro-farad capacitor. Supply frequency is 50 Hz. The potential difference across the coil is found to be equal to that across the capacitor. Calculate the resistance and the inductance of the coil. Calculate the net power factor.*

[Nagpur University, November 1997]

Solution. $X_C = 1/(3.14 \times C) = 28.952$ ohms

$\therefore$ Coil Impedance, $Z = 28.952\ \Omega$

Coil resistance = $28.952 \times 0.8 = 23.162\ \Omega$

Coil reactance = 17.37 ohms

Coil-inductance = 17.37/314 = 55.32 milli-henrys

Total impedance, $Z_T = 23.16 + j\,17.37 - j\,28.952 = 23.162 - j\,11.582 = 25.9$ ohms

Net power-factor = 23.162/25.9 = 0.8943 leading

Example 13.26. *For the circuit shown in Fig. 13.21 (c), find the values of R and C so that $V_b = 3V_a$, and V_b and V_a are in phase quadrature. Find also the phase relationships between V_s and V_b, and V_b and I.* **[Rajiv Gandhi Technical University, Bhopal, Summer 2001]**

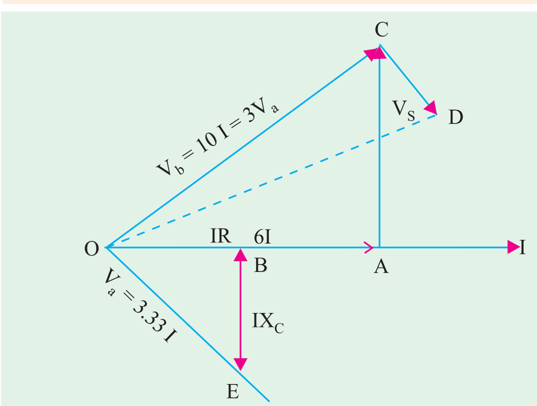

Fig. 13.21 (*b*)

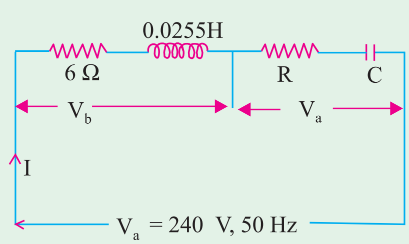

Fig . 13.21 (*c*)

Solution. $\angle COA = \phi = 53.13°$

$\angle BOE = 90° - 53.13° = 36.87°$

$\angle DOA = 34.7°$ Angle between V and I

Angle between V_s and $V_b = 18.43°$

$X_L = 314 \times 0.0255 = 8$ ohms

$Z_b = 6 + j\,8 = 10 \angle 53.13°$ ohms

$V_b = 10\,I = 3\,V_a$, and hence $V_a = 3.33\,I$

In phasor diagram, I has been taken as reference. V_b is in first quadrant. Hence V_a must be in the fourth quadrant, since Z_a consists of R and X_c. Angle between V_a and I is then 36.87°. Since Z_a and Z_b are in series, V is represented by the phasor OD which is at angle of 34.7°.

$$|V| = \sqrt{10}\,V_a = 10.53\,I$$

Thus, the circuit has a total effective impedance of 10.53 ohms.

In the phasor diagram, $OA = 6\,I$, $AC = 8\,I$, $OC = 10\,I = V_b = 3\,V_a$

Hence, $V_a = OE = 3.33\,I$,

Since $BOE = 36.87°$, $OB - RI = OE \times \cos 36.87° = 3.33 \times 0.8 \times I = 2.66\,I.$

Hence, $R = 2.66$

And $BE = OE \sin 36.87° = 3.33 \times 0.6 \times I = 2\,I$

Hence $X_c = 2$ ohms. For $X_c = 2$ ohms, $C = 1/(314 \times 2) = 1592$ µF

Horizontal component of $OD = OB + OA = 8.66\,I$

Vertical component of $OD = AC - BE = 6\,I$

$OD = 10.54\,I = V_s$

Hence, the total impedance = 10.54 ohms = $8.66 + j\,6$ ohms

Angle between V_s and $I = \angle DOA = \tan^{-1}(6/866) = 34.7°$

Example 13.27. *A coil is connected in series with a pure capacitor. The combination is fed from a 10 V supply of 10,000 Hz. It was observed that the maximum current of 2 Amp flows in the circuit when the capacitor is of value 1 microfarad. Find the parameters (R and L) of the coil.* **[Nagpur University April 1996]**

Solution. This is the situation of resonance in A.C. Series circuit, for which $X_L = X_C$

$Z = R = V/I = 10/2 = 5$ ohms

If ω_0 is the angular frequency, at resonance, L and C are related by $\omega_0^2 = 1/(LC)$,

which gives $L = 1/(\omega_0^2 C) = 2.5 \times 10^{-4}$ H = 0.25 mH

Example 13.28. *Two impedances consist of (resistance of 15 ohms and series-connected inductance of 0.04 H) and (resistance of 10 ohms, inductance of 0.1 H and a capacitance of 100 µF, all in series) are connectd in series and are connected to a 230 V, 50 Hz a.c. source. Find :* ***(i)*** *Current drawn,* ***(ii)*** *Voltage across each impedance,* ***(iii)*** *Individual and total power factor. Draw the phasor diagram.* **[Nagpur University, Nov. 1996]**

Solution. Let suffix 1 be used for first impedance, and 2 for the second one. At 50 Hz,

$Z_1 = 15 + j\,(314 \times 0.04) = 15 + j\,12.56$ ohms

$Z_1 = \sqrt{15^2 + 12.56^2} = 19.56$ ohms,

Impedance-angle, $\theta_1 = \cos^{-1}(15/19.56) = +40°$,

$Z_2 = 10 + j\,(314 \times 0.1) - j\,\{1/(314 \times 100 \times 16^{-6})\}$

$= 10 + j\,31.4 - j\,31.85 = 10 - j\,0.45$

$= 10.01$ ohms, Impedance angle, $\theta_2 = -2.56°$,

Total Impedance, $Z = Z_1 = Z_2 = 15 + j\,12.56 + 10 - j\,0.85$

$= 25 + j\,12.11 = 27.78\,\underline{|\mathbf{25.85°}}$

For this, Phase-angle of + 25.85°, the power-factor of the total impedance

$= \cos 25.85° = 0.90$, Lag.

Current drawn $= 230/27.78 = 8.28$ Amp, at 090 lagging p.f.

$V_1 = 8.28 \times 19.56 = 162$ Volts

$V_2 = 8.28 \times 10.01 = 82.9$ Volts

Individual Power-factor

$\cos \theta_1 = \cos 40° = 0.766$ Lagging

$\cos \theta_2 = \cos 2.56° = 0.999$ leading

Phasor diagram : In case of a series circuit, it is easier to treat the current as a reference. The phasor diagram is drawn as in Fig. 13.22.

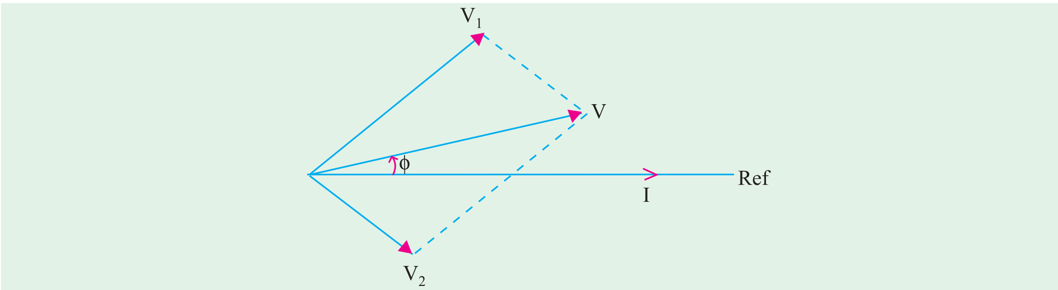

Fig. 13.22

Example 13.29. *Resistor (= R), choke-coil (r, L), and a capacitor of 25.2 μF are connected in series. When supplied from an A.C. source, in takes 0.4 A. If the voltage across the resistor is 20 V, voltage across the resistor and choke is 45 volts, voltage across the choke is 35 volts, and voltage across the capacitor is 50 V.*

Find : **(*a*)** *The values of r, L* **(*b*)** *Applied voltage and its frequency*, **(*c*)** *P.F. of the total circuit and active power consumed. Draw the phasor diagram.* **[Nagpur Univ. April 1998]**

Solution.

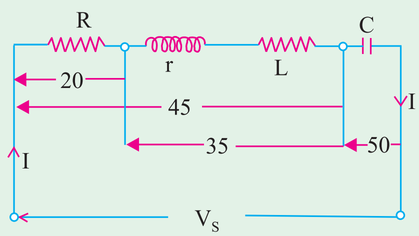

Fig. 13.23 (*a*)

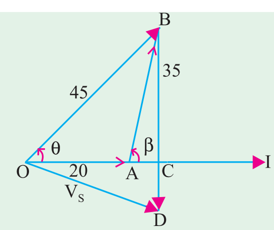

Fig. 13.23 (*b*)

(*b*) Since the current I is 0.4 amp, and voltage drop across R is 20 V,

$$R = 20/0.4 = 50 \text{ ohms}$$

Similarly, Impedance of the coil, $Z_1 = 35/0.4 = 87.5$ ohms

Capacitive reactance $X_c = 50/0.4 = 125$ ohms

With a capacitor of 25.5 μF, and supply angular frequency of ω radians/sec

$$\frac{1}{\omega} = X_C \,.\, C = 125 \times 25.5 \times 10^{-6}, \text{ which gives } \omega = 314 \text{ ra./sec.}$$

The corresponding source frequency, $f = 50$ Hz

(*c*) The phasor diagram is drawn in Fig. 13.23 (*b*), taking I as the reference.

Solving triangle OAB,

$$\cos\phi = \frac{45^2 + 20^2 - 35^2}{2\times 45\times 20} = 0.667, \text{ and hence } \phi = 48.2°$$

Similarly, $\cos(180° - \beta) = \dfrac{400 + 1225 - 2025}{2\times 20\times 35} = 106.6°$

This gives $\beta = 73.4°$. From the phasor diagram is Fig. 13.23 (*b*),

$$OC = OA + AC = 20 + 35\cos\beta = 30$$

$$BC = 35\sin\beta = 33.54. \text{ The capacitive–reactance drop is } BD.$$

Since $BD = 50$, $CD = 16.46$ volts.

$$V_s = \sqrt{OC^2 + CD^2} = 34.22 \text{ volts}$$

$$\angle COD = \phi = \cos^{-1}(OC/OD) = 28.75°$$

The power-factor of the total circuit = cos φ = 0.877, Leading,

Since I Leads V_s in Fig. 13.23 (*b*).

(*a*) For the coil, *ACB* part of the phasor diagram is to be observed.

$$r = AC/I = 10/0.4 = 25 \text{ ohms}$$

$$X_L = BC/I = 33.54/0.4 = 83.85 \text{ ohms}$$

Hence, coil-inductance, $L = 83.85/314 = 267$ milli-henry.

$$P = \text{Active Power Consumed} = V_s I\cos\phi = 12 \text{ watts}$$

or $P = (0.4)^2\times (R + r) = 12$ watts

13.6. Power in an Iron-cored Choking Coil

Total power, P taken by an iron-cored choking coil is used to supply

(*i*) power loss in ohmic resistance *i.e.* $I^2 R$. **(*ii*)** iron-loss in core, P_i

∴ $P = I^2 R + P_i$ or $\dfrac{P}{I^2} = R + \dfrac{P_1}{I^2}$ is known as the **effective resistance*** of the choke.

∴ effective resistance = true resistance = equivalent resistance $\dfrac{P_i}{I^2}$ $\therefore R_{eff} = \dfrac{P}{I^2} = R + \dfrac{P_i}{I^2}$

Example 13.30. *An iron-cored choking coil takes 5 A when connected to a 20-V d.c. supply and takes 5 A at 100 V a.c. and consumes 250 W. Determine* **(a)** *impedance* **(b)** *the power factor* **(c)** *the iron loss* **(d)** *inductance of the coil.* **(Elect. Engg. & M.A.S.I. June, 1991)**

Solution. (*a*) $Z = 100/5 =$ **20 Ω**

(*b*) $P = VI\cos\phi$ or $250 = 100\times 5\times\cos\phi$ ∴ $\cos\phi = 250/500 =$ **0.5**

(*c*) Total loss = loss in resistance + iron loss ∴ $250 = 20\times 5 + P_i$ ∴ $P_i = 250 - 100 =$ **150 W**

(*d*) Effective resistance of the choke is $\dfrac{P}{I^2} = \dfrac{250}{25} = 10\ \Omega$

∴ $X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(400 - 100)} =$ **17.32 Ω**

Example 13.31. *An iron-cored choking coil takes 5 A at a power factor of 0.6 when supplied at 100-V, 50 Hz. When the iron core is removed and the supply reduced to 15 V, the current rises to 6 A at power factor of 0.9.*

Determine **(a)** *the iron loss in the core* **(b)** *the copper loss at 5 A* **(c)** *the inductance of the choking coil with core when carrying a current of 5 A.*

* At higher frequencies like radio frequencies, there is skin-effect loss also.

Solution. When core is removed, then $Z = 15/6 = 2.5\ \Omega$

True resistance, $R = Z\cos\phi = 2.5 \times 0.9 = 2.25\ \Omega$

With Iron Core

Power input = $100 \times 5 \times 0.6 = 300$ W

Power wasted in the true resistance of the choke when current is 5 A = $5^2 \times 2.25 = 56.2$ W

(*a*) Iron loss = 300 − 56.2 = **244 W (approx)** (*b*) Cu loss at 5 A = **56.2 W**

(*c*) $Z = 100/5 = 20\ \Omega$; $X_L = Z\sin\phi = 20 \times 0.8 = 16\Omega \therefore 2\pi \times 50 \times L = 16 \quad \therefore L =$ **0.0509 N**

Tutorial Problem No. 13.1

1. The voltage applied to a coil having $R = 200\ \Omega$, $L = 638$ mH is represented by $e = 20 \sin 100\pi t$. Find a corresponding expression for the current and calculate the average value of the power taken by the coil. **[$i = 0.707 \sin (100\pi t - \pi/4)$; 50 W] (*I.E.E. London*)**
2. The coil having a resistance of 10 Ω and an inductance of 0.2 *H* is connected to a 100-V, 50-Hz supply. Calculate (*a*) the impedance of the coil (*b*) the reactance of the coil (*c*) the current taken and (*d*) the phase difference between the current and the applied voltage. **[(*a*) 63.5 Ω (*b*) 62.8 Ω (*c*) 1.575 A (*d*) 80°57′]**
3. An inductive coil having a resistance of 15 Ω takes a current of 4 A when connected to a 100-V, 60 Hz supply. If the coil is connected to a 100 -V, 50 Hz supply, calculate (*a*) the current (*b*) the power (*c*) the power factor. Draw to scale the vector diagram for the 50-Hz conditions, showing the component voltages. **[(*a*) 4.46 A (*b*) 298 W (*c*) 0.669]**
4. When supplied with current at 240-V, single-phase at 50 Hz, a certain inductive coil takes 13.62 A. If the frequency of supply is changed to 40 Hz, the current increases to 16.12 A. Calculate the resistance and inductance of the coil. **[17.2 W, 0.05 H] (*London Univ.*)**
5. A voltage $v(t) = 141.4 \sin (314t + 10°)$ is applied to a circuit and a steady current given by $i(t) = 14.4 \sin (314t - 20°)$ is found to flow through it. Determine (*i*) the p.f. of the circuit and (*ii*) the power delivered to the circuit. **[0.866 (lag); 866 W]**
6. A circuit takes a current of 8 A at 100 V, the current lagging by 30° behind the applied voltage. Calculate the values of equivalent resistance and reactance of the circuit. **[10.81 Ω; 6.25 Ω]**
7. Two inductive impedance *A* and *B* are connected in series. *A* has $R = 5\ \Omega$, $L = 0.01$ *H;* B has $R = 3\ \Omega$, $L = 0.02$ H. If a sinusoidal voltage of 230 V at 50 Hz is applied to the whole circuit calculate (*a*) the current (*b*) the power factor (*c*) the voltage drops. Draw a complete vector diagram for the circuit. **[(*a*) 18.6 (*b*) 0.648 (*c*) V_A = 109.5 V, V_B = 129.5 V] (*I.E.E. London*)**
8. A coil has an inductance of 0.1 H and a resistance of 30 Ω at 20°C. Calculate (*i*) the current and (*ii*) the power taken from 100-V, 50-Hz mains when the temperature of the coil is 60° C, assuming the temperature coefficient of resistance to be 0.4% per°C from a basic temperature of 20°C. **[(*i*) 2.13 A (*ii*) 158.5 W] (*London Univ.*)**
9. An air-cored choking coil takes a current of 2 A and dissipates 200 W when connected to a 200-V, 50-Hz mains. In other coil, the current taken is 3 A and the power 270 W under the same conditions. Calculate the current taken and the total power consumed when the coils are in series and connected to the same supply. **[1.2, 115 W] (*City and Guilds, London*)**
10. A circuit consists of a pure resistance and a coil in series. The power dissipated in the resistance is 500 W and the drop across it is 100 V. The power dissipated in the coil is 100 W and the drop across it is 50 V. Find the reactance and resistance of the coil and the supply voltage. **[9.168Ω; 4Ω; 128.5V]**
11. A choking coil carries a current of 15 A when supplied from a 50-Hz, 230-V supply. The power in the circuit is measured by a wattmeter and is found to be 1300 watt. Estimate the phase difference between the current and p.d. in the circuit. **[0.3768] (*I.E.E. London*)**
12. An ohmic resistance is connected in series with a coil across 230-V, 50-Hz supply. The current is 1.8

A and p.ds. across the resistance and coil are 80 V and 170 V respectively. Calculate the resistance and inductance of the coil and the phase difference between the current and the supply voltage.
[61.1 Ω, 0.229 H, 34°20′] (*App. Elect. London Univ.*)

13. A coil takes a current of 4 A when 24 V d.c. are applied and for the same power on a 50-Hz a.c. supply, the applied voltage is 40. Explain the reason for the difference in the applied voltage. Determine (*a*) the reactance (*b*) the inductance (*c*) the angle between the applied p.d. and current (*d*) the power in watts. **[(*a*) 8 Ω (*b*) 0.0255 H (*c*) 53°7′ (*d*) 96 W]**

14. An inductive coil and a non-inductive resistance *R* ohms are connected in series across an a.c. supply. Derive expressions for the power taken by the coil and its power factor in terms of the voltage across the coil, the resistance and the supply respectively. If $R = 12\ \Omega$ and the three voltages are in order, 110 V, 180 V and 240 V, calculate the power and the power factor of the coil. **[546 W; 0.331]**

15. Two coils are connected in series. With 2 A d.c. through the circuit, the p.ds. across the coils are 20 and 30 V respectively. With 2 A a.c. at 40 Hz, the p.ds. across the coils are 140 and 100 V respectively. If the two coils in series are connected to a 230-V, 50-Hz supply, calculate (*a*) the current (*b*) the power (*c*) the power factor. **[(*a*) 1.55 A (*b*) 60 W (*c*) 0.1684]**

16. It is desired to run a bank of ten 100-W, 10-V lamps in parallel from a 230-V, 50-Hz supply by inserting a choke coil in series with the bank of lamps. If the choke coil has a power factor of 0.2, find its resistance, reactance and inductance. **[*R* = 4.144 Ω, *X* = 20.35 Ω , *L* = 0.065 H] (*London Univ.*)**

17. At a frequency for which ω= 796, an e.m.f. of 6 V sends a current of 100 mA through a certain circuit. When the frequency is raised so that ω= 2866, the same voltage sends only 50 mA through the same circuit. Of what does the circuit consist ? **[*R* = 52 Ω, L = 0.038 H in series] (*I.E.E. London*)**

18. An iron-cored electromagnet has a d.c. resistance of 7.5 Ω and when connected to a 400-V 50-Hz supply, takes 10 A and consumes 2 kW. Calculate for this value of current (*a*) power loss in iron core (*b*) the inductance of coil (*c*) the power factor (*d*) the value of series resistance which is equivalent to the effect of iron loss. **[1.25 kW, 0.11 H, 0.5; 12.5 Ω] (*I.E.E. London*)**

13.7. A.C. Through Resistance and Capacitance

The circuit is shown in Fig. 13.24 (*a*). Here $V_R = IR$ = drop across *R* in phase with *I*.

$V_C = IX_C$ = drop across capacitor –lagging *I* by π/2

As capacitive reactance X_C is taken negative, V_C is shown along negative direction of *Y*-axis in the voltage triangle [Fig. 13.24 (*b*)]

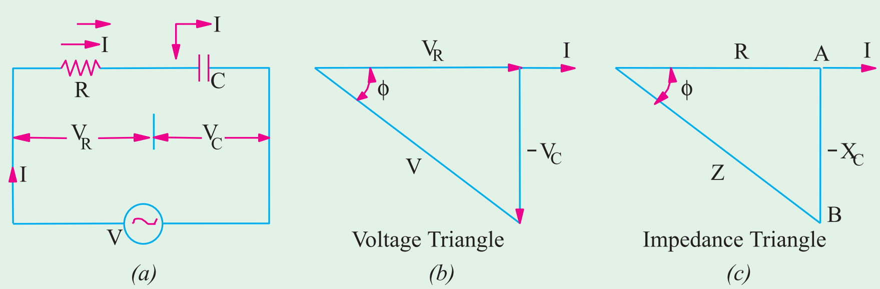

Fig. 13.24

Now $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$ or $I = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{V}{Z}$

The denominator is called the ***impedance*** of the circuit. So, $Z = \sqrt{R^2 + X_C^2}$

Impedance triangle is shown in Fig. 13.24 (*c*)

From Fig. 13.24 (*b*) it is found that *I* leads *V* by angle ϕ such that $\tan\phi = -X_C/R$

Hence, it means that if the equation of the applied alternating voltage is $v = V_m \sin \omega t$, the equation of the resultant current in the *R-C* circuit is $i = I_m \sin(\omega t + \phi)$ so that current ***leads*** the applied voltage by an angle ϕ. This fact is shown graphically in Fig. 13.25.

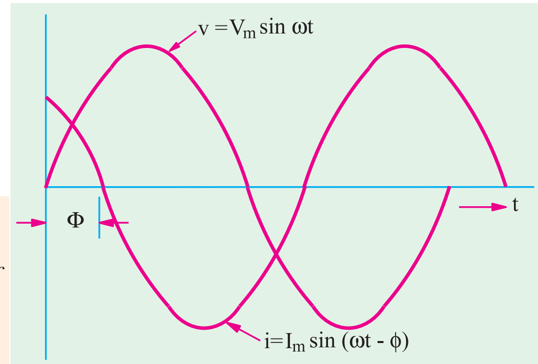

Fig. 13.25

Example 13.32. *An a.c. voltage (80 + j 60) volts is applied to a circuit and the current flowing is (–4 + j 10) amperes. Find **(i)** impedance of the circuit **(ii)** power consumed and **(iii)** phase angle.* **[Elect. Technology, Indore, Univ., Bombay Univ. 1999]**

Solution. $\mathbf{V} = (80 + j\,60) = 100 \angle 36.9°$;

$\mathbf{I} = -4 + j\,10 = 10.77 \angle \tan^{-1}(-2.5) = 10.77 \angle (180° - 68.2°) = 10.77 \angle 111.8°$

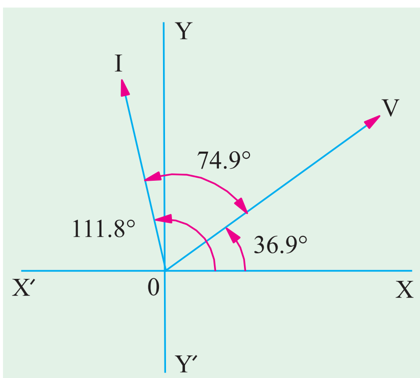

Fig. 13.26

(i) $\mathbf{Z} = \mathbf{V/I} = 100 \angle 36.9° / 10.77 \angle 111.8°$

$= 9.28 \angle -74.9°$

$= 9.28(\cos 74.9° - j \sin 74.9°) = 2.42 - j\,8.96\ \Omega$

Hence $R = 2.42\ \Omega$ and $X_C = 8.96\ \Omega$ capacitive

(ii) $P = I^2 R = 10.77^2 \times 2.42 =$ **2.81 W**

(iii) Phase angle between voltage and current = 74.9° with current ***leading*** as shown in Fig. 13.26.

Alternative Method for Power

The method of conjugates will be used to determine the real power and reactive volt-ampere. It is a convenient way of calculating these quantities when both voltage and current are expressed in cartesian form. If the conjugate of *current* is multiplied by the voltage in cartesian form, the result is a complex quantity, the real part of which gives the real power *j* part of which gives the reactive volt-amperes (*VAR*). It should, however, be noted that real power as obtained by this method of conjugates is the same regardless of whether *V* or *I* is reversed although sign of voltamperes will depend on the choice of *V* or *I*.*

Using current conjugate, we get $P_{VA} = (80 + j\,60)(-4 - j\,10) = 280 - j\,1040$

$\therefore$ Power consumed = **280 W**

Example 13.33. *In a circuit, the applied voltage is 100 V and is found to lag the current of 10 A by 30°. **(i)** Is the p.f. lagging or leading ? **(ii)** What is the value of p.f. ?*

***(iii)** Is the circuit inductive or capacitive ? **(iv)** What is the value of active and reactive power in the circuit ?* **(Basic Electricity, Bombay Univ.)**

Solution. The applied voltage lags behind the current which, in other words, means that current leads the voltage.

(i) $\therefore$ p.f. is **leading** ***(ii)*** p.f. = $\cos\phi = \cos 30°$ = **0.866 (lead)** ***(iii)*** Circuit is **capacitive** ***(iv)*** Active power = $VI \cos\phi = 100 \times 10 \times 0.866 =$ **866 W**

Reactive power $= VI \sin\phi = 100 \times 10 \times 0.5 =$ **500 VAR (lead)**

or $VAR = \sqrt{(VA)^2 - W^2} = \sqrt{(100 \times 10)^2 - 866^2} =$ **500 (lead)**

* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs are positive.

Example 13.34. *A tungsten filament bulb rated at 500-W, 100-V is to be connected to series with a capacitance across 200-V, 50-Hz supply. Calculate :*

(a) *the value of capacitor such that the voltage and power consumed by the bulb are according to the rating of the bulb.* ***(b)*** *the power factor of the current drawn from the supply.* ***(c)*** *draw the phasor diagram of the circuit.* **(Elect. Technology-1, Nagpur Univ. 1991)**

Solution. The rated values for bulb are : voltage = 100 V and current $I = W/V = 500/100$ = 5A. Obviously, the bulb has been treated as a pure resistance :

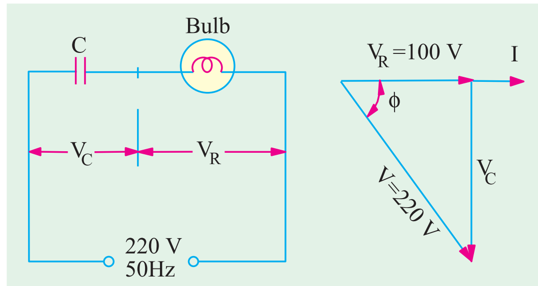

Fig 13.27

(a) $V_C = \sqrt{220^2 - 100^2} = 196\ V$

Now, $IX_C = 196$ or $5\ X_C = 196$, $X_C = 39.2\ \Omega$

$\therefore I/\omega C = 39.2$ or $C = 1/314 \times 39.2 =$ **81 μF**

(b) p.f. = $\cos\phi = V_R/V = 100/200 =$ **0.455 (lead)**

(c) The phasor diagram is shown in Fig 13.27.

Example 13.35. *A pure resistance of 50 ohms is in series with a pure capacitance of 100 microfarads. The series combination is connected across 100-V, 50-Hz supply. Find* ***(a)*** *the impedance (b) current* ***(c)*** *power factor* ***(d)*** *phase angle* ***(e)*** *voltage across resistor* ***(f)*** *voltage across capacitor. Draw the vector diagram.* **(Elect. Engg.-1, JNT Univ. Warrangel)**

Solution. $X_C = 10^6\pi/2\pi \times 50 \times 100 = 32\ \Omega\ ; R = 50\ \Omega$

(a) $Z = \sqrt{50^2 + 32^2}$ **59.4 Ω** **(b)** $I = V/Z = 100/59.4 =$ **1.684 A**

(c) p.f. = $R/Z = 50 = 59.4 =$ **0.842 (lead)** **(d)** $\phi = \cos^{-1}$ (0.842)= **32°36′**

(e) $V_R = IR = 50 \times 1.684 =$ **84.2 V** **(f)** $V_C = IX_C = 32 \times 1.684 =$ **53.9 V**

Example 13.36. *A 240-V, 50-Hz series R-C circuit takes an r.m.s. current of 20 A. The maximum value of the current occurs 1/900 second before the maximum value of the voltage. Calculate* ***(i)*** *the power factor* ***(ii)*** *average power* ***(iii)*** *the parameters of the circuit.*

(Elect. Engg.-I, Calcutta Univ.)

Solution. Time-period of the alternating voltage is 1/50 second. Now a time interval of 1/50 second corresponds to a phase difference of 2 π radian or 360°. Hence, a time interval of 1/900 second corresponds to a phase difference of $360 \times 50/900 = 20°$.

Hence, current leads the voltage by 20º.

(i) power factor = cos 20° = **0.9397 (lead)**

(ii) averge power = $240 \times 20 \times 0.9397 =$ **4,510 W**

(iii) $Z = 240/20 = 12\ \Omega$; $R = Z\cos\Phi = 12 \times 0.9397 =$ **11.28 Ω**

$X_C = Z\sin\Phi = 12 \times \sin 20° = 12 \times 0.342 = 4.1\ \Omega$

$C = 10^6/2\pi \times 50 \times 4.1 =$ **775 μF**

Example 13.37. *A voltage v = 100 sin 314 t is applied to a circuit consisting of a 25 Ω resistor and an 80 μF capacitor in series. Determine :* ***(a)*** *an expression for the value of the current flowing at any instant* ***(b)*** *the power consumed* ***(c)*** *the p.d. across the capacitor at the instant when the current is one-half of its maximum value.*

Solution. $X_C = 1/(314 \times 80 \times 10^{-6}) = 39.8\ \Omega$, $Z = \sqrt{25^2 + 39.8^2} = 47\ \Omega$

$I_m = V_m/Z = 100/47 = 2.13$ A

$\phi = \tan^{-1}(39.8/25) = 57°52' = 1.01$ radian (lead)

(a) Hence, equation for the instantaneous current

$i =$ **2.13 sin (314 t + 1.01)** **(b)** Power $= I^2 R = (2.13/\sqrt{2})^2 \times 25$ = **56.7 W** **(c)** The voltage across the capacitor lags the circuit current by π/2 radians. Hence, its equation is given by

$$v_c = V_{cm} \sin\left(314\,t + 1.01 - \frac{\pi}{2}\right) \text{ where } V_{cm} = I_m \times X_C = 2.13 \times 39.8 = 84.8\,V$$

Now, when i is equal to half the maximum current (say, in the positive direction) then

$$i = 0.5 \times 2.13 \text{ A}$$

∴ $0.5 \times 2.13 = 2.13 \sin(314\,t + 1.01)$ or $314\,t + 1.01 = \sin^{-1}(0.5) = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ radian

∴ $v_C = 84.8 \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = 84.8 \sin(-\pi/3) = -73.5$ V

or $v_c = 34.8 \sin\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) = 84.4 \sin \pi/3 =$ **73.5 V**

Hence, p.d. across the capacitor is **73.5 V**

Example 13.38. *A capacitor and a non-inductive resistance are connected in series to a 200-V, single-phase supply. When a voltmeter having a non-inductive resistance of 13,500 Ω is connected across the resistor, it reads 132 V and the current then taken from the supply is 22.35 mA.*

Indicate on a vector diagram, the voltages across the two components and also the supply current ***(a)*** *when the voltmeter is connected and* ***(b)*** *when it is disconnected.*

Solution. The circuit and vector diagrams are shown in Fig. 13.28 (*a*) and (*b*) respectively.

(a) $V_C = \sqrt{200^2 - 132^2} =$ **150 V**

It is seen that $\phi = \tan^{-1}(150/132) = 49°$ in Fig. 13.28 (*b*). Hence

(i) Supply voltage lags behind the current by 49°. **(ii)** V_R leads supply voltage by 49° (*iii*) V_C lags behind the supply voltage by (90° −49°) = 41°

The supply current is, as given equal to **22.35 mA.** The value of unknown resistance R can be found as follows :

Current through voltmeter = 132/13,500 = 9.78 mA

∴ Current through $R = 22.35 - 9.78 = 12.57$ mA ∴ $R = 132/12.57 \times 10^{-3} = 10{,}500\ \Omega$;

$X_C = 150/22.35 \times 10^{-3} = 6.711\ \Omega$

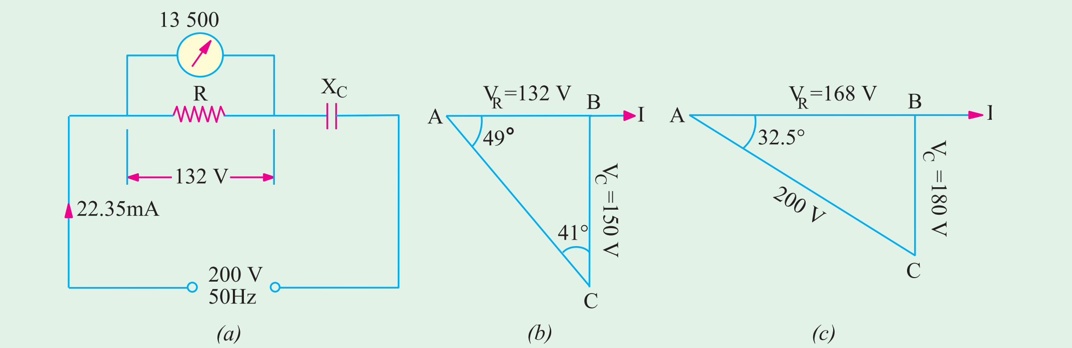

Fig. 13.28

(b) When voltmeter is disconnected, $Z = \sqrt{R^2 + X_L^2} = \sqrt{10{,}500^2 + 6{,}730^2} = 12{,}500\ \Omega$

Supply current = 200/12,461 = **16.0 mA**

In this case, $V_R = 16.0 \times 10^{-3} \times 10{,}500 =$ **168 V**

$V_C = 16.0 \times 10^{-3} \times 6711 =$ **107.4 V;** $\tan\phi = 107.4/168$

∴ $\phi = 32.5°$.

In this case, the supply voltage lags the circuit current by 32.5° as shown in Fig. 13.28 (*c*).

Example 13.39. *It is desired to operate a 100-W, 120-V electric lamp at its current rating from a 240-V, 50-Hz supply. Give details of the simplest manner in which this could be done using* **(a)** *a resistor* **(b)** *a capacitor and* **(c)** *an indicator having resistance of 10 Ω. What power factor would be presented to the supply in each case and which method is the most economical of power.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. Rated current of the bulb is = 100/120 = 5/6 A

The bulb can be run at its correct rating by any one of the three methods shown in Fig. 13.29. (*a*) With reference to Fig. 13.29 (*a*), we have

P.D. across $R = 240 - 120 = 120$ V

∴ $R = 120/(5/6) =$ **144 Ω**

Power factor of the circuit is **unity.** Power consumed = 240 × 5/6 = **200 W**

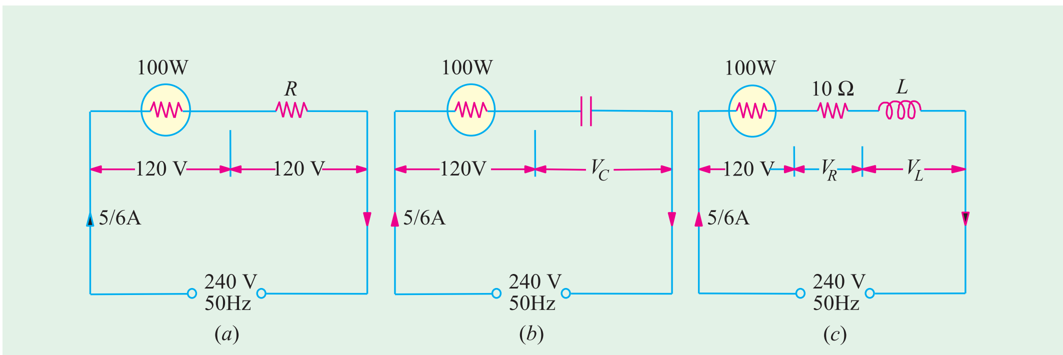

Fig. 13.29

(b) Referring to Fig. 13.29 (*b*), we have

$V_C = \sqrt{240^2 - 120^2} = 207.5\ V;\ X_C = 207.5\ (5/6) = 249\ \Omega$

∴ $1/314\ C = 249$ or $C =$ **12.8 μF;** p.f. = cos φ = 120/240 = **0.5 (lead)**

Power consumed = 240 × (5/6) × 0.5 = **100 W**

(c) The circuit connections are shown in Fig 13.29 (*c*)

$$V_R = (5/6) \times 10 = 25/3 \text{ V} \quad \therefore\ V_L = \sqrt{240^2 - \left(120 + \frac{25}{3}\right)^2} = 203\ V$$

∴ $314\,L \times (5/6) = 203$ ∴ $L =$ **0.775 H**

Total resistive drop = 120 + (25/3) = 128.3 V; cos φ = 128.3/240 = **0.535 (lag)**

Power consumed = 240 × (5/6) × 0.535 = **107 W**

Method (*b*) is most economical because it involves least consumption of power.

Example 13.40. *A two-element series circuit consumes 700 W and has a p.f. = 0.707 leading. If applied voltage is v = 141.1 sin (314 t + 30°), find the circuit constants.*

Solution. The maximum value of voltage is 141.4 V and it leads the reference quantity by 30°. Hence, the given sinusoidal voltage can be expressed in the phase form as

V $= (141.4/\sqrt{2})\angle 30° = 100\angle 30°$ now, $P = VI\cos\phi$ ∴ $700 = 100 \times I \times 0.707$; $I = 10$ A.

Since p.f. = 0.707 (lead); $\phi = \cos^{-1}(0.707) = 45°$ (lead).

It means that current leads the given voltage by 45° for it ***leads the common reference quantity***

by (30° + 45°) = 75°. Hence, it can be expressed as $\mathbf{I} = 10 \angle 75°$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \frac{100 \quad 30}{10 \quad 75} \quad 10 \qquad j\,7.1 \quad R \quad \mathbf{7.1\,\Omega}$$

Since $X_C = 7.1$ ∴ $1/314\,C = 7.1$; ∴ $C = \mathbf{450\ \mu F}$

13.8. Dielectric Loss and Power Factor of a Capacitor

An ideal capacitor is one in which there are no losses and whose current leads the voltage by 90° as shown in Fig. 13.30 (*a*). In practice, it is impossible to get such a capacitor although close approximation is achieved by proper design. In every capacitor, there is always some dielectric loss and hence it absorbs some power from the circuit. Due to this loss, the phase angle is somewhat less than 90° [Fig. 13.30 (*b*)]. In the case of a capacitor with a poor dielectric, the loss can be considerable and the phase angle much less than 90°. This dielectric loss appears as heat. By ***phase difference*** is meant the difference between the ideal and actual phase angles. As seen from Fig. 13.30 (*b*), the phase difference ψ is given by $\psi = 90 - \phi$ where ϕ is the actual phase angle, $\sin \psi = \sin (90 - \phi) = \cos \phi$ where $\cos \phi$ is the power factor of the capacitor.

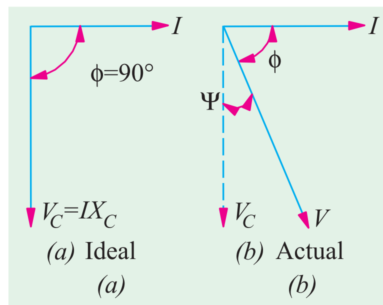

Fig. 13.30

Since ψ is generally small, $\sin \psi = \psi$ (in radians) ∴ $\tan \psi = \psi = \cos \phi$.

It should be noted that dielectric loss increases with the frequency of the applied voltage. Hence phase difference increases with the frequency *f*.

The dielectric loss of an actual capacitor is allowed for by imagining it to consist of a pure capacitor having an equivalent resistance either in series or in parallel with it as shown in Fig. 13.31. These resistances are such that $I^2 R$ loss in them is equal to the dielectric loss in the capacitor.

As seen from Fig. (13.27*b*), $\tan \psi = \dfrac{IR_{se}}{IX_C} \quad \dfrac{R_{se}}{1/\ \ C} \quad CR_{se}$ ∴ $R_{se} = \tan \psi / \omega C = \text{p.f.}/\omega C$

Similarly, as seen from Fig. 13.31 (*d*), $\tan \psi = \dfrac{I_1}{I_2} \quad \dfrac{V/R_{sh}}{V/X_C} \quad \dfrac{X_C}{R_{sh}} \quad \dfrac{1}{CR_{sh}}$

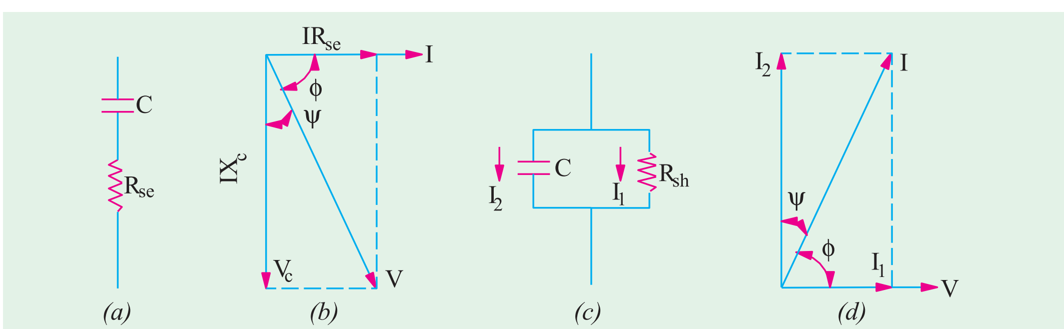

Fig. 13.31

$$R_{sh} = \frac{1}{\omega C \,.\, \tan \psi} = \frac{1}{\omega C \times \text{power factor}} = \frac{1}{\omega C \times \text{p.f.}}$$

The power loss in these resistances is $P = V^2/R_{sh} = \omega C V^2 \tan \psi = \omega\, C V^2 \times$ p.f.

or $= I^2 R_{se} = (I^2 \times \text{p.f.})/\omega C$

where p.f. stands for the power factor of the capacitor.

Note. (*i*) In case, ψ is not small, then as seen from Fig. 13.41 (*b*) $\tan \phi = \dfrac{X_C}{R_{se}}$ (Ex. 13.41) $R_{se} = X_C/\tan \phi$

From Fig. 13.31 (*d*), we get $\tan\phi = \dfrac{I_2}{I_1} = \dfrac{V/X_C}{V/R_{sh}} = \dfrac{R_{sh}}{X_C}$ $\therefore$ $R_{sh} = X_C \tan\phi = \tan\phi/\omega C$

(ii) It will be seen from above that both R_{se} and R_{sh} vary inversely as the frequency of the applied voltage. In other words, the resistance of a capacitor decreases in proportion to the increase in frequency.

$$\frac{R_{se1}}{R_{se2}} = \frac{f_2}{f_1}$$

Example 13.41. *A capacitor has a capacitance of 10 μF and a phase difference of 10°. It is inserted in series with a 100 Ω resistor across a 200-V, 50-Hz line, Find* ***(i)*** *the increase in resistance due to the insertion of this capacitor* ***(ii)*** *power dissipated in the capacitor and* ***(iii)*** *circuit power factor.*

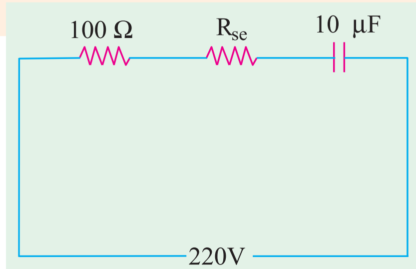

Fig. 13.32

Solution. $X_C = \dfrac{10^6}{2\pi \times 50 \times 10} = 318.3\ \Omega$

The equivalent series resistance of the capacitor in Fig. 13.32 is $R_{se} = X_C/\tan\phi$

Now $\phi = 90 - \psi = 90° - 10° = 80°$

$\tan\phi = \tan 80° = 5.671$

$\therefore$ $R_{se} = 318.3/5.671 = 56.1\ \Omega$

(i) Hence, resistance of the circuit increases by **56.1 Ω**

(ii) $Z = \sqrt{(R + R_{se})^2 + X_C^2} = \sqrt{156.1^2 + 318.3^2} = 354.4\ \Omega\ ;\ I = 220/354 = 0.62\ \text{A}$

Power dissipated in the capacitor $= I^2 R_{se} = 0.62^2 \times 56.1 =$ **21.6 W**

(iii) Circuit power factor is $= (R + R_{se})/Z = 156.1/354.4 =$ **0.44 (lead)**

Example 13.42. *Dielectric heating is to be employed to heat a slab of insulating material 2 cm thick and 150 sq. cm in area. The power required is 200 W and a frequency of 30 MHz is to be used. The material has a relative permittivity of 5 and a power factor of 0.03. Determine the voltage necessary and the current which will flow through the material. If the voltage were to be limited to 600-V, to what would the frequency have to be raised ?*

[Elect. Engg. AMIETE (New Scheme) June 1992]

Solution. The capacitance of the parallel-plate capacitor formed by the insulating slab is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 5 \times 150 \times 10^{-4}}{2 \times 10^{-2}} = 33.2 \times 10^{-12}\ F$$

As shown in Art. 13.8 $R_{sh} = \dfrac{1}{\omega C \times p.f.} = \dfrac{1}{(2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 0.05} = 3196\ \Omega$

Now, $P = V^2/R_{sh}$ or $V = \sqrt{P \times R_{sh}} = \sqrt{200 \times 3196} = 800\ V$

Current $I = V/X_C = \omega CV = (2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 800 = 5\ A$

Now, as seen from above $P = \dfrac{V^2}{R_{sh}} = \dfrac{V^2}{1/\omega C \times p.f.} = V^2 \omega C \times \text{p.f.}$ or $P \propto V^2 f$

$\therefore$ $800^2 \times 30 = 600^2 \times f$ or $f = \left(\dfrac{800}{600}\right)^2 \times 30 =$ **53.3 MHz**

Tutorial Problem No. 13.2

1. A capacitor having a capacitance of 20 μF is connected in series with a non-inductive resistance of 120 Ω across a 100-V, 50-Hz supply, Calculate (*a*) voltage (*b*) the phase difference between the current and the supply voltage (*c*) the power. Also draw the vector diagram.

[(*a*) 0.501A (*b*) 52.9° (*c*) 30.2 W]

2. A capacitor and resistor are connected in series to an a.c. supply of 50 V and 50 Hz. The current is 2 A and the power dissipated in the circuit is 80 W. Calculate the resistance of the resistor and the capacitance of the capacitor. **[20 Ω ; 212 μF]**
3. A voltage of 125 *V* at 50 Hz is applied to a series combination of non-inductive resistor and a lossless capacitor of 50 μF. The current is 1.25 A. Find (*i*) the value of the resistor (*ii*) power drawn by the network (*iii*) the power factor of the network. Draw the phasor diagram for the network.
[(*i*) 77.3 Ω (*ii*) 121 W (*iii*) 0.773 (lead)] (*Electrical Technology-1, Osmania Univ.*)
4. A black box contains a two-element series circuit. A voltage (40 –*j*30) drives a current of (40 –*j*3) A in the circuit. What are the values of the elements ? Supply frequency is 50 Hz.
[*R* = 1.05 ; *C* = 4750 μF] (*Elect. Engg. and Electronics Bangalore Univ.*)
5. Following readings were obtained from a series circuit containing resistance and capacitance :
$V = 150\ V$; $I = 2.5$ A; $P = 37.5\ W$, $f = 60$ Hz.
Calculate (*i*) Power factor (*ii*) effective resistance (*iii*) capacitive reactance and (*iv*) capacitance.
[(*i*) 0.1 (*ii*) 6 Ω (*iii*) 59.7 Ω (*iv*) 44.4 μF]
6. An alternating voltage of 10 volt at a frequency of 159 kHz is applied across a capacitor of 0.01 μF. Calculate the current in the capacitor. If the power dissipated within the dielectric is 100 μW, calculate (*a*) loss angle (*b*) the equivalent series resistance (*c*) the equivalent parallel resistance.
[0.A (*a*) 10^{-4} radian (*b*) 0.01 Ω (*c*) 1 MΩ]

13.9. Resistance, Inductance and Capacitance in Series

The three are shown in Fig. 13.33 (*a*) joined in series across an a.c. supply of r.m.s. voltage *V*.

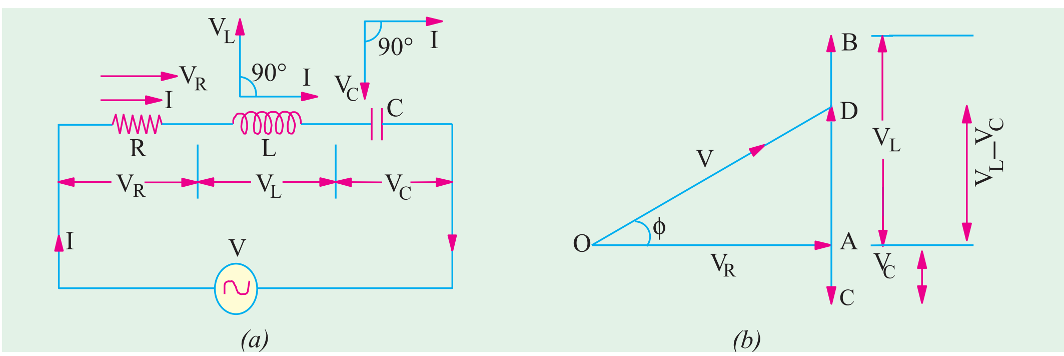

Fig. 13.33

Let $\quad V_R = IR$ = voltage drop across *R* —in phase with *I*

$V_L = I.X_L$ = voltage drop across *L* —leading *I* by π/2

$V_C = I.X_C$ = voltage drop across *C* —lagging *I* by π/2

In voltage triangle of Fig. 13.33 (*b*), *OA* represents V_R, *AB* and *AC* represent the inductive and capacitive drops respectively. It will be seen that V_L and V_C are 180° out of phase with each other *i.e.* they are in direct opposition to each other.

Subtracting *BD* (= *AC*) from *AB*, we get the net reactive drop $AD = I\,(X_L - X_C)$

The applied voltage *V* is represented by *OD* and is the vector sum of *OA* and *AD*

$$\therefore \quad OD = \sqrt{OA^2 \quad AD^2} \text{ or } V \quad \sqrt{(IR)^2 \quad (IX_L \quad IX_C)^2} \quad I\sqrt{R^2 \quad (X_L \quad X_C)^2}$$

$$\text{or} \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of the circuit. Obviously,

(impedance)^2 = (resistance)^2 + (net reactance)^2

or $\quad Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

where X is the net reactance (Fig. 13.33 and 13.34).

Phase angle ϕ is given by $\tan\phi = (X_L - X_C)/R = X/R$ = net reactance/resistance

Power factor is $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{R}{\sqrt{R^2 + X^2}}$

Hence, it is seen that if the equation of the applied voltage is $v = V_m \sin \omega t$, then equation of the resulting current in an *R-L-C* circuit is given by $i = I_m \sin(\omega t \pm \phi)$

The + ve sign is to be used when current leads *i.e.* $X_C > X_L$.

The −ve sign is to be used when current lags *i.e.* when $X_L > X_C$.

In general, the current lags or leads the supply voltage by an angle ϕ such that $\tan\phi = X/R$

Using symbolic notation, we have (Fig. 13.35), $\mathbf{Z} = R + j(X_L - X_C)$

Numerical value of impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

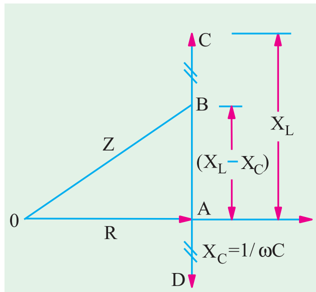

Fig. 13.34

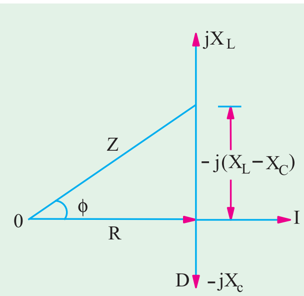

Fig. 13.35

Its phase angle is $\Phi = \tan^{-1}[X_L - X_C/R]$

$Z = Z \angle \tan^{-1}[(X_L - X_C)/R] = Z \angle \tan^{-1}(X/R)$

If $\mathbf{V} = V \angle 0$, then, $\mathbf{I = V/Z}$

Summary of Results of Series AC Circuits

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90°$ lag	1 > p.f. > 0 lag
Resistance and Capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90°$ lead	1 > p.f. > 0 lead
R-L-C	$\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

Example 13.43. *A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across 220-V, 50-Hz mains. Determine the following* ***(a)*** *impedance* ***(b)*** *current* ***(c)*** *voltage across R, L and C* ***(d)*** *power in watts and VA* ***(e)*** *p.f. and angle of lag.*

(Elect. Engg. A.M.Ae S.I. 1992)

Solution. $X_C = 0.2 \times 314 = 63\ \Omega$, $C = 10\ \mu F = 100 \times 10^{-6} = 10^{-4}$ farad

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 10^{-4}} = 32\ \Omega$, $X = 63 - 32 =$ **31 Ω (*inductive*)**

(*a*) $Z = \sqrt{(20^2 + 31)^2} =$ **37 Ω** **(*b*)** $I = 220/37 =$ **6 A (approx)**

(*c*) $V_R = I \times R = 6 \times 20 =$ **120 V;** $V_L = 6 \times 63 =$ **278 V,** $V_C = 6 \times 32 =$ **192 V**

(*d*) Power in $VA = 6 \times 220 =$ **1320**

Power in watts $= 6 \times 220 \times 0.54 =$ **713 W**

(*e*) p.f. $= \cos\phi = R/Z = 20/37 =$ **0.54;** $\phi = \cos^{-1}(0.54) =$ **57°18′**

Example 13.44. *A voltage e(t) = 100 sin 314 t is applied to series circuit consisting of 10 ohm resistance, 0.0318 henry inductance and a capacitor of 63.6 μF. Calculate* **(i)** *expression for i (t)* **(ii)** *phase angle between voltage and current* **(iii)** *power factor* **(iv)** *active power consumed* **(v)** *peak value of pulsating energy.* **(Elect. Technology, Indore Univ.)**

Solution. Obviously, $\omega = 314$ rad/s ; $X_l = \omega L = 314 \times 0.0318 = 10\ \Omega$

$X_C = 1/\omega C = 1/314 \times 63.6 \times 10^{-6} = 50\ \Omega$; $X = X_L - X_C = (10 - 50) = -40\ \Omega$ (capacitive)

$\mathbf{Z} = 10 - j\,40 = 41.2\ \angle -76°$;

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \quad \frac{(100/\sqrt{2})}{} \quad 1.716$$

$$I_m = I \times \sqrt{2} = 1.716 \times \sqrt{2} = 2.43\ A$$

(i) $i\,(t) = 2.43 \sin (314\,t + 76°)$

(ii) $\phi = 76°$ with current leading

(iii) p.f. = $\cos\phi = \cos 76° = $ **0.24 (lead)**

(iv) Active power, $P = VI \cos\phi$

$= (100/\sqrt{2})\,(2.43/\sqrt{2}) \times 0.24 = $ **29.16 W**

(v) As seen from Fig. 13.36, peak value of pulsating energy is $\dfrac{V_m I_m}{2} + \dfrac{V_m I_m}{2}\cos\phi$

$$= \frac{V_m I_m}{2}(1 + \cos\phi) = \frac{100 \times 2.43}{2}(1 + 0.24) = \mathbf{151\ W}$$

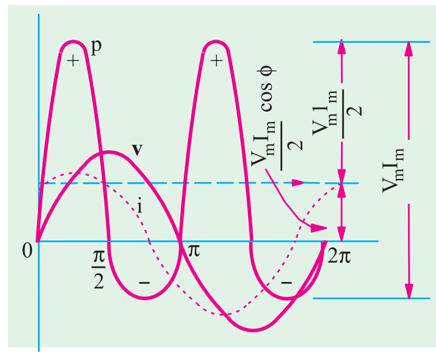

Fig. 13.36

Example 13.45. *Two impedances Z_1 and Z_2 when connected separately across a 230-V, 50-Hz supply consumed 100 W and 60 W at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply, find :*

(i) *total power absorbed and overall p.f.* **(ii)** *the value of the impedance to be added in series so as to raise the overall p.f. to unity.* **(Elect. Circuits-I, Bangalore Univ.)**

Solution. Inductive Impedance $V_1 I \cos\phi_1 = $ power; $230 \times I_1 \times 0.5 = 100$; $I_1 = 0.87$ A

Now, $I_1^2 R_1 = $ power or $0.87^2 R_1 = 100$; $R_1 = 132\ \Omega$; $Z_1 = 230/0.87 = 264\ \Omega$

$$X_L = \sqrt{Z_1^2 - R_1^2} = \sqrt{264^2 - 132^2} = 229\ \Omega$$

Capacitance Impedance $I_2 = 60/230 \times 0.6 = 0.434$ A ; $R_2 = 60/0.434^2 = 318\ \Omega$

$$Z_2 = 230/0.434 = 530\ \Omega\ ;\ X_C = \sqrt{530^2 - 318^2} = 424\ \Omega \text{ (capacitive)}$$

When Z_1 and Z_2 are connected in series

$$R = R_1 + R_2 = 132 + 318 = 450\ \Omega;\ X = 229 - 424 = -195\ \Omega \text{ (capacitive)}$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{450^2 + (-195)^2} = 490\ \Omega,\ I = 230/490 = 0.47\ \text{A}$$

(i) Total power absorbed = $I^2 R = 0.47^2 \times 450 = $ **99 W**, $\cos\phi = R/Z = 450/490 = $ **0.92 (lead)**

(ii) Power factor will become unity when the net capacitive reactance is neutralised by an equal inductive reactance. The reactance of the required series pure inductive coil is **195 Ω**

Example 13.46. *A resistance R, an inductance L = 0.01 H and a capacitance C are connected in series. When a voltage v = 400 cos (300 t –10°) votls is applied to the series combination, the current flowing is $10\sqrt{2}\cos(3000\,t - 55°)$ amperes. Find R and C.*

(Elect. Circuits Nagpur Univ. 1992)

Solution. The phase difference between the applied voltage and circuit current is (55° –10°) = 45° with current lagging. The angular frequency is $\omega = 3000$ radian/second. Since current lags, $X_L > X_C$.

Net reactance $X = (X_L - X_C)$. Also $X_L = \omega L = 3000 \times 0.01 = 30\ \Omega$

$\tan\phi = X/R$ or $\tan 45° = X/R$ $\therefore X = R$ Now, $Z = \dfrac{V_m}{I_m} = \dfrac{400}{10\sqrt{2}} = 28.3\ \Omega$

$Z^2 = R^2 + X^2 = 2R^2$ $\therefore$ $R = Z/\sqrt{2} = 28.3/\sqrt{2} = 20\ \Omega;\ X = X_L - X_C = 30 - X_C = 20$

$X_C = 10\ \Omega$ or $\dfrac{1}{\omega C} = 10$ or $\dfrac{1}{3000\ C} =$ or $C =$ **33 μF**

Example 13.47. *A non-inductive resistor is connected in series with a coil and a capacitor. The circuit is connected to a single-phase a.c. supply. If the voltages are as indicated in Fig. 13.37 when current flowing through the circuit is 0.345 A, find the applied voltage and the power loss in coil.* **(Elect. Engg. Pune Univ.)**

Solution. It may be kept in mind that the coil has not only inductance L but also some resistance r which produces power loss. In the voltage vector diagram, AB represents drop across R = 25 V. Vector BC represents drop across coil which is due to L and r. Which value is 40 V and the vector BC is at any angle of ϕ with the current vector. AD represents 50 V which is the drop across R and coil combined. AE represents the drop across the capacitor and leads the current by 90°.

It will be seen that the total horizontal drop in the circuit is AC and the vertical drop is AG. Their vector sum AF represents the applied voltage V.

From triangle ABD, we get $50^2 = 40^2 + 25^2 + 2 \times 25 \times 40 \times \cos\phi \therefore \cos\phi = 0.1375$ and $\sin\phi = 0.99$. Considering the coil, $IZ_L = 40$ $\therefore$ $Z_L = 40/0.345 = 115.94\ \Omega$

Now $r = Z_L \cos\phi = 115.94 \times 0.1375 = 15.94\ \Omega$

Power loss in the coil $= I^2r = 0.345^2 \times 15.94 =$ **1.9 W**

$BC = BD\cos\phi = 40 \times 0.1375 = 5.5$ V; $CD = BD\sin\phi = 40 \times 0.99 = 39.6$ V

$AC = 25 + 5.5 = 30.5$ V; $AG = AE - DC = 55 - 39.6 = 15.4$ V

$$AF = \sqrt{AC^2 + CF^2} = \sqrt{30.5^2 + 15.4^2} = \textbf{34.2 V}$$

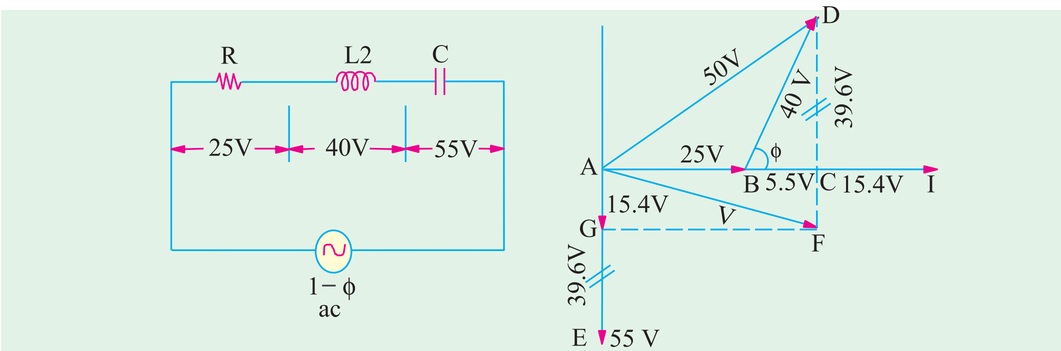

Fig. 13.37

Example 13.48. *A 4.7 H inductor which has a resistance of 20 Ω, a 4-μF capacitor and a 100-Ω non-inductive resistor are connected in series to a 100-V, 50-Hz supply. Calculate the time interval between the positive peak value of the supply voltage and the next peak value of power.*

Solution. Total resistance = 120 Ω; $X_L = 2\pi \times 50 \times 4.7 = 1477\ \Omega$

$X_C = 10^6/2\pi \times 50 \times 4 = 796\ \Omega$; $X = 1477 - 798 = 681\ \Omega$;

$$Z = \sqrt{120^2 + 681^2} = 691.3\ \Omega$$

$\cos\phi = R/Z = 120/691.3 = 0.1736$; $\phi = 80°$

Now, as seen from Fig. 13.38, the angular displace-

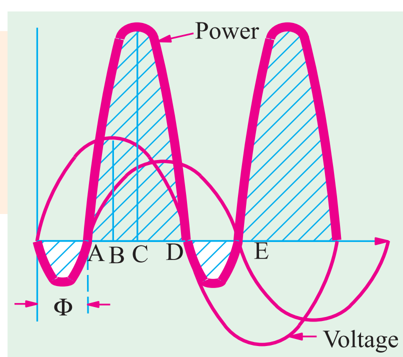

Fig. 13.38

ment between the peak values of supply voltage and power cycles is $BC = \phi/2$ because $AB = 90 - \phi$ and $AD = 180 - \phi$.

Hence $AC = 90 - \phi/2$

$\therefore \quad BC = AC - AB = (90 - \phi/2) - (90 - \phi) = \phi/2$

Angle difference = $\phi/2 = 80°/2 = 40°$

Since a full cycle of 360° corresponds to a time interval of 1.50 second

$\therefore$ 40° angular interval $= \frac{40}{50 \times 360} =$ **2.22 ms.**

Example 13.49. *A coil is in series with a 20 μF capacitor across a 230-V, 50-Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is* **(*i*)** *leading* **(*ii*)** *lagging.*

Sketch a vector diagram for each condition and calculate the coil power factor in each case.

(Elect. Engg.-I Nagpur Univ. 1993)

Solution. (*i*) Since power factor is leading, net reactance $X = (X_C - X_L)$ as shown in Fig. 13.39 (*a*).

$$I^2R = 200 \text{ or } 8^2 \times R = 200; \quad \therefore R = 200/64 = 25/8\ \Omega = 3.125\ \Omega$$

$$Z = V/I = 230/8 = 28.75\ \Omega;\ X_C = 10^6/2\pi \times 50 \times 20 = 159.15\ \Omega$$

$R_2 + X_2 = 28.75^2 \quad \therefore X = 28.58\ \Omega \quad \therefore (X_C - X_L) = 28.58$ or $159.15 - X_L = 28.58$

$\therefore \quad X_L = 130.57\ \Omega$ or $2\pi \times 50 \times L = 130.57 \quad \therefore L = 0.416\ H$

If θ is the p.f. angle of the coil, then $\tan\theta = R/X_L = 3.125/130.57 = 0.024$; θ = 1.37°, p.f. of the coil = 0.9997

(*ii*) When power factor is lagging, net reactance is $(X_L - X_C)$ as shown in Fig. 13.39 (*b*).

$\therefore \quad X_L - 159.15 = 28.58$ or $X_L = 187.73\ \Omega \quad \therefore 187.73 = 2\pi \times 50 \times L$ or $L = 0.597$ H.

In this case, $\tan\theta = 3.125/187.73 = 0.0167$; θ = 0.954° $\therefore \cos\theta = 0.9998$.

The vector diagrams for the two conditions are shown in Fig. 13.35.

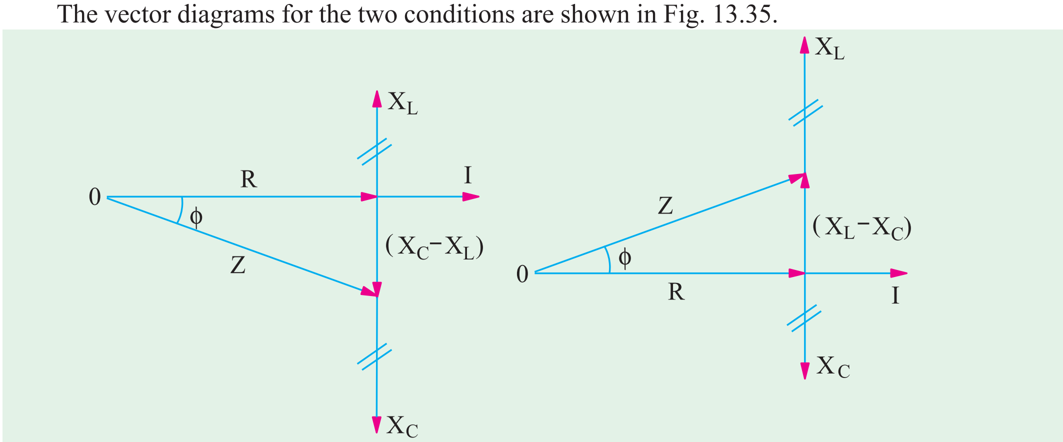

Fig. 13.39

Example 13.50. *In Fig. 13.40, calculate* **(*i*)** *current* **(*ii*)** *voltage drops V_1, V_2, and V_3 and* **(*iii*)** *power absorbed by each impedance and total power absorbed by the circuit. Take voltage vector along the reference axis.*

Solution. $\mathbf{Z_1} = (4 + j3)\ \Omega$; $\mathbf{Z_2} = (6 - j8)\ \Omega$; $\mathbf{Z_3} = (4 + j0)\Omega$

$\mathbf{Z} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} = (4 + j3) + (6 - j8) + (4 + j0) = (14 - j5)\Omega$

Taking $\mathbf{V} = V \angle 0° = 100 \angle 0° = (100 + j0)$

$$\therefore \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \quad \frac{100}{(14 \quad j5)} \quad \frac{100\,(14 \quad j5)}{(14 \quad j5)\,(14 \quad j5)} \quad 6.34 \quad j2.26$$

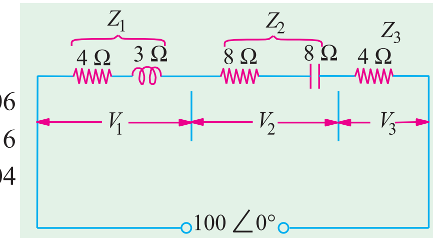

Fig. 13.40

(i) Magnitude of the current

$= \sqrt{(6.34^2 + 2.26^2)} = \mathbf{6.73\ A}$

(ii) $\mathbf{V_1 = IZ_1} = (6.34 + j\,2.26)\,(4 + j3) = 18.58 + j\,28.06$

$\mathbf{V_2 = IZ_2} = (6.34 + j\,2.26)\,(6 - j8) = 56.12 - j\,37.16$

$\mathbf{V_3 = IZ_3} = (6.34 + j\,2.26)\,(4 + j0) = 25.36 + j\,9.04$

$\mathbf{V} = 100 + j\,0$ (check)

(iii) $P_1 = 6.73^2 \times 4 = 181.13$ W.

$P_2 = 6.73^2 \times 6 = 271.74$ W, $P_3 = 6.73^2 \times 4 = 181.13$ W,

Total = 34 W

Otherwise $\quad \mathbf{P_{VA}} = (100 + j\,0)\,(6.34 - j\,2.26)$ (using current conjugate)

$= 634 - j\,226$

real power = 634 W (as a check)

Example 13.51. *Draw a vector for the circuit shown in Fig. 13.41 indicating the resistance and reactance drops, the terminal voltages V_1 and V_2 and the current. Find the values of **(i)** the current I **(ii)** V_1 and V_2 and **(iii)** p.f.* **(Elements of Elect Engg-I, Bangalore Univ.)**

Solution. $L = 0.05 + 0.1 = 0.15$ H; $X_L = 314 \times 0.5 = 47.1\ \Omega$

$X_C = 10^6/314 \times 50 = 63.7\ \Omega$; $X = 47.1 - 63.7 = -16.6\ \Omega$, $R = 30\ \Omega$,

$Z = \sqrt{30^2 + (-16.6)^2} = 34.3\ \Omega$

(i) $I = V/Z = 200/34.3 = \mathbf{5.83\ A}$, from Fig. 13.41 (*a*)

(ii) $X_{L1} = 314 \times 0.05$

$= 15.7\ \Omega$

$Z_1 = \sqrt{10^2 + 15.7^2} = 18.6\ \Omega$

$V_1 = IZ_1 = 5.83 \times 18.6$

$= \mathbf{108.4\ V}$

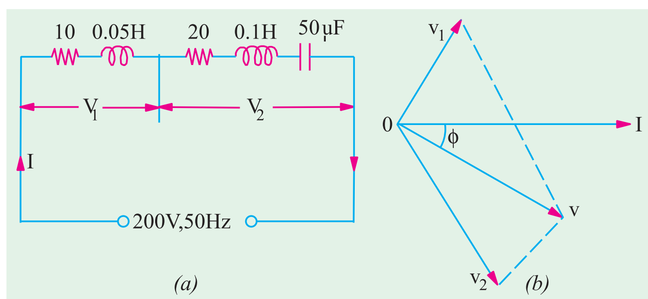

Fig. 13.41

$\phi_1 = \cos^{-1}(10/18.6) = 57.5°$ (lag)

$X_{L2} = 314 \times 0.1 = 31.4\ \Omega$, $X_c = -63.7\ \Omega$, $X = 31.4 - 63.7 = -32.2\ \Omega$, $Z_2 = \sqrt{20^2 + (-32.3)^2} = \mathbf{221\ V}$

$\phi_2 = \cos^{-1}(20/38) = 58.2$ (lead)

(iii) Combined p.f. = $\cos\phi = R/Z = 30/34.3 = \mathbf{0.875}$ **(lead)**, from Fig. 13.41 (*b*).

Example 13.52. *In a circuit, the applied voltage is found to lag the current by 30°.*

(a) *Is the power factor lagging or leading ? **(b)** What is the value of the power factor ? (c) Is the circuit inductive or capacitive ?*

In the diagram of Fig 13.42, the voltage drop across Z_1 is $(10 + j0)$ volts. Find out

(i) *the current in the circuit **(ii)** the voltage drops across Z_2 and Z_3 **(iii)** the voltage of the generator.*

(Elect. Engg.-I, Bombay Univ. 1991)

Solution. (*a*) Power factor is ***leading*** because current leads the voltage.

(*b*) p.f. = cos 30° = **0.86 (lead) (*c*)** The circuit is **capacitive.**

(i) Circuit current can be found by dividing voltage drop $\mathbf{V_1}$ by $\mathbf{Z_1}$

$$\mathbf{I} = \frac{10 + j\,0}{3 + j\,4} = \frac{10\angle 0°}{5\angle 53.1°} = 2\angle{-53.1°} = 2(\cos 53.1° - j\sin 53.1°)$$

$$= 2\,(0.6 - j\,0.8) = 1.2 - j\,1.6$$

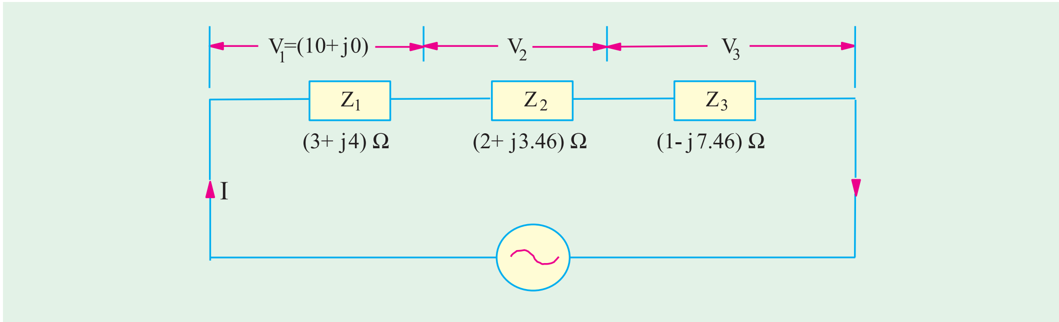

Fig. 13.42

$$\mathbf{Z_2} = 2 + j\,3.46;\ \mathbf{V_2 = IZ_2} = (1.2 - j\,1.6)\,(2 + j3.46) = (7.936 + j\,0.952) \text{ volt}$$

$$\mathbf{V_3} = (1.2 - j\,1.6)\,(1 - j\,7.46) = (-10.74 - j\,10.55) \text{ volt}$$

(*ii*) $$\mathbf{V} = \mathbf{V_1 + V_2 + V_3} = (10 + j\,0) + (7.936 + j\,0.952) + (-10.74 - j\,10.55)$$

$$= (7.2 - j\,9.6) = 12\,\angle -53.1°$$

Incidentally, it shows that current **I** and voltage **V** are in phase with each other.

Example 13.53. *A 230-V, 50-Hz alternating p.d. supplies a choking coil having an inductance of 0.06 henry in series with a capacitance of 6.8 μF, the effective resistance of the circuit being 2.5 Ω. Estimate the current and the angle of the phase difference between it and the applied p.d. If the p.d. has a 10% harmonic of 5 times the fundamental frequency, estimate* **(*a*)** *the current due to it and* **(*b*)** *the p.d. across the capacitance.* **(Electrical Network Analysis, Nagpur Univ. 1993)**

Solution. Fundamental Frequency : For the circuit in Fig. 13.43,

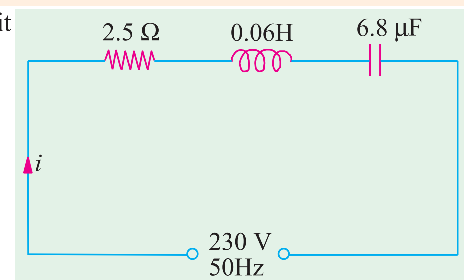

Fig. 13.43

$$X_L = \omega L = 2\pi \times 50 \times 0.06 = 18.85\ \Omega$$

$$X_C = \frac{10^6}{2\pi \times 50 \times 6.8} = 648\ \Omega$$

$$\therefore \quad X = 18.85 - 468 = -449.15\ \Omega$$

$$Z = \sqrt{2.5^2 + (-449.15)^2} = 449.2\ \Omega$$

Current $I_f = 230/449.2 \quad 0.512$ A, phase angle $\tan^{1} \dfrac{449.2}{2.5} \quad 89\ .42$

$\therefore$ current leads p.d. by 89° 42′ .

Fifth Harmonic Frequency $X_L = 18.85 \times 5 = 94.25\ \Omega$; $X_C = 468 + 5 = 93.6\ \Omega$

$X = 94.25 - 93.6 = 0.65\ \Omega$; **Z** $\sqrt{2.5^2 \quad 0.65} \quad 2.585$, Harmonic p.d. = 230 × 10/100 = 23 V

$\therefore$ Harmonic current $I_h = 23/2.585 = 8.893$ A

P.D. across capacitor at harmonic frequency is, $V_h = 8.893 \times 93.6 = 832.6$ V

The total current flowing through the circuit, due to the complex voltage wave form, is found from the fundamental and harmonic components thus. Let,

I = the r.m.s. value of total circuit current,

I_f = r.m.s. value of fundamental current,

I_h = r.m.s. value of fifth harmonic current,

(*a*) $\therefore$ $$I = \sqrt{I_f^2 + I_h^2} = \sqrt{0.512^2 + 8.893^2} = \mathbf{8.9\ A}$$

(*b*) The r.m.s. value of p.d. across capacitor is found in a similar way.

$$V_f = 0.512 \times 468 = 239.6 \text{ V}$$

$$\therefore \quad V = \sqrt{(V_f^2 + V_h^2)} = \sqrt{239.6^2 + 832.6^2} = \mathbf{866.4\ V}$$

Tutorial Problem No. 13.3

1. An e.m.f. represented by $e = 100 \sin 100\,\pi\, t$ is impressed across a circuit consisting of 40-Ω resistor in series with a 40-μ*F* capacitor and a 0.25 H indicator.
   Determine (*i*) the r.m.s. value of the current (*ii*) the power supplied (*iii*) the power factor.
   [(*i*) 1.77 A (*ii*) 125 W (*iii*) 1.0] *(London Univ.)*
2. A series circuit with a resistor of 100 Ω capacitor of 25 μ*F* and inductance of 0.15 H is connected across 220-V, 60-Hz supply. Calculate (*i*) current (*ii*) power and (*iii*) power factor in the circuit.
   [(*i*) 1.97 A; (*ii*) 390 W (*iii*) 0.9 (lead)] *(Elect. Engg. and Electronics Bangalore Univ.)*
3. A series circuit with $R = 10\ \Omega$, $L = 50\ mH$ and $C = 100\ \mu F$ is supplied with 200 V/50 Hz. Find (*i*) the impedance (*ii*) current (*iii*) power (*iv*) power factor.
   [(*i*) 18.94 Ω (*ii*) 18.55 A (*iii*) 1966 W (*iv*) 0.53 (leading)]
   (Elect. Engg. & Electronics Bangalore Univ.)
4. A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150-μ*F* capacitor across a 200-V, 50-Hz supply. Calculate (*a*) the inductive reactance, (*b*) the capacitive reactance, (*c*) the impedance (*d*) the current, (*e*) the power factor (*f*) the voltage across the coil and the capacitor respectively. **[(*a*) 31.4 Ω (*b*) 21.2 Ω (*c*) 14.3 Ω (*d*) 14 A (*e*) 0.7 lag (*f*) 460 V, 297 V]**
5. A circuit is made up of 10 Ω resistance, 12 mH inductance and 281.5 μ*F* capacitance in series. The supply voltage is 100 V (constant). Calculate the value of the current when the supply frequency is (*a*) 50 Hz and (*b*) 150 Hz. **[8 A leading; 8 A lagging]**
6. A coil having a resistance of 10 Ω and an inductance of 0.2 H is connected in series with a capacitor of 59.7 μ*F*. The circuit is connected across a 100-V, 50-Hz a.c. supply. Calculate (*a*) the current flowing (*b*) the voltage across the capacitor (*c*) the voltage across the coil. Draw a vector diagram to scale. **[(*a*) 10 A (*b*) 628 V (*c*) 635 V]**
7. A coil is in series with a 20 μ*F* capacitor across a 230-V , 50-Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is (*a*) leading and (*b*) lagging.
   Sketch a vector diagram for each condition and calculate the coil power factor in each case.
   [0.415 H; 0.597 H; 0.0238 ; 0.0166]
8. A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115-V, 50-Hz supply. Another circuit takes a current, of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230-V, 50Hz supply, calculate (*a*) the current (*b*) the power consumed and (*c*) the power factor.
   [(*a*) 5.5 A (*b*) 1.188 kW (*c*) 0.939 lag]
9. A coil of insulated wire of resistance 8 ohms and inductance 0.03 H is connected to an a.c. supply at 240 V, 50-Hz. Calculate (*a*) the current, the power and power factor (*b*) the value of a capacitance which, when connected in series with the above coil, causes no change in the values of current and power taken from the supply. **[(*a*) 19.4 A, 3012 W, 0.65 lag (*b*) 168.7 μF] *(London Univ.)***
10. A series circuit, having a resistance of 10 Ω, an inductance of 0.025 H and a variable capacitance is connected to a 100-V, 25-Hz single-phase supply. Calculate the capacitance when the value of the current is 8 A. At this value of capacitance, also calculate (*a*) the circuit impedance (*b*) the circuit power factor and (*c*) the power consumed. **[556 μF (*a*) 1.5 Ω (*b*) 0.8 leading (*c*) 640 W]**
11. An alternating voltage is applied to a series circuit consisting of a resistor and iron-cored inductor and a capacitor. The current in the circuit is 0.5 A and the voltages measured are 30 V across the resistor, 48 V across the inductor, 60 V across the resistor and inductor and 90 V across the capacitor. Find (*a*) the combined copper and iron losses in the inductor (*b*) the applied voltage.
    [(*a*) 3.3 W (*b*) 56 V] (*City & Guilds, London*)
12. When an inductive coil is connected across a 250-V, 50-Hz supply, the current is found to be 10 A and the power absorbed 1.25 kW. Calculate the impedance, the resistance and the inductance of the coil. A capacitor which has a reactance twice that of the coil, is now connected in series with the coil across the same supply. Calculate the p.d. across the capacitor. **[25 Ω; 12.5 Ω; 68.7 mH; 433 V]**

13. A voltage of 200 V is applied to a series circuit consisting on a resistor, an inductor and a capacitor. The respective voltages across these components are 170, 150 and 100 V and the current is 4 A. Find the power factor of the inductor and of the circuit. **[0.16; 0.97]**
14. A pure resistance R, a choke coil and a pure capacitor of 50μ F are connected in series across a supply of V volts, and carry a current of 1.57 A. Voltage across R is 30 V, across choke coil 50 V and across capacitor 100 V. The voltage across the combination of R and choke coil is 60 volt. Find the supply voltage V, the power loss in the choke, frequency of the supply and power factor of the complete circuit. Draw the phasor diagram. **[60.7 V; 6.5 W; 0.562 lead] (*F.E. Pune Univ.*)**

13.10. Resonance in R-L-C Circuits

We have seen from Art. 13.9 that net reactance in an *R-L-C* circuit of Fig. 13.40 (*a*) is

$$X = X_L - X_C \text{ and } Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{R^2 + X^2}$$

Let such a circuit be connected across an a.c. source of constant voltage V but of frequency varying from zero to infinity. There would be a certain frequency of the applied voltage which would make X_L equal to X_C in magnitude. In that case, $X = 0$ an $Z = R$ as shown in Fig. 13.40 (*c*). Under this condition, the circuit is said to be in electrical resonance.

As shown in Fig. 13.40 (*c*), $V_L = I.\ X_L$ and $V_C = I.\ X_C$ and the two are equal in magnitude but opposite in phase. Hence, they cancel each other out. The two reactances taken together act as a short-circuit since no voltage develops across them. Whole of the applied voltage drops across R so that $V = V_R$. The circuit impedance $Z = R$. The phasor diagram for series resonance is shown in Fig. 13.40 (*d*).

Calculation of Resonant Frequency

The frequency at which the net reactance of the series circuit is zero is called the resonant frequency f_0. Its value can be found as under : $X_L - X_C = 0$ or $X_L = X_C$ or $\omega_0 L = 1/\omega_0 C$

or $$\omega_0^2 = \frac{1}{LC} \text{ or } (2\pi f_0)^2 = \frac{1}{LC} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

If L is in henry and C in farad, then f_0 is given in *Hz*.

When a series *R-L-C* circuit is in resonance, it possesses minimum impedance $Z = R$. Hence, circuit current is maximum, it being limited by value of R alone. The current $I_0 = V/R$ and is in phase with V.

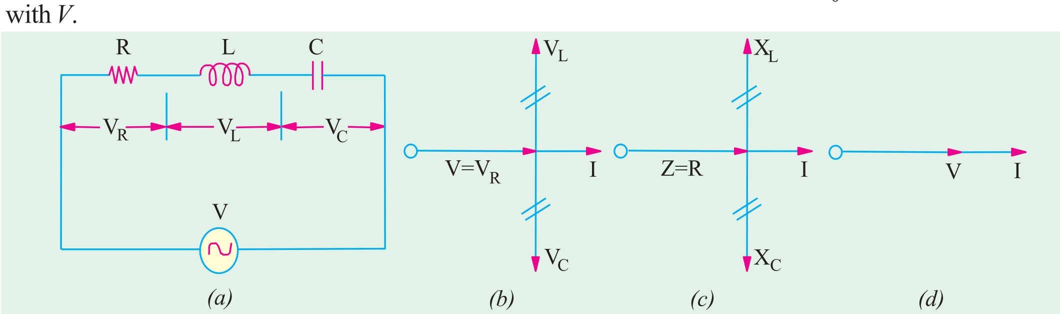

Fig. 13.44

Since circuit current is maximum, it produces large voltage drops across L and C. But these drops being equal and opposite, cancel each other out. Taken together, L an C from part of a circuit across which no voltage develops, however, large the current flowing. If it were not for the presence of R, such a resonant circuit would act like a short-circuit to currents of the frequency to which it resonates. Hence, a series resonant circuit is sometimes called ***acceptor*** circuit and the series resonance is often referred to as voltage resonance.

In fact, at resonance the series *RLC* circuit is reduced to a purely resistive circuit, as shown in Fig. 13.44.

Incidentally, it may be noted that if X_L and X_C are shown at any frequency f, that the value of the resonant frequency of such a circuit can be found by the relation $f_0 = f\sqrt{X_C/X_L}$.

Summary

When an R-L-C circuit is in resonance

1. net reactance of the circuit is zero *i.e.* $(X_L - X_C) = 0$. or $X = 0$.
2. circuit impedance is minimum *i.e.* $Z = R$. Consequently, circuit admittance is maximum.
3. circuit current is maximum and is given by $I_0 = V/Z_0 = V/R$.
4. power dissipated is maximum *i.e.* $P_0 = I_0^2\, R = V^2/R$.
5. circuit power factor angle $\theta = 0$. Hence, power factor $\cos\theta = 1$.
6. although $V_L = V_C$ yet V_{coil} is greater than V_C because of its resistance.
7. at resonance, $\omega^2 LC = 1$
8. $Q = \tan\theta = \tan 0° = 0$*.

13.11. Graphical Representation of Resonance

Suppose an alternating voltage of constant magnitude, but of varying frequency is applied to an R-L-C circuit. The variations of resistance, inductive reactance X_L and capacitive reactance X_C with frequency are shown in Fig. 13.45 (*a*).

(i) Resistance : It is independent of f, hence, it is represented by a straight line.

(ii) Inductive Reactance : It is given by $X_L = \omega L = 2\pi fL$. As seen, X_L is directly proportional to f *i.e.* X_L increases linearly with f. Hence, its graph is a straight line passing through the origin.

(iii) Capacitive Reactance : It is given by $X_C = 1/\omega C = 1/2\pi fC$. Obviously, it is inversely proportional to f. Its graph is a rectangular hyperbola which is drawn in the fourth quadrant because X_C is regarded negative. It is asymptotic to the horizontal axis at high frequencies and to the vertical axis at low frequencies.

(iv) Net Reactance : It is given by $X = X_L \sim X_C$. Its graph is a hyperbola (not rectangular) and crosses the X-axis at point A which represents resonant frequency f_0.

(v) Circuit Impedance : It is given by $Z = \sqrt{[R^2 + (X_L \sim X_C)^2]} = \sqrt{R^2 + X^2}$

At low frequencies Z is large because X_C is large. Since $X_C > X_L$, the net circuit reactance X is capacitive and the p.f. is leading [Fig. 13.45 (*b*)]. At high frequencies, Z is again large (because X_L is large) but is inductive because $X_L > X_C$. Circuit impedance has minimum values at f_0 given by $Z = R$ because $X = 0$.

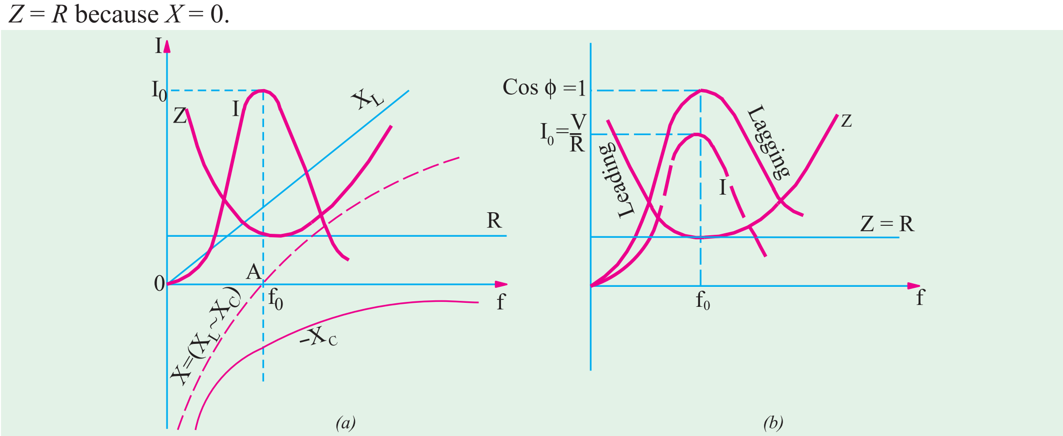

Fig. 13.45

(vi) Current I_0 : It is the reciprocal of the circuit impedance. When Z is low, I_0 is high and vice versa. As seen, I_0 has low value on both sides of f_0 (because Z is large there) but has maximum value of $I_0 = V/R$ at resonance. Hence, maximum power is dissipated by the series circuit under resonant conditions. At frequencies below and above resonance, current decreases as shown in Fig. 13.45

* However, value of Q_0 is as given in Art 13.5, 13.9 and 13.17.

(b). Now, $I_0 = V/R$ and $I = V/Z = V/\sqrt{(R^2 + X^2)}$. Hence $I/I_0 = R/Z = V/\sqrt{(R^2 + X^2)}$ where X is the net circuit reactance at any frequency f.

(vii) *Power Factor*

As pointed out earlier, X is capacitive below f_0. Hence, current leads the applied voltage. However, at frequencies above f_0, X is inductive. Hence, the current lags the applied voltage as shown in Fig. 13.45. The power factor has maximum value of unity at f_0.

13.12. Resonance Curve

The curve, between circuit current and the frequency of the applied voltage, is known as resonance curve. The shapes of such a curve, for different values of R are shown in Fig. 13.46. For smaller values of R, the resonance curve is sharply peaked and such a circuit is said to be sharply resonant or highly selective. However, for larger values of R, resonance curve is flat and is said to have poor selectivity. The ability of a resonant circuit to discriminate between one particular frequency and all others is called its selectivity. The selectivities of different resonant circuits are compared in terms of their half-power bandwidths (Art. 13.13).

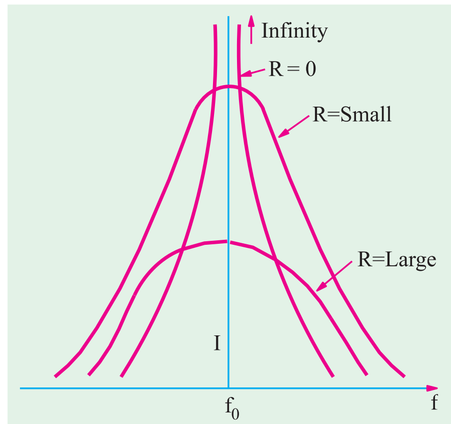

Fig. 13.46

13.13. Half-Power Bandwidth of a Resonant-Circuit

A discussed earlier, in an R-L-C circuit, the maximum current at resonance is solely determined by circuit resistance R ($\because X = 0$) but at off-resonance frequencies, the current amplitude depends on Z (where $X \neq 0$). The half-wave bandwidth of a circuit is given by the band of fequencies which lies between two points on either side of f_0 where current falls to $I_0/\sqrt{2}$. Narrower the bandwidth, higher the selectivity of the circuit and vice versa. As shown in Fig. 13.47 the half-power bandwidth AB is given by

$AB = \Delta f = f_2 - f_1$ or $AB = \Delta\omega = \omega_2 - \omega_1$ where f_1 and f_2 are the corner or edge frequencies.

As seen, $P_0 = I_0^2\,R$. However, power at either of the two points A and B is

$$P_1 = P_2 = I^2 R$$

$$= (I_0/\sqrt{2})^2\,R = I_0^2 R/2 = \frac{1}{2}\,I_0^2 R = \frac{1}{2} \times \text{power at resonance}$$

That is why the two points A and B on the resonance curve are known as half-power points* and the corresponding value of the bandwidth is called half-power bandwidth B_{hp}. It is also called –3dB* bandwidth. The following points regarding half-power point A and B are worth noting. At these points,

1. current is $I_0/\sqrt{2}$
2. impedance is $\sqrt{2}.\,R$ or $\sqrt{2}.\,Z_0$
3. $P_1 = P_2 = P_0/2$
4. the circuit phase angle is $\theta = \pm 45°$
5. $Q = \tan\theta = \tan 45° = 1$

* The decibel power responses at these points, in terms of the maximum power at resonance, is

$$10 \log_{10} P/P_0 = 10 \log_{10} \frac{I_m^2 R/2}{I_m^2 R} = 10 \log_{10} \frac{1}{2} = -10 \log_{10} 2 = -3\,dB$$

Hence, the half-power points are also referred to as –3 dB points.

6. $B_{hp} = f_2 - f_1 = f_0/Q_0 = \sqrt{f_1 f_2}/Q_0 = R/2\pi L.$

It is interesting to note that B_{hp} is independent of the circuit capacitance.

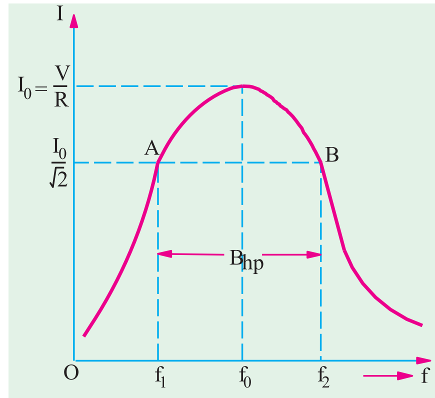

Fig. 13.47

13.14. Bandwidth B at any Off-resonance Frequency

It is found that the bandwidth of a given *R-L-C* circuit at any off-resonance frequencies f_1 and f_2 is given by

$B = f_0\, Q/Q_0 = \sqrt{f_1 f_2};.\, Q/Q_0 = f_2 - f_1$

where f_1 and f_2 are any frequencies (not necessarily half-power frequencies) below and above f_0.

Q = tangent of the circuit phase angle at the off-resonance frequencies f_1 and f_2.

Q_0 = quality factor at resonance $= \dfrac{\omega_0 L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

13.15. Determination of Upper and Lower Half-power Frequencies

As mentioned earlier, at lower half-power frequencies, $\omega_1 < \omega_0$ so that $\omega_1 L < 1/\omega_1 C$ and $\phi = -45°$

$$\therefore \quad \frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{or} \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

Putting $\dfrac{\omega_0}{Q_0} = \dfrac{R}{L}$ and $\omega_0^2 = \dfrac{1}{LC}$ in the above equation, we get $\omega_1^2 + \dfrac{\omega_0}{Q_0}\omega_1 - \omega_0^2 = 0$

The positive solution of the above equation is, $\omega_1 = \omega_0\left[\sqrt{\left(1 + \dfrac{1}{4Q_0^2}\right)} - \dfrac{1}{2Q_0}\right]$

Now at the upper half-power frequency, $\omega_2 > \omega_0$ so that $\omega_2 > 1/\omega_2 C$ and $\phi = +45°$

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{or} \quad \omega_2^2 - \frac{\omega_0}{Q_0}\omega_2 - \omega_0^2 = 0$$

The positive solution of the above equation is $\omega_2 = \omega_0\left[\sqrt{\left(1 + \dfrac{1}{4Q_0^2}\right)} + \dfrac{1}{2Q_0}\right]$

In case $Q_0 > 1$; then the term $1/4\,Q_0^2$ is negligible as compared to 1.

Hence, in that case $\quad \omega_1 \cong \omega_0\left(1 - \dfrac{1}{2Q_0}\right)$ and $\omega_2 \cong \omega_0\left(1 + \dfrac{1}{2Q_0}\right)$

Incidentally, it may be noted from above that $\omega_2 - \omega_1 = \omega_0/Q_0$.

13.16. Values of Edge Frequencies

Let us find the values of ω_1 and ω_2, $I_0 = V/R$...at resonance

$$I = \frac{V}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}} \text{ ... at any frequency}$$

At points *A* and *B*, $I \quad \dfrac{I_0}{\sqrt{2}} \quad \dfrac{1}{\sqrt{2}}\cdot\dfrac{V}{R}$

$$\therefore \quad \frac{1}{\sqrt{2}}\cdot\frac{V}{R} \quad \frac{V}{[(R^2 \quad (\quad L \quad 1/ \quad C)^2]^{1/2}} \text{ or } R \quad (\quad L \quad 1/ \quad C) \quad X$$

It shows that at half-power points, net reactance is equal to the resistance.

(Since resistance equals reactance, p.f. of the circuit at these points is = $1/\sqrt{2}$ *i.e.* 0.707, though leading at point *A* and lagging at point *B*).

Hence $R^2 = (\omega L - 1/\omega C)^2 \quad \therefore \ \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = \pm \alpha \pm \sqrt{\alpha^2 + \omega_0^2}$

where $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{L\,C}}$

Since $R^2/4L^2$ is much less then $1/\sqrt{LC}$ $\therefore \ \omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm \frac{R}{2L} \pm \omega_0$

Since only positive values of ω_0 are considered, $\omega = \omega_0 \pm R/2L = \omega_0 \pm \alpha$

$$\therefore \qquad \omega_1 = \omega_0 - \frac{R}{2\,L} \text{ and } \omega_2 = \omega_0 + \frac{R}{2\,L}$$

$$\therefore \qquad \Delta\omega = \ _2 \quad _1 \quad \frac{R}{L} \text{ rad/s and } f \quad f_2 \quad f_1 \quad \frac{R}{2\ \ L} \text{ Hz} \quad \frac{f_0}{Q_0} \text{ Hz}$$

$$\text{Also} \qquad f_1 = f_0 \quad \frac{R}{4\ \ L} \text{ Hz and } f_2 \quad f_0 \quad \frac{R}{4\ \ L} \text{ Hz}$$

It is obvious that f_0 is the *centre* frequency between f_1 and f_2.

Also, $\omega_1 = \omega_0 - \frac{1}{2}\Delta\omega$ and $\omega_2 = \omega_0 + \frac{1}{2}\Delta\omega$

As stated above, bandwidth is a measure of circuits selectivity. Narrower the bandwidth, higher the selectivity and vice versa.

13.17. Q-Factor of a Resonant Series Circuit

The *Q*-factor of an *R-L-C* series circuit can be defined in the following different ways.

(i) it is given by the voltage magnification produced in the circuit at resonance.

We have seen that at resonance, current has maximum value $I_0 = V/R$. Voltage across either coil or capacitor = $I_0 X_{L_0}$ or $I_0 X_{C_0}$, supply voltage $V = I_0\,R$

$$\therefore \quad \text{Voltage magnification} = \frac{V_{L_0}}{V} = \frac{I_0 X_{L_0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

$$\text{or} \qquad = \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{C0}}{R} = \frac{\text{reactance}}{\text{resistance}} = \frac{1}{\omega_0\,CR}$$

$$\therefore \quad Q\text{-factor}, \qquad Q_0 = \frac{\omega_0\,L}{R} = \frac{2\pi\,f_0\,L}{R} = \tan\phi \qquad \text{...}(i)$$

where ϕ is power factor of the coil.

(ii) The *Q*-factor may also be defined as under.

$$Q\text{-factor} = 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}} \qquad \text{...in the circuit}$$

$$= 2\pi \frac{\frac{1}{2} L I_0^2}{I^2\,R\,T_0} = 2\pi \frac{\frac{1}{2} L(\sqrt{2}I)^2}{I^2 R(1/f_0)} = \frac{I^2\,2\pi\,f_0 L}{I^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \qquad \text{...}(T_0 = 1/f_0)$$

In other words, $Q_0 = \frac{\text{energy stored*}}{\text{energy lost}}$ **...in the circuit**

---

* The author often jokingly tells students in his class that these days the quality of a person is also measured in terms of a quality factor given by

$$Q = \frac{\text{money earned}}{\text{money spent}}$$

Obviously, a person should try to have a high quality factor as possible by minimising the denominator and/or maximizing the numerator.

(*iii*) We have seen above that resonant frequency, $f_0 = \dfrac{1}{2\pi\sqrt{(LC)}}$ or $2\pi f_0 = \dfrac{1}{\sqrt{(LC)}}$

Substituting this value in Eq. (*i*) above, we get the *Q*-factor, $Q_0 = \dfrac{1}{R}\sqrt{\left(\dfrac{L}{C}\right)}$

(*iv*) In the case of series resonance, higher *Q*-factor means not only higher voltage magnification but also higher selectivity of the tuning coil. In fact, *Q*-factor of a resonant series circuit may be written as

$$Q_0 = \frac{\omega_0}{\text{bandwidth}} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{...as before}$$

Obviously, *Q*-factor can be increased by having a coil of large inductance but of small ohmic resistance.

(*v*) In summary, we can say that

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{X_{L0} X_{C0}}{R}} = \frac{f_0}{B_{hp}} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

13.18. Circuit Current at Frequencies Other Than Resonant Frequencies

At resonance, $I_0 = V/R$

At any other frequency above the resonant frequency, the current is given by *I*

$$\frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

This current lags behind the applied voltage by a certain angle ϕ

$$\therefore \quad \frac{I}{I_0} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \times \frac{R}{V} = \frac{1}{\sqrt{1 + \dfrac{1}{R^2}(\omega L - 1/\omega C)^2}} = \frac{1}{\left[1 + \left(\dfrac{\omega_0 L}{R}\right)^2 \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2\right]^{1/2}}$$

Now, $\omega_0 L/R = Q_0$ and $\omega/\omega_0 = f/f_0$ hence, $\dfrac{I}{I_0} = \dfrac{1}{\left[1 + Q_0^2\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2\right]^{1/2}}$

13.19. Relation Between Resonant Power P_0 and Off-resonant Power P

In a series *RLC* resonant circuit, current is maximum *i.e.* I_0 at the resonant frequency f_0. The maximum power P_0 is dissipated by the circuit at this frequency where X_L equals X_C. Hence, circuit impedance $Z_0 = R$.

$$\therefore \quad P_0 = I_0^2 R = (V/R)^2 \times R = V^2/R$$

At any other frequency either above or below f_0 the power is (Fig. 13.48).

$$P = I^2 R = \left(\frac{V}{Z}\right)^2 \times R = \frac{V^2 R}{Z^2} = \frac{V^2 R}{R^2 + X^2} = \frac{V^2 R}{R^2 + X^2 R^2/R^2}$$

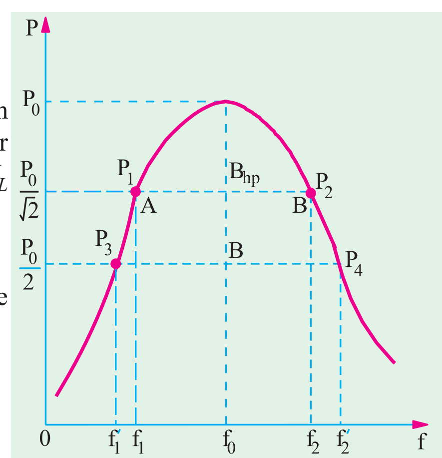

Fig. 13.48

$$= \frac{V^2R}{R^2 + R^2Q^2} = \frac{V^2R}{R^2\,(1+Q^2)} = \frac{V^2}{R\,(1+Q^2)} = \frac{P_0}{(1+Q^2)}$$

The above equation shows that any frequency other than f_0, the circuit power P is reduced by a factor of $(1 + Q^2)$ where Q is the tangent of the circuit phase angle (and not Q_0). At resonance, circuit phase angle $\theta = 0$, and $Q = \tan\theta = 0$. Hence, $P = P_0 = V^2/R$ (values of Q_0 are given in Art.)

Example 13.54. *For a series R.L.C circuit the inductor is variable. Source voltage is $200\sqrt{2}$ sin $100\pi t$. Maximum current obtainable by varying the inductance is 0.314 A and the voltage across the capacitor then is 300 V. Find the circuit element values.*

(Circuit and Field Theory, A.M.I.E. Sec B, 1993)

Solution. Under resonant conditions, $I_m = V/R$ and $V_L = V_C$.

$\therefore \quad R = V/I_m = 200/0.314 = 637\ \Omega,\ V_C = I_m \times X_{CD} = I_m/\omega_0 C$

$\therefore \quad C = I_m/\omega_0 V_C = 0.314/100\,\pi \times 300 = 3.33\ \mu F.$

$V_L = I_m \times X_L = I_m\,\omega_0 L;\ L = V_1/\omega_0 I_m = 300/100\,\pi \times 0.314 =$ **3.03 H**

Example 13.55. *A coil having an inductance of 50 mH and resistance 10 Ω is connected in series with a 25 μF capacitor across a 200 V ac supply. Calculate* **(a)** *resonance frequency of the circuit* **(b)** *current flowing at resonance and* **(c)** *value of Q_0 by using different data.*

(Elect. Engg. A.M.Ae. S.I, June 1991)

Solution. (a) $f_0 = \frac{1}{2\ \sqrt{LC}} \quad \frac{1}{2\ \sqrt{50\ \ 10^{3}\ \ 25\ \ 10^{6}}} \quad 142.3$ Hz

(b) $I_0 = V/R = 200/10 = 20$ A

(c) $Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi \times 142.3 \times 50 \times 10^{-3}}{10} = 4.47$

$$Q_0 = \frac{1}{\omega_0 CR} = \frac{1}{2\pi \times 142.3 \times 25 \times 10^{-6} \times 10} = 4.47$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50 \times 10^{-3}}{25 \times 10^{-6}}} = 4.47$$

Example 13.56. *A 20-Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25-V variable frequency supply. When the frequency is 400-Hz, the current is at its maximum value of 0.5 A and the potential difference across the capacitor is 150 V. Calculate*

(a) *the capacitance of the capacitor*

(b) *the resistance and inductance of the inductor.*

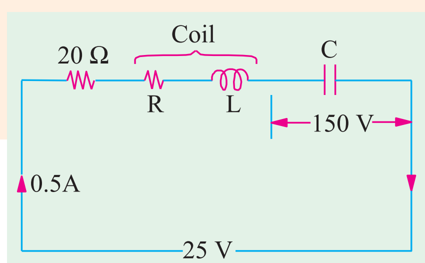

Fig. 13.49

Solution. Since current is maximum, the circuit is in resonance.

$X_C = V_C/I = 150/0.5 = 300\ \Omega$

(a) $X_C = 1/2\pi fC$ or $300 = 1/2\pi \times 400 \times C$

$\therefore \quad C = 1.325 \times 10^{-6}\ F =$ **1.325 μF**

(b) $X_L = X_C = 300\ \Omega$

$\therefore \quad 2\pi \times 400 \times L = 300 \quad \therefore\ L =$ **0.119 H**

(c) Now, at resonance,

circuit resistance = circuit impedance or $20 + R = V/I = 25/0.5 \quad \therefore\ R = 30\ \Omega$ ...Fig.13.49

Example 13.57. *An R-L-C series circuit consists of a resistance of 1000 Ω, an inductance of 100 mH an a capacitance of 10 μμ F. If a voltage of 100 V is applied across the combination, find* **(*i*)** *the resonance frequency* **(*ii*)** *Q-factor of the circuit and* **(*iii*)** *the half-power points.*

(Elect. Circuit Analysis, Bombay Univ.)

Solution. (*i*)
$$f_0 = \frac{1}{2\pi\sqrt{10^{-1}\times 10^{-11}}} = \frac{10^6}{2\pi} = \mathbf{159\ kHz}$$

(*ii*)
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\times\sqrt{\frac{10^{-1}}{10^{-11}}} = \mathbf{100}$$

(*iii*)
$$f_1 = f_0 - \frac{R}{4\pi L} = 159\times 10^3 - \frac{1000}{4\pi\times 10^{-1}} = \mathbf{158.2\ kHz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 159\times 10^3 + \frac{1000}{4\pi\times 10^{-1}} = \mathbf{159.8\ kHz}$$

Example 13.58. *A series R-L-C circuit consists of R = 1000 Ω, L = 100 mH and C = 10 picofarads. The applied voltage across the circuit is 100 V.*

(*i*) *Find the resonant frequency of the circuit.*

(*ii*) *Find the quality factor of the circuit at the resonant frequency.*

(*iii*) *At what angular frequencies do the half power points occur ?*

(*iv*) *Calculate the bandwidth of the circuit.*

(Networks-I, Delhi Univ. & U.P. Tech. Univ. 2001)

Solution. (*i*)
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times 10^{-3}\times 10\times 10^{-12}}} = 159.15\ \text{kHz}$$

(*ii*)
$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \frac{1}{1000} \quad \sqrt{\frac{100 \quad 10^{\ 3}}{10 \quad 10^{\ 12}}} \quad 100$$

(*iii*)
$$B_{hp} = \frac{R}{2\pi L} = \frac{1000}{2\pi\times 100\times 10^{-3}} = 1591.5\ \text{Hz}$$

Also. $B_{hp} = f_0/Q_0$ = 159.15 kHz/100 = 1.5915 kHz = **1591.5 Hz** ...as above

(*iv*)
$$\omega_1 = {}_0\ 1 \quad \frac{1}{2Q_0} \quad 2 \quad 159.15\ \ 1 \quad \frac{1}{2 \quad 100} \quad 994.969\ \text{radia/sec.}$$

$$\omega_2 = {}_0\ 1 \quad \frac{1}{2\,Q_0} \quad 2 \quad 159.15\ \ 1 \quad \frac{1}{2 \quad 100} \quad 1004.969\ \text{rad/sec}$$

(*v*) Band width $= (\omega_2 - \omega_1) = 1004.969 - 994.969 = 10.00$ rad/sec.

Example 13.59. *An R-L-C series resonant circuit has the following parameters :*

Resonance frequency = 5000/2π Hz; impedance at resonance = 56 Ω and Q-factor = 25.

Calculate the capacitance of the capacitor and the inductance of the inductor.

Assuming that these values are independent of the frequency, find the two frequencies at which the circuit impedance has a phase angle of π/4 radian.

Solution. Here $\omega_0 = 2\pi f_0 = 2\pi \times 5000/2\pi = 5000$ rad/s

Now, $Q = \dfrac{\omega_0 L}{R}$ or $25 = \dfrac{5000\ L}{56}$ or $L = 0.28$ H

Also at resonance $\omega_0 L = 1/\omega_0 C$ or $5000 \times 0.28 = 1/5000 \times C$ ∴ $C = 0.143\ \mu$F

The circuit impedance has a phase shift of 45° and the two half-power frequencies which can be found as follows :

$$BW = \frac{f_0}{Q} = \frac{5000/2\pi}{25} = \mathbf{31.83\ Hz}$$

Therefore lower half-power frequency = (f_0 −31.83/2) = 5000/2π− 15.9 = **779.8 Hz.**

Upper half-power frequency = (f_0 + 31.83/2) = 5000/2π + 15.9 = **811.7 Hz.**

Example 13.60. *An R-L-C series circuit is connected to a 20-V variable frequency supply. If R = 20 Ω, L = 20 mH and C = 0.5 μF, calculate the following :*

(a) *resonant frequency f_0* **(b)** *resonant circuit Q_0 using L/C ratio* **(c)** *half-power bandwidth using f_0 and Q_0* **(d)** *half-power bandwidth using the general formula for any bandwidth* **(e)** *half-power bandwidth using the given component values* **(f)** *maximum power dissipated at f_0.*

Solution. (a) $f_0 = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{(20\times10^{-3}\times0.5\times10^{-6})} =$ **159 Hz**

(b) $Q_0 = \dfrac{1}{R}\cdot\sqrt{\dfrac{L}{C}} = \dfrac{1}{20}\cdot\sqrt{\dfrac{20\times10^{-3}}{0.5\times10^{-6}}} =$ **10**

(c) $B_{hp} = f_0/Q_0$ = 1591/10 = **159.1 Hz**

(d) $B_{hp} = f_0\,Q/Q_0$ = 1591 × tan 45°/10 = **159.1 Hz**

It is so because the power factor angle at half-power frequencies is ±45°.

(e) $B_{hp} = R/2\pi L = 20/2\pi\times20\times10^{-3}$ = **159.1 Hz**

(f) $f_0 = V^2/R = 20^2/20$ = **20 W**

Example 13.61. *An inductor having a resistance of 25 Ω and a Q_0 of 10 at a resonant frequency of 10 kHz is fed from a 100 ∠ 0° supply. Calculate*

(a) *Value of series capacitance required to produce resonance with the coil*

(b) *the inductance of the coil* **(c)** *Q_0 using the L/C ratio* **(d)** *voltage across the capacitor* **(e)** *voltage across the coil.*

Solution. (a) $X_{L0} = Q_0R = 10\times25 = 250\ \Omega$

Now, $X_{C0} = X_{L0} = 250\ \Omega$

Hence, $C = 1/2\pi f_0\times X_{C0} = 1/2\pi\times10^4\times250 = 63.67\times10^{-9}\,F =$ **63.67 μF**

(b) $L = X_{L0}/2\pi f_0 = 250/2\pi\times10^4 =$ **3.98 mH**

(c) $Q_0 = \dfrac{1}{R}\cdot\sqrt{\dfrac{L}{C}}$

Now, $\dfrac{L}{C} = \dfrac{3.98\times10^{-3}}{63.67\times10^{-9}} = 6.25\times10^4$

∴ $Q_0 = \dfrac{1}{25}\ \sqrt{6.25\ \ 10^4}$ **10** (verification)

(d) $V_{C0} = -jQ_0V = -j\,100\angle0°\times10 = -j\,1000V =$ **−100 ∠− 90° V**

(e) Since $V_{L0} = V_{C0}$ in magnitude, hence, $V_{L0} = +j\,1000$ V

$= 1000\angle90°$V; Also, $V_R = V = 100\angle0°$

Hence, $V_{coil} = V_R + V_{L0}$

$= 100 + 1000\angle90° = 100 + j\,1000 =$ **1005 ∠ 84.3°**

Example 13.62. *A series L.C circuit has L = 100 pH, C = 2500 μF and Q = 70. Find* **(a)** *resonant frequency f_0* **(b)** *half-power points and* **(c)** *bandwidth.*

Solution. (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{10^9}{2\pi\sqrt{100\times2500}} =$ **318.3 kHz**

(b) $f_2 - f_1 = \Delta f = f_0/Q = 318.3/70 =$ **4.55 kHz**

(c) $f_1f_2 = f_0^2 = 318.3^2;\ f_2 - f_1 = 4.55$ kHz

Solving for f_1 and f_2, we get, f_1 = **3104 kHz** and f_2 = **320.59 kHz**

Note. Since Q is very high, there would be negligible error in assuming that the half-power points are equidistant from the resonant frequency.

Example 13.63. *A resistor and a capacitor are connected in series across a 150 V ac supply. When the frequency is 40 Hz, the circuit draws 5 A. When the frequency is increased to 50 Hz, it draws 6 A. Find the values of resistance and capacitance. Also find the power drawn in the second case.* **[Bombay University, 1997]**

Solution. Suffix 1 for 40 Hz and 2 for 50 Hz will be given.

$Z_1 = 150/5 = 30$ ohms at 40 Hz

or $R^2 + X_{C2}^2 = 900$

Similarly, $R^2 + X_{C2}^2 = 625$ at 50 Hz, since $Z_2 = 25\ \Omega$

Further, capacitive reactance is inversely proportional to the frequency.

$X_{C1}/X_{C2} = 50/40$ or $X_{C1} = 1.25\ X_{C2}$

$X_{C1}^2 - X_{C2}^2 = 900 - 625 = 275$

$X_{C1}^2\ (1.25^2 - 1) = 275$ or $X_{C2}^2 = 488.9$, $X_{C2} = 22.11$

$X_{C1} = 1.25 \times 22.11 = 27.64$ ohms

$R^2 = 900 - X_{C1}^2 = 900 - 764 = 136$, $R = 11.662$ ohms

$$C \quad \frac{1}{2 \quad 40 \quad 27.64} \quad 144 \quad \text{F}$$

Power drawn in the second case = $6^2 \times 11.662 = 420$ watts

Example 13.64. *A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set 500 pF, the current has its maximum value while it is reduced to one-half when the capacitance is 600 pF. Find*

(*i*) *the resistance,* (*ii*) *the inductance,* (*iii*) *the Q-factor of the inductor.*

[Bombay University, 1996]

Solution. Resonance takes palce at 1 MHz, for C = 500 pF.

$$LC = \frac{1}{\omega_0\ 2} = \frac{1}{(2\pi \times 10^6)^2} = \frac{10^{-12}}{4\pi^2}$$

$$L = 10^{-12}/(4 \times \pi^2 \times 500 \times 10^{-12}) = 1/(4 \times \pi^2 \times 500)$$

$$= 50.72 \text{ mH}$$

$$Z_1 = \text{Impedance with 500 pF capacitor} = R + j\ \omega L - j\ 1/\omega C$$

$$= R + j\ (2p \times 10^6 \times 50.72 \times 10^{-6}) - j\ \frac{1}{2\pi \times 10^6 \times 500 \times 10^{-12}}$$

$$= R, \text{ since resonance occurs.}$$

$$Z_2 = \text{Impedance with 600 pF capacitor}$$

$$|Z|_2 = R + j\ \omega L - j\ \frac{1}{\omega \times 600 \times 10^{-12}} = 2\ R, \text{ since current is halved.}$$

$$\omega L - 1/\omega C = \sqrt{3}\ R$$

$$\sqrt{3}\ R = 2\pi \times 10^6 \times 50.7 \times 10^{-6} - \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}}$$

$$= 2\pi \times 50.72\ \frac{10^6}{2\pi\ 600}$$

$$= 318.52 - 265.4 = 53.12$$

$$R = 30.67 \text{ ohms}$$

$$Q \text{ Factor of coil} = (\omega_0 L)/R$$

$$= 50.72 \times 10^{-6} \times 2\pi \times 10^{6}/30.67 = 50.72 \times 2\pi/30.67 = 10.38$$

Example 13.65. *A large coil of inductance 1.405 H and resistance 40 ohms is connected in series with a capacitor of 20 microfarads. Calculate the frequency at which the circuit resonates.*

If a voltage of 100 V at the corresponding frequency is applied to the circuit, calculate the current drawn from the supply and the voltages across the coil and across the capacitor.

[Nagpur University Nov. 1999]

Solution.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.405 \times 20 \times 10^{-6}}} = 188.65 \text{ radians/sec.}$$

$$f_0 = \frac{188.65}{2\pi} = 30.04 \text{ Hz}$$

Reactance at 30.04 Hz have to be calculated for voltages across the coil and the capacitor

$$X_L = \omega_0 L = 188.65 \times 1.405 = 265\ \Omega$$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{188.65 \times 20 \times 10^{-6}} = 265\ \Omega$$

Coil Impedance $= \sqrt{40^2 + 265^2} = 268\ \Omega$

Impedance of the total circuit

$$= 40 + j\,265 - j\,265 = 40\ \Omega$$

Supply Current $= \dfrac{100}{40} = 2.5$ amp, at unity p.f.

Voltage across the coil = 2.5 × 268 = 670 V

Voltage across the capacitor = 2.5 × 265 = 662.5 V

The phasor diagram is drawn below; in Fig. 13.50 (*a*) for the circuit in Fig. 13.50 (*b*)

$$OB = V,\ BC = V_C$$

$$AB = IX_L$$

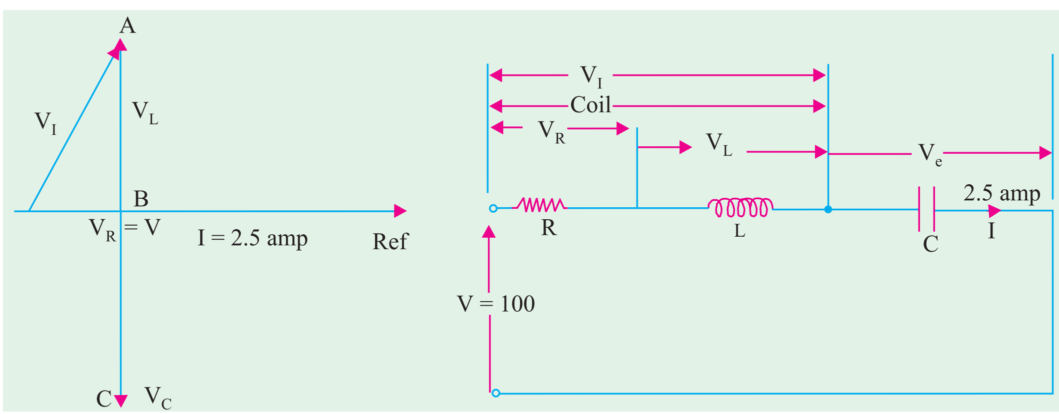

Fig. 13.50 (*a*) Phasor diagram

Fig. 13.50 (*b*) Series resonating circuit

Example 13.66. *A series R-L-C circuit is excited from a constant-voltage variable frequency source. The current in the circuit becomes maximum at a frequency of 600/2π Hz and falls to half the maximum value at 400/2π Hz. If the resistance in the circuit is 3 Ω, find L and C.*

(Grad. I.E.T.E. Summer 1991)

Solution. Current at resonance is $I_0 = V/R$

Actual current at any other frequency is $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$$\therefore \quad \frac{I}{I_0} = \frac{V}{\left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}} \cdot \frac{R}{V} = \frac{1}{\left[1 + \dfrac{1}{R^2}\left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{1}{\left[1 + \left(\dfrac{\omega_0 L}{R}\right)^2 \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2\right]^{1/2}}$$

Now $\quad Q = \dfrac{\omega_0 L}{R}$ and $\dfrac{\omega}{\omega_0} = \dfrac{f}{f_0}$, hence $\dfrac{I}{I_0} = \dfrac{1}{\left[1 + Q^2\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2\right]^{1/2}}$

In the present case, $f_0 = 600/2\pi$ Hz, $f = 400/2\pi$ Hz and $I/I_0 = 1/2$

$$\therefore \quad \frac{1}{2} = \frac{1}{\left[1 + Q^2\left(\dfrac{400}{600} - \dfrac{600}{400}\right)^2\right]^{1/2}} = \frac{1}{\left[1 + Q^2\left(\dfrac{2}{2} - \dfrac{3}{2}\right)^2\right]^{1/2}}$$

or $\quad \dfrac{1}{4} = \dfrac{1}{1 + 25\,Q^2/36} \quad \therefore\ Q = 2.08$

Now, $\quad Q = \dfrac{1}{\omega_0 RC}$ or $2.08 = \dfrac{1}{600 \times 3 \times C} \quad \therefore\ C = 267 \times 10^{-6}\ F =$ **267 mF**

Also $\quad Q = \omega_0\ L/R \ \therefore\ 2.08 = \dfrac{600L}{R} = \dfrac{600L}{3} \quad \therefore\ L =$ **10.4 mH**

Example 13.67. *Discuss briefly the phenomenon of electrical resonance in simple R-L-C circuits.*

A coil of inductance L and resistance R in series with a capacitor is supplied at constant voltage from a variable-frequency source. Call the resonance frequency ω_0 and find, in terms of L, R and ω_0, the values of that frequency at which the circuit current would be half as much as at resonance.

(Basic Electricity, Bombay Univ.)

Solution. For discussion of resonance, please refer to Art. 13.10.

The current at resonance is maximum and is given by $I_0 = V/R$. Current at any other frequency is

$$I = \frac{V}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}$$

$$\therefore \quad \frac{I_0}{I} = \frac{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}{R}$$

or $$N = \left[1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

Now $$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$\therefore \quad R = \frac{\omega_0 L}{Q} = \frac{1}{\omega_0 CQ}$$

Substituting this value in the above equation, we get

$$N = \left[1 + Q^2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2\right]^{1/2}.$$

or $$\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2 = \frac{N^2 - 1}{Q^2} \quad \therefore \quad \frac{\sqrt{(N^2 - 1)}}{Q} = \pm\left(\frac{f}{f_0} - \frac{f_0}{f}\right)$$

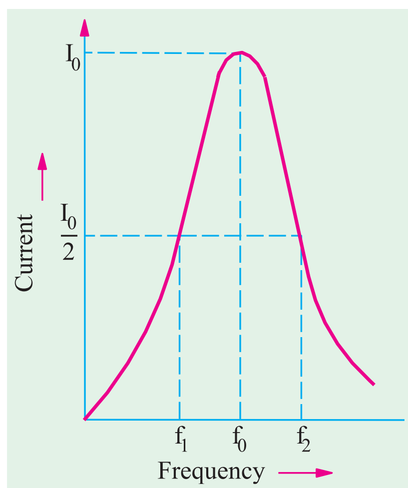

Fig. 13.51

or $$\frac{\sqrt{(N^2-1)}}{Q} = \frac{f_2}{f_0} - \frac{f_0}{f_2} = \frac{f_0}{f_1} - \frac{f_1}{f_0}$$

where $f_2 > f_0$ and $f_1 < f_0$ are the two frequencies at which the current has fallen to 1/N of the resonant value.

In the present case, $N = 2$ (Fig. 13.50) $\therefore \frac{f_2}{f_0} - \frac{f_0}{f_2} = \frac{\sqrt{3}}{Q}$ and $\frac{f_0}{f_1} - \frac{f_1}{f_0} = \frac{\sqrt{3}}{Q}$

From these equations, f_1 and f_2 may be calculated.

Example 13.68. *A coil of inductance 9 H and resistance 50 Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current of 1A occurs at 75 Hz, find the frequency when the current is 0.5 A.*

(Principles of Elect. Engg. Delhi Univ.)

Solution. Here, $N = I_0/I = I/0.5 = 2$; $Q = \omega_0 L/R = 2\pi \times 75 \times 9/50 = 84.8$

Let f_1 and f_2 be the frequencies at which current falls to half its maximum value at resonance frequency. Then, as seen from above

$$\frac{f_0}{f_1} - \frac{f_1}{f_0} = \frac{\sqrt{3}}{Q} \text{ or } \frac{75}{f_1} - \frac{f_1}{75} = \frac{\sqrt{3}}{84.4}$$

or $$(75^2 - f_1^2)/75 f_1 = 0.02 \text{ or } f_1^2 + 1.5 f_1 - 5625 = 0 \text{ or } f_1 = \mathbf{74.25\ Hz}$$

Also $$\frac{f_2}{75} - \frac{75}{f_2} = \frac{\sqrt{3}}{84.4} \text{ or } f_2^2 - 1.5 f_2 - 5625 = 0 \text{ or } f_2 = \mathbf{75.75\ Hz.}$$

Example 13.69. *Using the data given in Ex. 13.45 find the following when the power drops to 4 W on either side of the maximum power at resonance.*

(a) *circuit Q* (*b*) *circuit phase angle φ* **(c)** *4-W bandwidth B*

(d) *lower freqency f_1* **(e)** *upper frequency f_2.*

(f) *resonance frequency using the value of f_1 and f_2.*

Solution. (a) $$P = \frac{P_0}{(1+Q_0^2)} \quad \therefore Q = \sqrt{(P_0/P_1) - 1} = \sqrt{(20/4) - 1} = \mathbf{2}$$

(b) $$\tan(\pm\theta) = 2\ j \pm\theta/\tan^{-1} 2 = \pm\mathbf{63.4°}$$

(c) $$B_{hp} = \frac{f_0 Q}{Q_0} = \frac{1591 \times 2}{10} = 318.2 \text{ Hz}$$

(d) $$f_1 = f_0 - B/2 = 1591 - (318.2/2) = 1431.9 \text{ Hz}$$

(e) $$f_2 = f_0 + B/2 = 1591 + (318.2/2) = 1750.1 \text{ Hz}$$

(f) $$f_0 = \sqrt{f_1 f_2} = \sqrt{1431.9 \times 1750.2} = \mathbf{1591\ Hz.}$$

It shows that regardless of the bandwidth magnitude, f_0 is always the geometric mean of f_1 and f_2.

Example 13.70. *A constant e.m.f. source of variable frequency is connected to a series R.L.C. circuit of Fig. 13.51.*

(a) *Shown in nature of the frequency $-V_R$ graph*

(b) *Calculate the following* **(i)** *frequency at which maximum power is consumed in the 2 Ω resistor*

(ii) *Q-factor of the circuit at the above frequency* **(iii)** *frequencies at which the power consumed in 2 Ω resistor is one-tenth of its maximum value.*

(Network Analysis A.M.I.E Sec. B.W.)

Solution. (a) The graph of angular frequency ω versus voltage drop across *R i.e.* V_R is shown in Fig. 13.52. It is seen that as frequency of the applied voltage increases, V_R increases till it reaches its maximum value when the given *RLC* circuit becomes purely reactive *i.e.* when $X_L = X_C$ (Art. 13.10). (*b*) (*i*) maximum power will be consumed in the 2 Ω resistor when maximum current flows in the circuit under resonant condition.

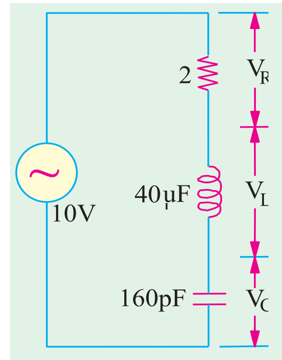

Fig. 13.52

For resonance $\omega_0 L = 1/\omega_0 X_C$ or

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{40 \times 10^{-6} \times 160 \times 10^{-12}} = 10^9/80 \text{ rad/s}$$

$\therefore \quad f_0 = \omega_0/2\pi = 10^9/2\pi \times 80 =$ **1.989 MHz**

(*ii*) *Q*-factor, $Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{10^9 \times 40 \times 10^{-6}}{80 \times 2} =$ **250**

(*iii*) Maximum current $I_0 = V/R$ (Art. 13.10). Current at any other frequency is $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

Power at any frequency $I^2R = \dfrac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} . R$

Maximum power $I_0^2 \, R = \left(\dfrac{V}{R}\right)^2 . R$

Hence, the frequencies at which power consume would be one-tenth of the maximum power will be given by the relation. $\dfrac{1}{10} . \left(\dfrac{V}{r}\right)^2 R = \dfrac{V^2}{R^2 + (\omega L - 1/\omega C)^2} . R$

or cross multiplying, we get $R^2 + (\omega L - 1/\omega C)^2 = 10R^2$ or $(\omega L - 1/\omega C^2 = 9 R^2$

$$\therefore \quad (\omega L - 1/\omega C) = \pm 3 R \text{ and } \omega_1 L - 1/\omega_1 C = 3R$$

$$\text{and } \omega_2 L - 1/\omega_2 C = -3 R$$

Adding the above two equations, we get

$$(\ _1 \quad _2) L \quad \frac{1}{C} \quad \frac{2 \quad 1}{1 \cdot \ 2} \quad 0 \text{ or } \quad 1 \ 2 \quad \frac{1}{LC} \quad \frac{2}{0}$$

Subtracting the same two equations, we have $L(\ _1 \quad _2) \quad \dfrac{1}{C} \quad \dfrac{1}{2} \quad \dfrac{1}{1} \quad 6R$

$$(\ _1 \quad _2) \quad \frac{1}{LC} \quad \frac{1 \quad 2}{1 \cdot \ 2} \quad \frac{6R}{L}$$

Substituting the value of $1/LC = \omega^2 = \omega_1\omega_2$, we get

$$(\omega_1 - \omega_2) + \omega_1\omega_2 \left(\frac{\omega_1 - \omega_2}{\omega_1\omega_2}\right) = \frac{6R}{L} \text{ or } (\omega_1 - \omega_2) = \Delta\omega = \frac{3R}{L}$$

Now, $\quad \omega_2 = \omega_0 + \dfrac{\Delta\omega}{2} = \omega_0 + \dfrac{1.5R}{L}$ and $\omega_1 = \omega_0 - \dfrac{1.5R}{L}$

$\therefore f_2 = f_0 + 1.5\ R/2\pi L = 1.989 \times 10^6 + 1.5 \times 2.2\pi \times 40 \times 10^{-6} = 1.989 \times 10^6 + 0.0119 \times 10^6 =$ **2 MHz**

$f_1 = f_0 - 1.5\ R/2\pi L = 1.989 \times 10^6 - 0.0119 \times 10^6 =$ **1.977 MHz**

Example 13.71. *Show that in R-L-C circuit, the resonant frequency ω_0 is the geometric mean of the lower and upper half-power frequencies ω_1 and ω_2 respectively.*

Solution. As stated earlier, at lower half-power radiant frequency ω_1; $X_C > X_L$ and at frequencies higher than half-power frequencies $X_L > X_C$. However, the difference between the two equals *R*.

$\therefore \quad$ at ω_1, $X_C - X_L = R$ or $1/\omega_1 C - \omega_1 L = R \quad$ ...(*i*)

At ω_2, $X_L - X_C = R$ or $\omega_2 L - 1/\omega_2 C = R$

Multiplying both sides of Eq. (*i*) by *C* and substituting $_0^2 \quad 1/\sqrt{LC}$, we get

$$\frac{1}{\ _1} \quad \frac{_1}{_0^2} \quad \frac{_2}{_0^2} \quad \frac{1}{\ _2} \text{ or } \frac{1}{\ _1} \quad \frac{1}{\ _2} \quad \frac{_1 \quad _2}{_0^2} \text{ or } _0 \quad \sqrt{_1 \ _2}$$

Example 13.72. *Prove that in a sereis R-L-C circuit, $Q_0 = \omega_0 L/R = f_0/bandwidth = f_0/BW$.*

Solution. As has been proved in Art. 14.13, at half power frequencies, net reactance equals resistance. Moreover, at ω_1, capacitive reactance exceeds inductive reactance whereas at ω_2, inductive reactance exceeds capacitive reactance.

$$\therefore \qquad \frac{1}{2\pi f_1 C} - 2\pi f_1 L = R \text{ or } f_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

Also
$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = \text{ or } f_2 = \frac{R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

Now, $BW = f_2 - f_1 = R/2\pi L$. Hence, $Q_0 = f_0/BW = 2\pi f_0 L/R = \omega_0 L/R$.

Tutorial Problem No. 13.4

1. An a.c. series circuit has a resistance of 10 Ω, an inductance of 0.2 H and a capacitance of 60 μF. Calculate
(*a*) the resonant frequency (*b*) the current and (*c*) the power at resonance.
Give that the applied voltage is 200 V. **[46 Hz; 20 A; 4 kW]**
2. A circuit consists of an inductor which has a resistance of 10 Ω and an inductance of 0.3 H, in series with a capacitor of 30 μ*F* capacitance. Calculate
(*a*) the impedance of the circuit to currents of 40 Hz (*b*) the resonant frequency (*c*) the peak value of stored energy in joules when the applied voltage is 200 V at the resonant frequency.
[58.31 Ω; 53 Hz; 120 J]
3. A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 240-V, 50-Hz supply, the maximum current given by varying the inductance is 0.5 A. At this current, the voltage across the capacitor is 250 V. Calculate the values of
(*a*) the resistance (*b*) the capacitance (*c*) the inductance. **[480 Ω, 6.36 μF; 1.59 H]**
Neglect the resistance of the inductor
4. A circuit consisting of a coil of resistance 12Ω and inductance 0.15 H in series with a capacitor of 12μF is connected to a variable frequency supply which has a constant voltage of 24 V.
Calculate (*a*) the resonant frequency (*b*) the current in the circuit at resonance (*c*) the voltage across the capacitor and the coil at resonance. **[(*a*) 153 Hz (*b*) 2 A (*c*) 224 V]**
5. A resistance, a capacitor and a variable inductance are connected in series across a 200-V, 50-Hz supply. The maximum current which can be obtained by varying the inductance is 314 mA and the voltage across the capacitor is then 300 V. Calculate the capacitance of the capacitor and the values of the inductance and resistance. **[3.33 μF, 3.04 H, 637 Ω] (*I.E.E. London*)**
6. A 250-V circuit, consisting of a resistor, an inductor and a capacitor in series, resonates at 50 Hz. The current is then 1A and the p.d. across the capacitor is 500 V. Calculate (*i*) the resistance (*ii*) the inductance and (*iii*) the capacitance. Draw the vector diagram for this condition and sketch a graph showing how the current would vary in a circuit of this kind if the frequency were varied over a wide range, the applied voltage remaining constant.
[(*i*) 250 Ω (*ii*) 0.798 H (*iii*) 12.72 μ F] (*City & Guilds, London*)
7. A resistance of 24 Ω, a capacitance of 150 μF and an inductance of 0.16 H are connected in series with each other. A supply at 240 V, 50 Hz is applied to the ends of the combination. Calculate (*a*) the current in the circuit (*b*) the potential differences across each element of the circuit (*c*) the frequency to which the supply would need to be changed so that the current would be at unity power-factor and find the current at this frequency.
[(*a*) 6.37 A (*b*) V_R = 152.9 V, V_C = 320 V, V_C = 123.3 V (*c*) 32 Hz; 10 A] (*London Univ.*)
8. A series circuit consists of a resistance of 10 Ω, an inductance of 8 mH and a capacitance of 500 μμ*F*. A sinusoidal E.M.F. of constant amplitude 5 V is introduced into the circuit and its frequency varied

over a range including the resonant frequency.
At what frequencies will current be (*a*) a maximum (*b*) one-half the-maximum ?
[(*a*) 79.6 kHz (*b*) 79.872 kHz, 79.528 kHz] (*App. Elect. London Univ.*)

9. A circuit consists of a resistance of 12 ohms, a capacitance of 320 µF and an inductance of 0.08 H, all in series. A supply of 240 V, 50 Hz is applied to the ends of the circuit. Calculate :
(*a*) the current in the coil.
(*b*) the potential differences across each element of the circuit.
(*c*) the frequency at which the current would have unity power-factor.
[(*a*) 12.4 A (*b*) 149 V, 311 V (*c*) 32 Hz] (*London Univ.*)

10. A series circuit consists of a reactor of 0.1 henry inductance and 5 ohms resistance and a capacitor of 25.5 µ F capacitance.
Find the resonance frequency and the precentage change in the current for a divergence of 1 percent from the resonance frequency.
[100 Hz, 1.96% at 99 Hz; 4.2% at 101 Hz] (*City and Guilds, London*)

11. The equation of voltage and currents in two element series circuit are :
$v(t) = 325.3 \sin(6.28\,kt + \pi/3)$ volts
$i(t) = 14.142 \sin(6.28\,kt + \pi/2)$ Amp.
(*i*) Plot the power p (t) on wave diagram. (*ii*) Determine power factor and its nature. (*iii*) Determine the elements value. ***(Nagpur University, Winter 2002)***

12. A pure capacitor is connected in series with practical inductor (coil). The voltage source is of 10 volts, 10,000 Hz. It was observed that the maximum current of 2 Amp. flows in the circuit when the value of capacitor is 1 micro-farad. Find the parameter (R & L) of the coil.
(Nagpur University, Winter 2002)

13. Draw the phasor diagram for each of the following combinations :
(*i*) Rand L in series and combination in parallel with C.
(*ii*) Rand C in series and combination in parallel with L.
(*iii*) R, Land C in series, with $X_L > X_C$, when ac source is connected to it.
(Nagpur University, Summer 2003)

14. Two choke coils are connected in series as shown in Fig. 13.53. :

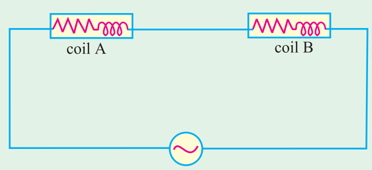

Fig. 13.53

Internal resistance and its inductive reactance of coil A is 4 Ω and 8 Ω resp. Supply voltage is 200 V. Total power consumed in the circuit is 2.2 kW and reactive power consumed is 1.5 kVAR. Find the internal resistance and inductive reactance of coil B. ***(Nagpur University, Summer 2004)***

15. A series circuit having a resistance of 10Ω, and inductance of (1/2π) H and a variable cap. is connected to 100 V, 50 Hz supply. Calculate the value of capacitor to form series resonance. Calculate resonant current, power and power factor. ***(Gujrat University, June/July 2003)***

16. Derive expressions for impedance, current and power factor for an R-L-C series circuit when applied with a.c. voltage. Draw also the vector diagram. ***(Gujrat University, June/July 2003)***

17. Explain the terms active power, reactive power and power factor. Also describe the series resonance of RLC circuit and list its important properties. ***(R.G.P.V. Bhopal University, June 2004)***

18. A 60 Hz sinusoidal voltage $v = 100 \sin \omega t$ is applied to a series R-L circuit. Assuming R = 10 Ω, L = 0.01 H, find the steady state current and relative phase angle. Also compute the effective magnitude and phase of voltage drops across each circuit elements.
(R.G.P.V. Bhopal University, June 2004)

19. With reference to Fig 13.54, find the values of R and X so that $V_1 = 3V_2$ and V_1 and V_2 are in quadrature. Applied voltage across AB is 240V. ***(Belgaum Karnataka University, February 2002)***

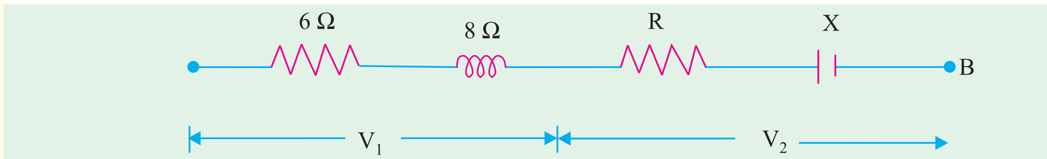

Fig. 13.54

20. A choke coil takes a current of 2 A lagging 60^0 behind the applied voltage of 220V at 50Hz. Calculate the inductance and resistance of the coil. ***(V.T.U., Belgaum Karnataka University Winter 2003)***
21. The instantaneous values ofthe voltage across a two element series circum and the current flowing through it are given by V = 100 sin $(314t - \pi/4)V$, $i = 20 sin\ (314t - 90^0)A$. Find the frequency and the circuit elements. ***(V.T.U., Belgaum Karnataka University, Winter 2003)***
22. Show that the power consumed in a pure inductance is zero. ***(U.P. TechnicalUniversity 2003) (RGPV Bhopal 2002)***
23. What do you understand by the terms power factor, active power and reactive power? ***(Mumbai University 2003) (RGPV Bhopal 2002)***
24. Series R-L-C circuit ***(Mumbai University 2003) (RGPV Bhopal 2002)***
25. Describe the properties of (*i*) Resistance (*ii*) Inductance and (*iii*) capacitance used in A.C. Circuit. ***(RGPV Bhopal June 2003)***
26. Define Apparant Power and Power factor in a.c. circuit. Describe parallel resonance and list its important properties. ***(Mumbai University 2003) (RGPV Bhopal December 2003)***

OBJECTIVE TESTS -13

1. In a series *R-L* circuit, V_L—V_R by—degrees.
   (*a*) lags, 45 (*b*) lags, 90
   (*c*) leads, 90 (*d*) leads, 45
2. The voltage applied across an *R-L* circuit is equal to—of V_R and V_L.
   (*a*) arithmetic sum (*b*) algebraic sum
   (*c*) phasor sum (*d*) sum of the squares.
3. The power in an a.c. circuit is given by
   (*a*) $VI \cos \phi$ (*b*) $VI \sin \phi$
   (*c*) I^2Z (*d*) I^2X_L
4. The p.f. of an *R-C* circuit is
   (*a*) often zero
   (*b*) between zero and 1
   (*c*) always unity
   (*d*) between zero and −1.0
5. In a series RLC circuit at resonance, the magnitude of the voltage developed across the capacitor
   (*a*) is always zero
   (*b*) can never be greater than the input voltage.
   (*c*) can be greater than the input voltage, however, it is 90^0 out of phase with the input voltage
   (*d*) can be greater than the input voltage,and in in phase with the input voltage. ***(GATE 2001)***
6. The total impedance Z (jω) of the circuit shown above is

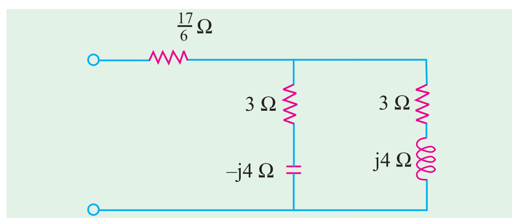

Fig. 13.55

   (*a*) (6 + *j*0) Ω (*b*) (7 + *j*0) Ω
   (*c*) (0 + *j*8) Ω (*d*) (6 + *j*8) Ω ***(ESE 2003)***
7. The impedance of a parallel RC network is $Z(s) = \dfrac{58}{s^2 + 0.5s + 100}$. Then the values of R, L and C are, respectively
   (*a*) $10\,\Omega, \frac{1}{20}$ H, $\frac{1}{5}$ F (*b*) $1\,\Omega, \frac{1}{2}$ H, $\frac{1}{5}$ F
   (*c*) $10\,\Omega, \frac{1}{20}$ H, $\frac{1}{5}$ F (*d*) $2\,\Omega, \frac{1}{20}$ H, $\frac{1}{5}$ F
   (Engineering Services Exam. 2003)

ANSWERS

1. *c* **2.** *c* **3.** *a* **4.** *b*

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