C H A P T E R 10

ELECTRICAL INSTRUMENTS AND MEASURE-MENTS

Learning Objectives

- Absolute and Secondary Instruments
- Deflecting Torque
- Controlling Torque
- Damping Torque
- Moving-iron Ammeters and Voltmeters
- Moving-coil Instruments
- Permanent Magnet Type Instruments
- Voltmeter Sensitivity
- Electrodynamic or Dynamometer Type Instruments
- Hot-wire Instruments
- Megger
- Induction Voltmeter
- Wattmeters
- Energy Meters
- Electrolytic Meter
- Ampere-hour Mercury Motor Meter
- Friction Compensation
- Commutator Motor Meters
- Ballistic Galvanometer
- Vibration Galvanometer
- Vibrating-reed Frequency Meter
- Electrodynamic Frequency Meter
- Moving-iron Frequency Meter
- Electrodynamic Power Factor Meter
- Moving-iron Power Factor Meter
- Nalder-Lipman Moving-iron Power Factor Meter
- D.C. Potentiometer
- A.C. Potentiometers
- Instrument Transformers
- Potential Transformers.

Electrical instruments help us to measure the changes in variables such as voltage, current and resistance

10.1. Absolute and Secondary Instruments


An absolute instrument

The various electrical instruments may, in a very broad sense, be divided into ***(i) absolute*** instruments and ***(ii) secondary*** **instruments. Absolute instruments** ***are those which give the value of the quantity to be measured, in terms of the constants of the instrument and their deflection only.*** No previous calibration or comparision is necessary in their case. The example of such an instrument is tangent galvanometer, which gives the value of current, in terms of the tangent of deflection produced by the current, the radius and number of turns of wire used and the horizontal component of earth's field.

Secondary instruments are those, in which the value of electrical quantity to be measured can be determined from the deflection of the instruments, only when they have been pre-calibrated by comparison with an absolute instrument. Without calibration, the deflection of such instruments is meaningless.

It is the secondary instruments, which are most generally used in everyday work; the use of the absolute instruments being merely confined within laboratories, as standardizing instruments.

10.2. Electrical Principles of Operation

All electrical measuring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are :

1. Magnetic effect - for ammeters and voltmeters usually.
2. Electrodynamic effect - for ammeters and voltmeters usually.
3. Electromagnetic effect - for ammeters, voltmeters, wattmeters and watthour meters.
4. Thermal effect - for ammeters and voltmeters.
5. Chemical effect - for d.c. ampere-hour meters.
6. Electrostatic effect - for voltmeters only.

Another way to classify secondary instruments is to divide them into ***(i) indicating instruments (ii) recording instruments and (iii) integrating instruments.***

Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured *at the time* at which it is being measured. Their indications are given by pointers moving over calibrated dials. Ordinary ammeters, voltmeters and wattmeters belong to this class.

Recording instruments are those, which, instead of indicating by means of a pointer and a scale the instantaneous value of an electrical quantity, give a ***continuous record*** or the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked pen which rests lightly on a chart or graph, that is moved at a uniform and low speed, in a direction perpendicular to that of the deflection of the pen. The path traced out by the pen presents a continuous record of the variations in the deflection of the instrument.

Integrating instruments are those which measure and register by a set of dials and pointers either the ***total*** quantity of electricity (in amp-hours) or the ***total*** amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time. This summation gives the product of time and the electrical quantity but gives no direct indication as to the ***rate*** at which the quantity or energy is being supplied because their registrations are independent of this rate provided the current flowing through the instrument is sufficient to operate it.

Ampere-hour and watt-hour meters fall in this class.

10.3. Essentials of Indicating Instruments

As defined above, indicating instruments are those which indicate the value of the quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale and which is attached to a moving system pivoted in jewelled bearings. The moving system is subjected to the following three torques :

1. A deflecting (or operating) torque
2. A controlling (or restoring) torque
3. A damping torque.

10.4. Deflecting Torque

The deflecting or operating torque (T_d) is produced by utilizing one or other effects mentioned in Art. 10.2 *i.e.* magnetic, electrostatic, electrodynamic, thermal or inductive etc. The actual method of torque production depends on the type of instrument and will be discussed in the succeeding paragraphs. This deflecting torque causes the moving system (and hence the pointer attached to it) to move from its '***zero***' position *i.e.* its position when the instrument is disconnected from the supply.

10.5. Controlling Torque

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque oppose the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques are equal. The deflecting torque ensures that currents of different magnitudes shall produce deflections of the moving system in proportion to their size. Without such at torque, the pointer would swing over to the maximum deflected position irrespective of the magnitude of the current to be measured. Moreover, in the absence of a restoring torque, the pointer once deflected, would not return to its zero position on removing the current. The controlling or restoring or balancing torque in indicating instruments is obtained either by a spring or by gravity as described below :

(*a*) Spring Control

A hair-spring, usually of phosphor-bronze, is attached to the moving system of the instrument as shown in Fig. 10.1 (*a*).

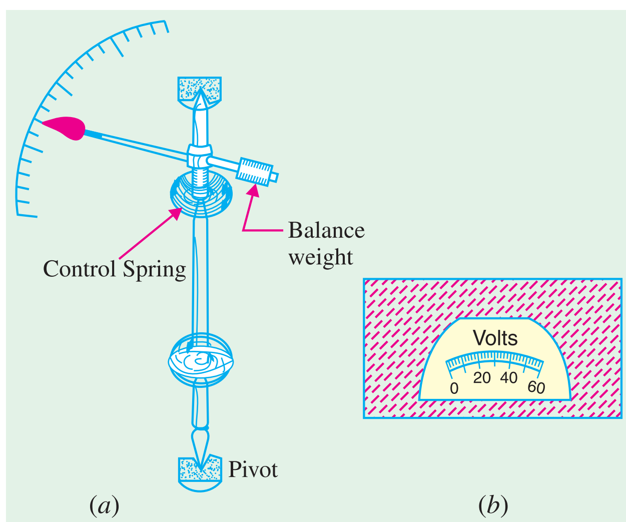


Fig. 10.1

With the deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring produces restoring torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to a position of rest (or equilibrium) when the deflecting torque (T_d) and controlling torque (T_c) are equal. For example, in permanent-magnet, moving-coil type of instruments, the deflecting torque is proportional to the current passing through them.

$\therefore \quad T_d \propto I$

and for spring control $T_c \propto \theta$

As $\quad T_c = T_d$

$\therefore \quad \theta \propto I$

Since deflection θ is directly proportional to current I, the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range as shown in Fig. 10.1 (*b*).

To ensure that controlling torque is proportional to the angle of deflection, the spring should have a fairly large number of turns so that angular deformation per unit length, on full-scale deflection, is small. Moreover, the stress in the spring should be restricted to such a value that it does not produce a permanent set in it.

Springs are made of such materials which

(i) are non-magnetic
(ii) are not subject to much fatigue
(iii) have low specific resistance-especially in cases where they are used for leading current in or out of the instrument
(iv) have low temperature-resistance coefficient.

The exact expression for controlling torque is $T_c = C\theta$ where C is *spring constant*. Its value is given by $C = \frac{Ebt^3}{L}$ N-m/rad. The angle θ is in radians.

(*b*) Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system such that the two exert torques in the opposite directions. The usual arrangements is shown in Fig. 10. 2(*a*).

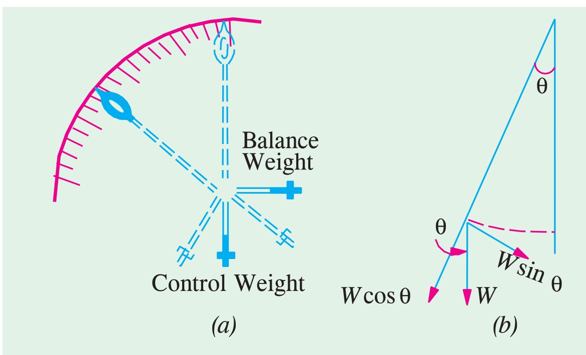


Fig. 10.2

It is seen from Fig. 10.2 (*b*) that the controlling or restoring torque is proportional to the sine of the angle of deflection *i.e.*

$$T_c \propto \sin\theta$$

The degree of control is adjusted by screwing the weight up or down the carrying system

It $\quad T_d \propto I$

then for position of rest

$$T_d = T_c$$

or $\quad I \propto \sin\theta$ (*not* θ)

It will be seen from Fig. 10.2 (*b*) that as θ approaches 90°, the distance AB increases by a relatively small amount for a given change in the angle than when θ is just increasing from its zero value. Hence, gravity-controlled instruments have scales which are not uniform but are cramped or crowded at their lower ends as shown in Fig. 10.3.

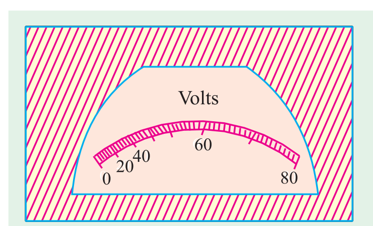


Fig. 10.3

As compared to spring control, the disadvantages of gravity control are :

1. it gives cramped scale
2. the instrument has to be kept vertical.

However, gravity control has the following advantages :

1. it is cheap
2. it is unaffected by temperature
3. it is not subjected to fatigue or deterioration with time.

Example 10.1 *The torque of an ammeter varies as the square of the current through it. If a current of 5 A produces a deflection of 90°, what deflection will occur for a current of 3 A when the instrument is* ***(i)*** *spring-controlled and* ***(ii)*** *gravity-controlled.*

(Elect. Meas. Inst and Meas. Jadavpur Univ.)

Solution. Since deflecting torque varies as (current)^2, we have $T_d \propto I^2$

For spring control, $T_c \propto \theta \quad \therefore \theta \propto I^2$

For gravity control, $T_c \propto \sin\theta \quad \therefore \sin\theta \propto I^2$

(*i*) **For spring control** $90^o \propto 5^2$ and $\theta \propto 3^2$; $\theta = 90° \times 3^2/5^2 =$ **32.4º**

(*ii*) **For gravity control** $\sin 90^o \propto 5^2$ and $\sin\theta \propto 3^2$

$\sin\theta = 9/25 = 0.36$; $\theta = \sin^{-1}(0.36) =$ **21.1º.**

10.6. Damping Torque

A damping force is one which acts on the moving system of the instrument ***only when it is moving*** and always opposes its motion. Such stabilizing or demping force is necessary to bring the pointer to rest ***quickly,*** otherwise due to inertia of the moving system, the pointer will oscillate about its final deflected position for quite some time before coming to rest in the steady position. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be ***dead-beat.*** Any increase of damping above this limit *i.e.* overdamping will make the instruments slow and lethargic. In Fig. 10.4 is shown the effect of damping on the variation of position with time of the moving system of an instrument.

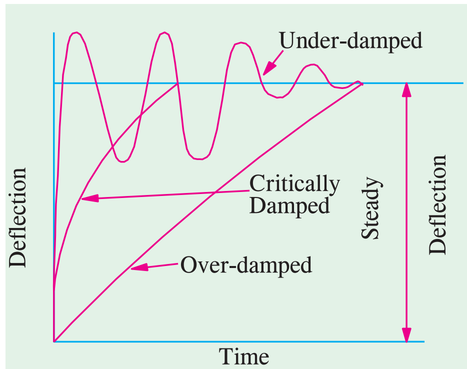


Fig. 10.4

The damping force can be produced by ***(i) air frictions (ii) eddy currents*** and ***(iii) fluid friction*** (used occasionally).

Two methods of air-friction damping are shown in Fig. 10.5 (*a*) and 10.5 (*b*). In Fig.. 10.5 (*a*), the light aluminium piston attached to the moving system of the instrument is arranged to travel with

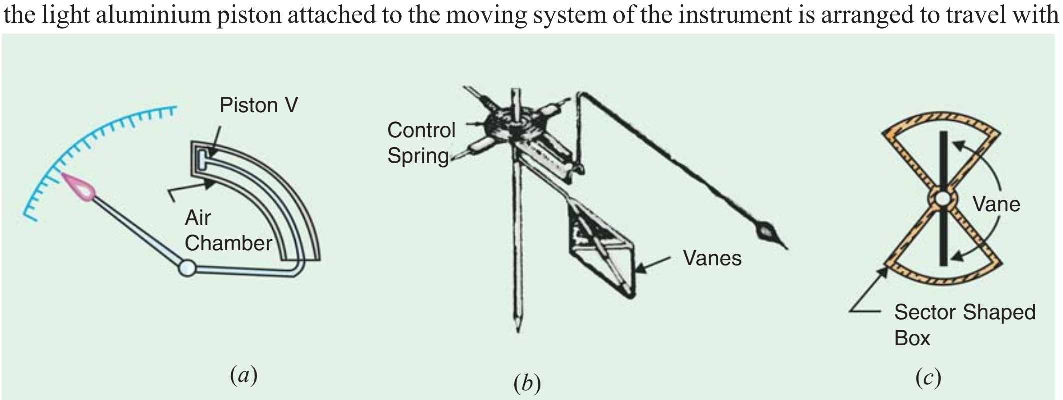


Fig. 10.5

a very small clearance in a fixed air chamber closed at one end. The cross-section of the chamber is either circular or rectangular. Damping of the oscillation is affected by the compression and suction actions of the piston on the air enclosed in the chamber. Such a system of damping is not much favoured these days, those shown in Fig. 10.5 (*b*) and (*c*) being preferred. In the latter method, one or two light aluminium vanes are mounted on the spindle of the moving system which move in a closed sector-shaped box as shown.

Fluid-friction is similar in action to the air friction. Due to greater viscosity of oil, the damping is more effective. However, oil damping is not much used because of several disadvantages such as objectionable creeping of oil, the necessity of using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The eddy-current form of damping is the most efficient of the three. The two forms of such a damping are shown in Fig. 10.6 and 10.7. In Fig. 10.6 (*a*) is shown a thin disc of a conducting but ***non-magnetic*** material like copper or aluminium mounted on the spindle which carries the moving system and the pointer of the instrument. The disc is so positioned that its edges, when in rotation, cut the magnetic flux between the poles of a permanent magnet. Hence, eddy currents are produced in the disc which flow and so produce a damping force in such a direction as to oppose the very cause producing them (Lenz's Law Art. 7.5). Since the cause producing them is the rotation of the disc, these eddy current retard the motion of the disc and the moving system as a whole.

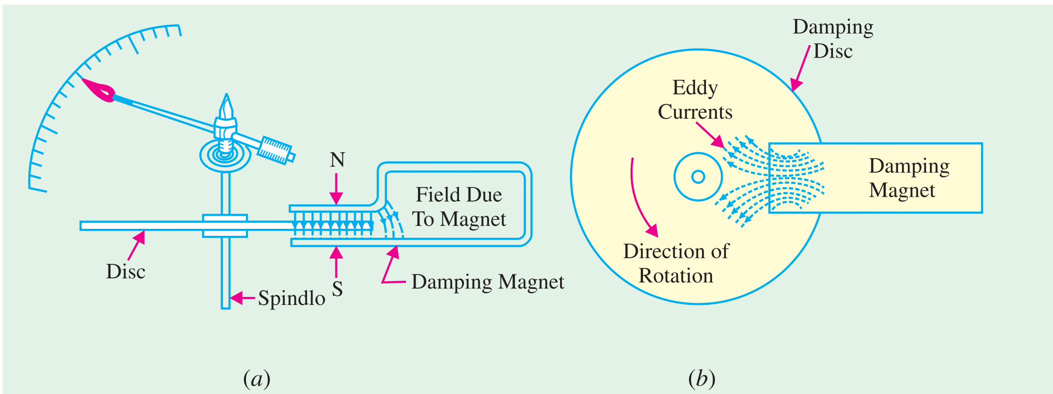


Fig. 10.6

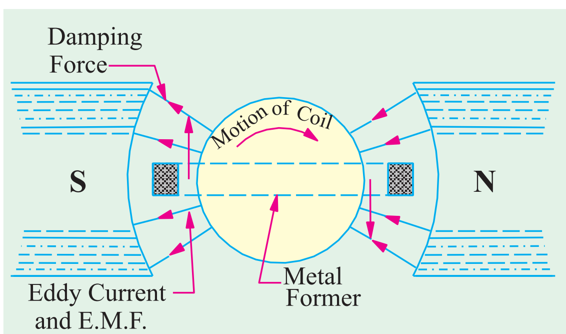


Fig. 10.7

In Fig. 10.7 is shown the second type of eddy-current damping generally employed in permanent-magnet moving coil instruments. The coil is wound on a thin light aluminium former in which eddy currents are produced when the coil moves in the field of the permanent magnet. The directions of the induced currents and of the damping force produced by them are shown in the figure.

The various types of instruments and the order in which they would be discussed in this chapter are given below.

Ammeters and voltmeters

1. Moving-iron type (both for D.C./A.C.)
   - ***(a) the attraction type***
   - ***(b) the repulsion type***
2. Moving-coil type
   - ***(a) permanent-magnet type*** (for D.C. only)
   - ***(b) electrodynamic or dynamometer type*** (*for D.C./A.C.*)
3. Hot-wire type (both for D.C./A.C.)
4. Induction type (for A.C. only)
   - ***(a) Split-phase type***
   - ***(b) Shaded-pole type***
5. Electrostatic type-for voltmeters only (for D.C./A.C.)

Wattmeter

6. ***Dynamometer type*** (both for D.C./A.C.),
7. ***Induction type*** (for A.C. only)
8. *Electrostatic type* (for D.C. only)

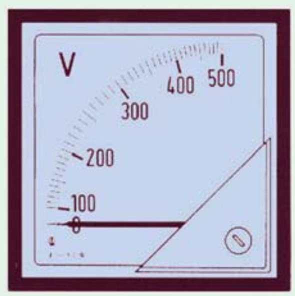
Energy Meters

9. ***Electrolytic type*** (for D.C. only)
10. **Motor Meters**
    - ***(i) Mercury Motor Meter.*** For d.c. work only. Can be used as amp-hour or watt-hour meter.
    - ***(ii) Commutator Motor Meter.*** Used on D.C./A.C. Can be used as Ah or Wh meter.

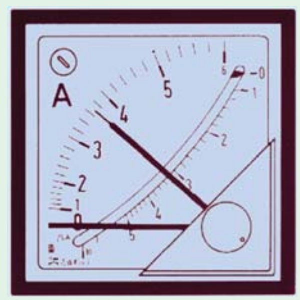
(iii) Induction type. For A.C. only.

11. Clock meters (as Wh-meters).

10.7. Moving-iron Ammeters and Voltmeters



Moving iron voltmeter



Moving iron ammeter

There are two basic forms of these instruments *i.e.* the ***attraction*** type and the ***repulsion*** type. The operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field and that of repulsion type depends on the repulsion of two adjacent pieces of iron magnetised by the same magnetic field. For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil. In case the instrument is to be used as an ammeter, the coil has comparatively fewer turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit. In case it is to be used as a voltmeter, the coil has high impedance so as to draw as small a current as possible since it is connected in parallel with the circuit. As the current through the coil is small, it has large number of turns in order to produce sufficient ampere-turns.

10.8. Attraction Type M.I. Insturments

The basic working principle of an attraction-type moving-iron instrument is illustrated in Fig. 10.8. It is well-known that if a piece of an unmagnetised soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet. Hence, if we pivot an oval-shaped disc of soft iron on a spindle between bearings near the coil (Fig. 10.8), the iron disc will swing into the coil when the latter has an electric current passing through it. As the field strength would be strongest at the centre of

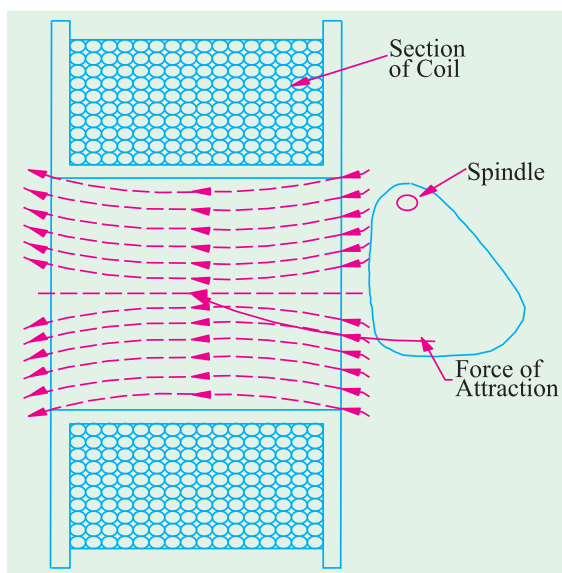


Fig. 10.8

the coil, the ovalshaped iron disc is pivoted in such a way that the greatest bulk of iron moves into the centre of the coil. If a pointer is fixed to the spindle carrying the disc, then the passage of current through the coil will cause the pointer to deflect. The amount of deflection produced would be greater when the current producing the magnetic field is greater. Another point worth noting is that ***whatever the direction of current through the coil, the iron disc would always be magnetised in such a way that it is pulled inwards.*** Hence, such instruments can be used both for direct as well as alternating currents.

A sectional view of the actual instrument is shown in Fig. 10.9. When the current to be measured is passed through the coil or solenoid, a magnetic field is produced, which attracts the eccentrically-mounted disc inwards, thereby deflecting the pointer, which moves over a calibrated scale.

Deflecting Torque

Let the axis of the iron disc, when in zero position, subtend an angle of ϕ with a direction perpendicular to the direction of the field H produced by the coil. Let the deflection produced be θ corresponding to a current I through the coil. The magnetisation of iron disc is proportional to the component of H acting along the axis of the disc *i.e.* proportional to $H \cos [90 - (\phi + \theta)]$ or $H \sin (\theta + \phi)$. The force F pulling the disc inwards is proportional to MH or $H^2 \sin (\theta + \phi)$. If the permeability of iron is assumed constant, then, $H \propto I$. Hence, $F \propto I^2 \sin (\theta + \phi)$. If this force acted at a distance of l from the pivot of the rotating disc, then deflecting torque $T_d = Fl \cos (\theta + \phi)$. Putting the value of F, we get

$$T_d \propto I^2 \sin (\theta + \phi) \times l \cos (\theta + \phi) \propto I^2 \sin 2 (\theta + \phi) = KI^2 \sin 2 (\theta + \phi) \qquad \text{...sin } l \text{ is constant}$$

If spring-control is used, then controlling torque $T_c = K' \theta$

In the steady position of deflection, $T_d = T_c$

$\therefore \quad KI^2 \sin 2 (\theta + \phi) = K' \theta$; Hence $\theta \propto I^2$

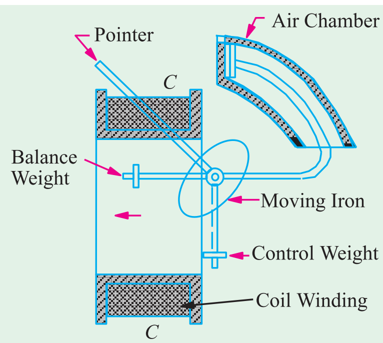


Fig. 10.9

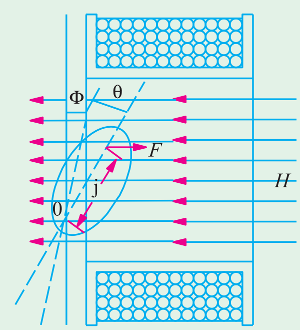


Fig. 10.10

If A.C. is used, then $\theta \propto I^2_{r.m.s.}$

However, if gravity-control is used, then $T_c = K_1 \sin \theta$

$\therefore \quad KI^2 \sin 2 (\theta + \phi) = K_1 \sin \theta \qquad \therefore \sin \theta \ \alpha I^2 \sin 2 (\theta + \phi)$

In both cases, the scales would be uneven.

Damping

As shown, air-friction damping is provided, the actual arrangement being a light piston moving in an air-chamber.

10.9. Repulsion Type M.I. Instruments

The sectional view and cut-away view of such an instrument are shown in Fig. 10.11 and 10.12. It consists of a fixed coil inside which are placed two soft-iron rods (or bars) A and B parallel to one another and along the axis of the coil. One of them *i.e.* A is fixed and the other B which is movable carries a pointer that moves over a calibrated scale. When the current to be measured is passed

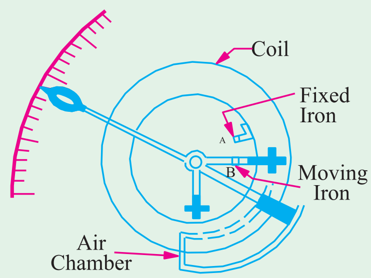


Fig. 10.11

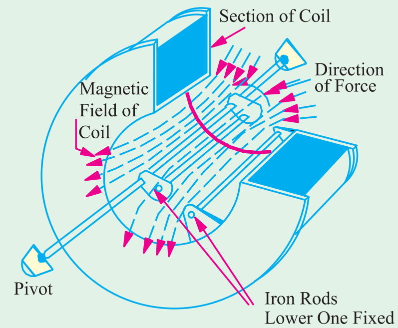


Fig. 10.12

through the fixed coil, it sets up its own magnetic field which magnetises the two rods similarly *i.e.* the adjacent points on the lengths of the rods will have the same magnetic polarity. Hence, they repel each other with the result that the pointer is deflected against the controlling torque of a spring or gravity. The force of repulsion is approximately proportional to the square of the current passing through the coil. Moreover, whatever may be the direction of the current through the coil, the two rods will be magnetised similaraly and hence will repel each other.

In order to achieve uniformity of scale, two tongue-shaped strips of iron are used instead of two rods. As shown in Fig. 10.13 (*a*), the fixed iron consists of a tongue-shaped sheet iron bent into a cylindrical form, the moving iron also consists of another sheet of iron and is so mounted as to move parallel to the fixed iron and towards its narrower end [Fig. 10.13 (*b*)].

Deflecting Torque

The deflecting torque is due to the repulsive force between the two similarly magnetised iron rods or sheets.

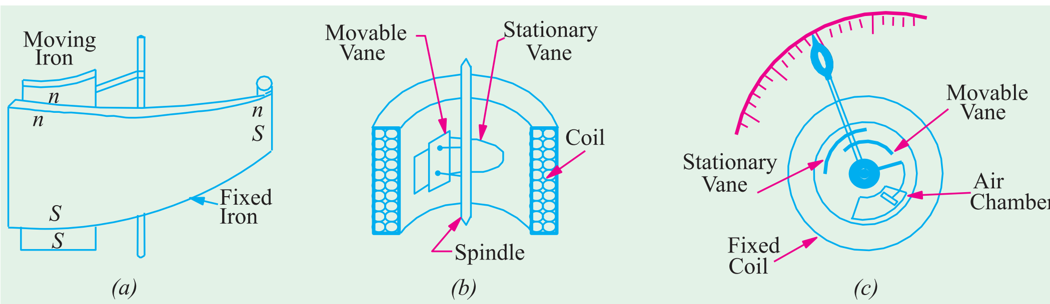


Fig. 10.13

Instantaneous torque $\propto$ repulsive force $\propto m_1 m_2$ ...product of pole strengths

Since pole strength are proportional to the magnetising force H of the coil,

$\therefore$ instantaneous torque $\propto H^2$

Since H itself is proportional to current (assuming ***constant*** permeability) passing through the coil, $\therefore$ instantaneous torque $\propto I^2$

Hence, the deflecting torque, which is proportional to the mean torque is, in effect, proportional to the mean value of I^2. Therefore, when used on a.c. circuits, the instrument reads the r.m.s. value of current.

Scales of such instruments are uneven if rods are used and uniform if suitable-shaped pieces of iron sheet are used.

The instrument is either gravity-controlled or as in modern makes, is spring-controlled.

Damping is pneumatic, eddy current damping cannot be employed because the presence of a permanent magnet required for such a purpose would affect the deflection and hence, the reading of the instrument.

Since the polarity of both iron rods reverses simultaneously, the instrument can be used both for a.c. and d.c. circuits *i.e.* instrument belongs to the unpolarised class.

10.10. Sources of Error

There are two types of possible errors in such instruments, firstly, those which occur both in a.c. and d.c. work and secondly, those which occur in a.c. work alone.

(*a*) Errors with both d.c. and a.c. work

(*i*) *Error due to hysteresis.* Because of hysteresis in the iron parts of the moving system, readings are higher for descending values but lower for ascending values.

The hysteresis error is almost completely eliminated by using Mumetal or Perm-alloy, which have negligible hysteresis loss.

(*ii*) *Error due to stray fields.* Unless shielded effectively from the effects of stray external fields, it will give wrong readings. Magnetic shielding of the working parts is obtained by using a covering case of cast-rion.

(*b*) Errors with a.c. work only

Changes of frequency produce **(*i*)** change in the impedance of the coil and **(*ii*)** change in the magnitude of the eddy currents. The increase in impedance of the coil with increase in the frequency of the alternating current is of importance in voltmeters (Ex. 10.2). For frequencies higher than the one used for calibration, the instrument gives lower values. However, this error can be removed by connecting a capacitor of suitable value in parallel with the swamp resistance R of the instrument. It can be shown that the impedance of the whole circuit of the instrument becomes independent of frequency if $C = L/R^2$ where C is the capacitance of the capacitor.

10.11. Advantages and Disadvantages

Such instruments are cheap and robust, give a reliable service and can be used both on a.c. and d.c. circuits, although they cannot be calibrated with a high degree of precision with d.c. on account of the effect of hysteresis in the iron rods or vanes. Hence, they are usually calibrated by comparison with an alternating current standard.

10.12. Deflecting Torque in terms of Change in Self-induction

The value of the deflecting torque of a moving-iron instrument can be found in terms of the variation of the self-inductance of its coil with deflection θ.

Suppose that when a direct current of I passes through the instrument, its deflection is θ and inductance L. Further suppose that when current changes from I to $(I + dI)$, deflection changes from θ to (θ + dθ) and L changes to $(L + dL)$. Then, the increase in the energy stored in the magnetic field is

$$dE = d\left(\frac{1}{2}LI^2\right) = \frac{1}{2}L2I.dI + \frac{1}{2}I^2dL = LI.\,dI + \frac{1}{2}I^2.\,dL \text{ joule.}$$

If $T\frac{1}{2}$ N-m is the controlling torque for deflection θ, then extra energy stored in the control system is $T \times d\theta$ joules. Hence, the total increase in the stored energy of the system is

$$LI.dI + \frac{1}{2}I^2.\,dL + T \times d\theta \qquad \text{...}(i)$$

The e.m.f. induced in the coil of the instrument is $e = N.\dfrac{d\Phi}{dt}$ volt

where $d\phi$ = change in flux linked with the coil due to change in the position of the disc or the bars

dt = time taken for the above change ; N = No. of turns in the coil

Now $L = NF/I \quad \therefore \quad \Phi = LI/N \quad \therefore \quad \dfrac{d\Phi}{dt} = \dfrac{1}{N}.\dfrac{d}{dt}(LI)$

Induced e.m.f. $e = N.\dfrac{1}{N}.\dfrac{d}{dt}(LI) = \dfrac{d}{dt}(LI)$

The energy drawn from the supply to overcome this back e.m.f is

$$= e.Idt = \frac{d}{dt}(LI).Idt = I.d(LI) = I(L.dI + I.dL) = LI.dI + I^2.dL \qquad \text{...}(ii)$$

Equating **(*i*)** and **(*ii*)** above, we get $LI.dI + \frac{1}{2}I^2 dL + T.d\theta = LI.dI + I^2.dL \quad \therefore \; T = \frac{1}{2}I^2\frac{dL}{d\theta}$ N-m

where $dL/d\theta$ is henry/radian and I in amperes.

10.13. Extension of Range by Shunts and Multipliers

(*i*) As Ammeter. The range of the moving-iron instrument, when used as an ammeter, can be extended by using a suitable shunt across its terminals. So far as the operation with direct current is concerned, there is no trouble, but with alternating current, the division of current between the instrument and shunt changes with the change in the applied frequency. For a.c. work, both the inductance and resistance of the instrument and shunt have to be taken into account.

Obviously, $\frac{\text{current through instruments, } i}{\text{current through shunt, } I_s} = \frac{R_s + j\omega L_s}{R + j\omega L} = \frac{\mathbf{Z_s}}{\mathbf{Z}}$

where R, L = resistance and inductance of the instrument

R_s, L_s = resistance and inductance of the shunt

It can be shown that above ratio *i.e.* the division of current between the instrument and shunt would be independent of frequency if the time-constants of the instrument coil and shunt are the same *i.e.* if $L/R = L_s/R_s$. The multiplying power (N) of the shunt is given by

$$N = \frac{I}{i} = 1 + \frac{R}{R_s}$$

where I = line current ; i = full-scale deflection current of the instrument.

(*ii*) As Voltmeter. The range of this instrument, when used as a voltmeter, can be extended or multiplied by using a high non-inductive resistance R connected in series with it, as shown in Fig. 10.14. This series resistance is known as 'multiplier' when used on d.c. circuits. Suppose, the range of the instrument is to be extended from ν to V. Then obviously, the excess voltage of $(V - \nu)$ is to be dropped across R. If i is the full-scale deflection current of the instrument, then

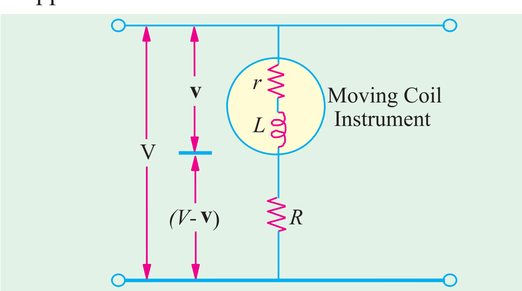


Fig. 10.14

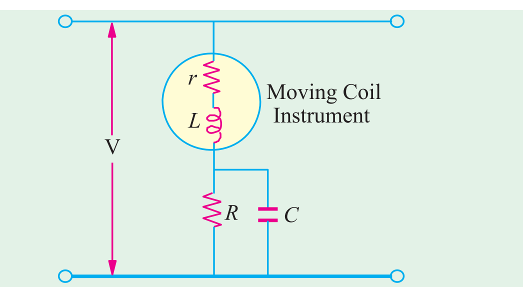


Fig. 10.15

$$iR = V - \nu; \quad R = \frac{V - \nu}{i} = \frac{V - ir}{i} = \frac{V}{i} - r$$

Voltage magnification = V/ν. Since $iR = V - \nu$; $\therefore \frac{iR}{\nu} = \frac{V}{\nu} - 1$

or $$\frac{iR}{ir} = \frac{V}{\nu} - 1 \qquad \therefore \quad \frac{V}{\nu} = \left(1 + \frac{R}{r}\right)$$

Hence, greater the value of R, greater is the extension in the voltage range of the instrument.

For d.c. work, the principal requirement of R is that its value should remain constant *i.e.* it should have low temperature-coefficient. But for a.c. work it is essential that total impedance of the voltmeter and the series resistance R should remain as nearly constant as possible at different frequencies. That is why R is made as non-inductive as possible in order to keep the inductance of the whole circuit to the minimum. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting R by a capacitor C as shown in Fig. 10.15. In case $r \ll R$, the impedance of the voltmeter circuit will remain practically constant (for frequencies upto 1000 Hz) provided.

$$C = \frac{L}{(1+\sqrt{2})R^2} = 0.41\frac{L}{R^2}$$

Example 10.2. *A 250-volt moving-iron voltmeter takes a current of 0.05 A when connected to a 250-volt d.c. supply. The coil has an inductance of 1 henry. Determine the reading on the meter when connected to a 250-volt, 100-Hz a.c. supply.* **(Elect. Engg., Kerala Univ.)**

Solution. When used on d.c. supply, the instrument offers ohmic resistance only. Hence, resistance of the instrument = 250/0.05 = 5000 Ω

When used on a.c. supply, the instrument offers ***impedance*** instead of ohmic resistance.

$$\text{impedance at 100 Hz} = \sqrt{5000^2 + (2\pi \times 100 \times 1)^2} = 5039.3\ \Omega$$

$\therefore$ voltage of the instrument = 250 × 5000/5039.3 = **248 V**

Example 10.3. *A spring-controlled moving-iron voltmeter reads correctly on 250-V d.c. Calculate the scale reading when 250-V a.c. is applied at 50 Hz. The instrument coil has a resistance of 500 Ω and an inductance of 1 H and the series (non-reactive) resistance is 2000 Ω.*

(Elect. Instru. & Measure. Nagpur Univ. 1992)

Solution. Total circuit resistance of the voltmeter is

$$= (r + R) = 500 + 2{,}000 = 2{,}500\ \Omega$$

Since the voltmeter reads correctly on direct current supply, its full-scale deflection current is = 250/2500 = 0.1 A.

When used on a.c. supply, instrument offers an impedance

$$Z = \sqrt{2500^2 + (2\pi \times 50 \times 1)^2} = 2.520\ \Omega \quad \therefore\ I = 0.099\ \text{A}$$

$\therefore$ Voltmeter reading on a.c. supply = 250 × 0.099/0.1 = **248 V***

Note. Since swamp resistance R = 2,000 Ω, capacitor required for compensating the frequency error is

$$C = 0.41\ L/R^2 = 0.41 \times 1/2000^2 = 0.1\ \mu\text{F}.$$

Example 10.4. *A 150-V moving-iron voltmeter intended for 50 Hz has an inductance of 0.7 H and a resistance of 3 kΩ. Find the series resistance required to extend the range of the instrument to 300 V. If this 300-V, 50-Hz instrument is used to measure a d.c. voltage, find the d.c. voltage when the scale reading is 200 V.* **(Elect. Measur, A.M.I.E. Sec B, 1991)**

Solution. Voltmeter reactance = 2π × 50 × 0.7 = 220 Ω

Impedance of voltmeter = (3000 + j 220) = 3008 Ω

When the voltmeter range is doubled, its impedance has also to be doubled in order to have the same current for full-scale deflection. If R is the required series resistance, then $(3000 + R)^2 + 220^2 = (2 \times 3008)^2$ $\therefore$ R = 3012 Ω

When used on d.c. supply, if the voltmeter reads 200 V, the actual applied d.c. voltage would be = 200 × (Total A.C. Impedance)/total d.c. resistance) = 200 × (2 × 3008)/(3000 + 3012) = 200 × (6016 × 6012) = 200.134 V.

Example 10.5. *The coil of a moving-iron voltmeter has a resistance of 5,000 Ω at 15ºC at which temperature it reads correctly when connected to a supply of 200 V. If the coil is wound with wire whose temperature coefficient at 15ºC is 0.004, find the percentage error in the reading when the temperature is 50ºC.*

In the above instrument, the coil is replaced by one of 2,000 Ω but having the same number of turns and the full 5,000 Ω resistance is obtained by connecting in series a 3,000 Ω resistor of negligible temperature-coefficient. If this instrument reads correctly at 15ºC, what will be its percentage error at 50ºC.

Solution. Current at 15ºC = 200/5,000 = 0.04 A

Resistance at 50ºC is $R_{50} = R_{15}(1 + \alpha_{15} \times 35)$

$\therefore$ R_{50} = 5,000 (1 + 35 × 0.004) = 5,700 Ω

* or reading = 250 × 2500/2520 = **248 V.**

$\therefore$ current at 50ºC = 200/5,700

$\therefore$ reading at 50ºC = $\dfrac{200 \times (200/5,700)}{0.04}$ = 175.4 V or = 200 × 5000/5700 = 175.4 V

$\therefore$ % error = $\dfrac{175.4 - 200}{200} \times 100$ = **−12.3%**

In the second case, swamp resistance is 3,000 Ω whereas the resistance of the instrument is only 2,000 Ω.

Instrument resistance at 50ºC = 2,000 (1 + 35 × 0.004) = 2,280 Ω

$\therefore$ total resistance at 50ºC = 3,000 + 2,280 = 5,280 Ω

$\therefore$ current at 50ºC = 200/5,280 A

$\therefore$ instrument reading = $200 \times \dfrac{200/5,280}{0.04}$ = 189.3 V

$\therefore$ percentage error = $\dfrac{189.3 - 200}{200} \times 100$ = **−5.4 %**

Example 10.6. *The change of inductance for a moving-iron ammeter is 2μH/degree. The control spring constant is 5 × 10^{-7} N-m/degree.*

The maximum deflection of the pointer is 100º, what is the current corresponding to maximum deflection ? **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. As seen from Art. 10.12 the deflecting torque is given by

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m}$$

Control spring constant = 5 × 10^{-7} N-m/degree

Deflection torque for 100° deflection = 5 × 10^{-7} × 100 = 5 × 10^{-5} N-m ; $dL/d\theta$ = 2 μH/degree = 2 × 10^{-6} H/degree.

$\therefore\ 5 \times 10^{-5} = \frac{1}{2} I^2 \times 2 \times 10^{-6} \quad \therefore\ I^2 =$ **50** and I = 7.07 A

Example 10.7. *The inductance of attraction type instrument is given by L = (10 + 5θ − θ²)μH where θ is the deflection in radian from zero position. The spring constant is 12 ×* 10^{-6} *N-m/rad. Find out the deflection for a current of 5 A.*

(Elect. and Electronics Measurements and Measuring Instruments Nagpur Univ. 1993)

Solution. $L = (10 + 5\,\theta - \theta^2) \times 10^{-6}$ H

$$\therefore \quad \frac{dL}{d\theta} = (0 + 5 - 2 \times \theta) \times 10^{-6} = (5 - 2\theta) \times 10^{-6} \text{ H/rad}$$

Let the deflection be θ radians for a current of 5A, then deflecting torque,

$$T_d = 12 \times 10^{-6} \times \theta \text{ N-m}$$

Also, $$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$ ...Art.

Equating the two torques, we get

$12 \times 10^{-6} \times \theta = \dfrac{1}{2} \times 5^2 \times (5 - 2\theta) \times 10^{-6} \quad \therefore\ \theta =$ **1.689 radian**

Tutorial Problems No. 10.1

1. Derive an expression for the torque of a moving-iron ammeter. The inductance of a certain moving-iron ammeter is (8 + 4θ − ½ θ²) μ*H* where θ is the deflection in radians from the zero position. The control-spring torque is 12 × 10^{-6} N-m/rad. Calaculate the scale position in radians for a current of 3A. **[1.09 rad] (*I.E.E. London*)**

2. An a.c. voltmeter with a maximum scale reading of 50-V has a resistance of 500 Ω and an inductance of 0.09 henry, The magnetising coil is wound with 50 turns of copper wire and the remainder of the circuit is a non-inductive resistance in series with it. What additional apparatus is needed to make this instrument read correctly on both d.c. circuits or frequency 60 ? **[0.44 μF in parallel with series resistance]**

3. A 10-V moving-iron ammeter has a full-scale deflection of 40 mA on d.c. circuit. It reads 0.8% low on 50 Hz a.c. Hence, calculate the inductance of the ammeter. **[115.5 mH]**

4. It is proposed to use a non-inductive shunt to increase the range of a 10-A moving iron ammeter to 100 A. The resistance of the instrument, including the leads to the shunt, is 0.06 Ω and the inductance is 15 μ*H* at full scale. If the combination is correct on a.d.c circuit, find the error at full scale on a 50 Hz a.c. circuit. **[3.5 %](*London Univ.*)**

10.14. Moving-coil Instruments

There are two types of such instruments ***(i) permanent-magnet type*** which can be used for d.c. work only and ***(ii)* the *dynamometer type*** which can be used both for a.c. and d.c. work.

10.15. Permanent Magnet Type Instruments

The operation of a permanent-magnet moving-coil type instrument is be based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

Construction

As its name indicates, the instrument consists of a permanent magnet and a rectangular coil of many turns wound on a light aluminium or copper former inside which is an iron core as shown in

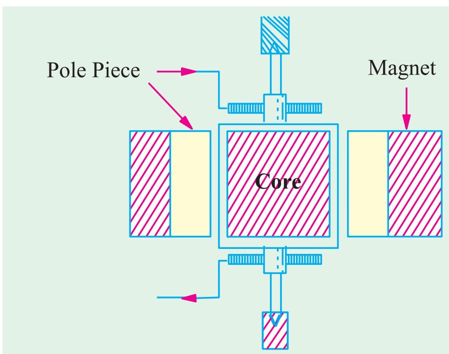


Fig. 10.16.

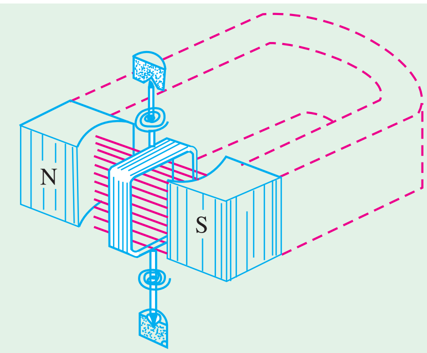


Fig. 10.17

Fig. 10.16. The powerfull U-shaped permanent magnet is made of Alnico and has soft-iron end-pole pieces which are bored out cylindrically. Between the magnetic poles is fixed a soft iron cylinder whose function is ***(i)*** to make the field radial and uniform and ***(ii)*** to decrease the reluctance of the air path between the poles and hence increase the magnetic flux. Surrounding the core is a rectangular coil of many turns wound on a light aluminium frame which is supported by delicate bearings and to which is attached a light pointer. The aluminium frame not only provides support for the coil but also provides damping by eddy currents induced in it. The sides of the coil are free to move in the two air-gaps between the poles and core as shown in Fig. 10.16 and Fig. 10.17. Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of lending the current in and out of the coil. The two springs are spiralled in opposite directions in order to neutralize the effects of temperature changes.

Deflecting Torque

When current is passed through the coil, force acts upon its both sides which produce a deflecting torque as shown in Fig. 10.18. Let

B = flux density in Wb/m^2

l = length or depth of the coil in metre

b = breadth of coil in metre

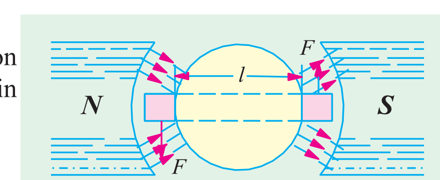


Fig. 10.18

N = number of turns in the coil

If I ampere is the current passing through the coil, then the magnitude of the force experienced by each of its sides is = BIl newton

For N turns, the force on each side of the coil is = $NBIl$ newton

$\therefore$ deflecting torque T_d = force $\times$ perpendicular distance

$= NBIl \times b = NBI(l \times b) = NBIA$ N-m

where A is the face area of the coil.

It is seen that if B is constant, then T_d is proportional to the current passing through the coil *i.e.* $T_d \propto I$.

Such instruments are invariable spring-controlled so that $T_c \propto$ deflection θ.

Since at the final deflected position, $T_d = T_c \quad \therefore \theta \propto I$

Hence, such instruments have uniform scales. Damping is electromagnetic *i.e.* by eddy currents induced in the metal frame over which the coil is wound. Since the frame moves in an intense magnetic field, the induced eddy currents are large and damping is very effective.

10.16. Advantage and Disadvantages

The permanent-magnet moving-coil (PMMC) type instruments have the following advantages and disadvantages :

Advantages

1. they have low power consumption.
2. their scales are uniform and can be designed to extend over an arc of 170° or so.
3. they possess high (torque/weight) ratio.
4. they can be modified with the help of shunts and resistances to cover a wide range of currents and voltages.
5. they have no hysteresis loss.
6. they have very effective and efficient eddy-current damping.
7. since the operating fields of such instruments are very strong, they are not much affected by stray magnetic fields.

Disadvantages

1. due to delicate construction and the necessary accurate machining and assembly of various parts, such instruments are somewhat costlier as compared to moving-iron instruments.
2. some errors are set in due to the aging of control springs and the parmanent magnets.

Such instruments are mainly used for d.c. work only, but they have been sometimes used in conjunction with rectifiers or thermo-junctions for a.c. measurements over a wide range or frequencies.

Permanent-magnet moving-coil instruments can be used as ammeters (with the help of a low resistance shunt) or as voltmeters (with the help of a high series resistance).

The principle of permanent-magnet moving-coil type instruments has been utilized in the construction of the following :

1. For a.c. galvanometer which can be used for detecting extremely small d.c. currents. A galvanometer may be used either as an ammeter (with the help of a low resistance) or as a voltmeter (with the help of a high series resistance). Such a galvanometer (of pivoted type) is shown in Fig. 10.19.
2. By eliminating the control springs, the instrument can be used for measuring the quantity of electricity passing through the coil. This method is used for *fluxmeters.*

Fig. 10.19

3. If the control springs of such an instrument are purposely made of large moment of inertia, then it can be used as ballistic galvanometer.

10.17. Extension of Range

(*i*) As Ammeter

When such an instrument is used as an ammeter, its range can be extended with the help of a low-resistance shunt as shown in Fig. 10.12 (*a*). This shunt provides a bypath for extra current because it is connected across (*i.e.* in parallel with) the instrument. These shunted instruments can be made to record currents many times greater than their normal full-scale deflection currents. The ratio of maximum current (with shunt) to the full-scale deflection current (without shunt) is known as the 'multiplying power' or 'multiplying factor' of the shunt.

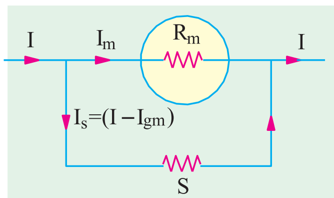


Fig. 10.20 (*a*)

Let R_m = instrument resistance

S = shunt resistance

I_m = full-scale deflection current of the instrument

I = line current to be measured

As seen from Fig. 10.20 (*a*), the voltage across the instrument coil and the shunt is the same since both are joined in parallel.

$$\therefore \quad I_m \times R_m = S\,I_s = S\,(I - I_m) \qquad \therefore\ S = \frac{I_m R_m}{(I - I_m)};\ \text{Also}\ \frac{I}{I_m} = \left(1 + \frac{R_m}{S}\right)$$

$$\therefore \quad \text{multiplying power} = \left(1 + \frac{R_m}{S}\right)$$

Obviously, lower the value of shunt resistance, greater its multiplying power.

(*ii*) As voltmeter

The range of this instrument when used as a voltmeter can be increased by using a high resistance in series with it [Fig. 10.20 (*b*)].

Let I_m = full-scale deflection current

R_m = galvanometer resistance

ν = $R_m I_m$ = full-scale p.d. across it

V = voltage to be measured

R = series resistance required

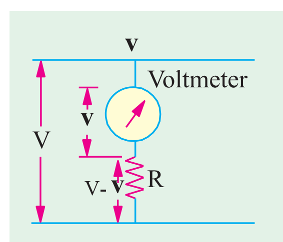


Fig. 10.20 (*b*)

Then it is seen that the voltage drop across R is $V - \nu$

$$\therefore \quad R = \frac{V - \nu}{I_m} \quad \text{or} \quad R \,.\, I_m = V - \nu$$

Dividing both sides by ν, we get

$$\frac{RI_m}{\nu} = \frac{V}{\nu} - 1 \quad \text{or} \quad \frac{R \,.\, I_m}{I_m R_m} = \frac{V}{\nu} - 1 \quad \therefore\ \frac{V}{\nu} = \left(1 + \frac{R}{R_m}\right)$$

$$\therefore \quad \text{voltage multiplication} = \left(1 + \frac{R}{R_m}\right)$$

Obviously, larger the value of R, greater the voltage multiplication or range. Fig. 10.20 (*b*) shown a voltmeter with a single multiplier resistor for one range. A multi-range voltmeter requires on multiplier resistor for each additional range.

Example 10.8. *A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil circuit rtesistance of R = 1 kΩ and need potential difference of 0.5 V across it for full-scale deflection.*

(1) *To what total current does this correspond ?*

(2) *Calculate the value of shunt to give full scale deflection when the total current is 10 A and 75 A.* **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. It should be noted that the shunt and the meter coil are in parallel and have a common p.d. of 0.5 V applied across them.

(1) $\therefore I_m = 0.5/1000 = 0.0005$ A; $I_s = 0.5./0.02 = 25$ A

$\therefore$ line current = **25.0005 A**

(2) When total current is 10 A, $I_s = (10 - 0.0005) = 9.9995$ A

$$\therefore \quad S = \frac{I_m R_m}{I_s} = \frac{0.0005 \times 1000}{9.9995} = 0.05\ \Omega$$

When total current is 75 A, $I_s = (75 - 0.0005) = 74.9995$ A

$\therefore \quad S = 0.0005 \times 1000/74.9995 =$ **0.00667 Ω**

Example 10.9. *A moving-coil instrument has a resistance of 10 Ω and gives full-scale deflection when carrying a current of 50 mA. Show how it can be adopted to measure voltage up to 750 V and currents upto 1000 A.* **(Elements of Elect. Engg.I, Bangalore Univ.)**

Solution. **(*a*) As Ammeter.** As discussed above, current range of the meter can be extended by using a shunt across it [Fig. 10.21 (*a*)].

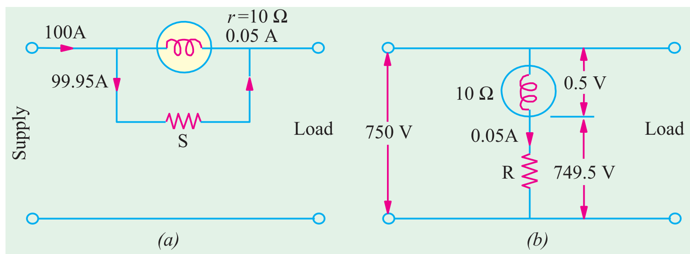


Fig. 10.21

Obviously,

$10 \times 0.05 = S \times 99.95$

$\therefore \quad S =$ **0.005 Ω**

(*b*) As Voltmeter. In this case, the range can be extended by using a high resistance R in series with it.

[Fig. 10.21 (*b*)]. Obviously, R must drop a voltage of $(750 - 0.5) = 749.5$ V while carrying 0.05 A.

$\therefore \quad 0.05\ R = 749.5$ or $R =$ **14.990 Ω**

Example 10.10. *How will you use a P.M.M.C. instrument which gives full scale deflection at 50 mV p.d. and 10 mA current as*

(1) *Ammeter 0 - 10 A range*

(2) *Voltmeter 0-250 V range* **(Elect. Instruments & Measurements Nagpur Univ. 1993)**

Solution. Resistance of the instrument $R_m = 50$ mV/10 mA = 5

(*i*) As Ammeter

full-scale meter current, $I_m = 10$ mA = 0.01 A

shunt current $I_s = I - I_m = 10 - 0.01 = 9.99$ A

$$\text{Reqd. shunt resistance, } S = \frac{I_m . R_m}{(I - I_m)} = \frac{0.01 \times 5}{9.99} = 0.0005\ \Omega$$

(*ii*) As Voltmeter

Full-scale deflection voltage, $v = 50$ mV = 0.05 V; $V = 250$ V

$$\text{Reqd. series resistance, } R = \frac{V - v}{I_m} = \frac{250 - 0.05}{0.01} = 24{,}995\ \Omega$$

Example 10.11. *A current galvanometer has the following parameters :*

$B = 10 \times 10^{-3}$ *Wb/m²* ; $N = 200$ *turns*, $l = 16$ *mm*;

$d = 16$ *mm*; $k = 12 \times 10^{-9}$ *Nm/radian.*

Calculate the deflection of the galvanometer when a current of 1 μA flows through it.

(Elect. Measurement Nagpur Univ. 1993)

Solution. Deflecting torque $T_d = NBIA$ N-m $= 200 \times (10 \times 10^{-3}) \times (1 \times 10^{-6}) \times (1 \times 10^{-3}) \times (16 \times 10^{-3})$ N-m $= 512 \times 10^{-12}$ N-m

Controlling torque T_c = controlling spring constant × deflection $= 12 \times 10^{-9} \times \theta$ N-m

Equating the deflecting and controlling torques, we have $12 \times 110^{-9} \times \theta = 512 \times 10^{-12}$

$\therefore \quad \theta = 0.0427$ radian = **2.45°**

Example 10.12. *The coil of a moving coil permanent magnet voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of 120 × 10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 0.5 Wb/m², estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.* **(Elect. Mesur. AMIE Sec. B Summer 1991)**

Solution. Let I be the current for full-scale deflection. Deflection torque $T_d = NBIA$

$= 100 \times 0.5 \times I \times (1200 \times 10^{-6}) = 0.06\, I$ N-m

Controlling torque $\quad T_c = 120 \times 10^{-6}$ N-m

In the equilibrium position, the two torques are equal *i.e.* $T_d = T_c$.

$\therefore \quad 0.06\, I = 120 \times 10^{-6} \quad \therefore\ I = 2 \times 10^{-3}$ A.

Since the instrument is meant to read 1 volt per division, its full-scale reading is 100 V.

Total resistance $= 100/2 \times 10^{-3} = 50{,}000\ \Omega$

Since voltmeter coil resistance is negligible, it represents the additional required resistance.

Example 10.13. *Show that the torque produced in a permanent-magnet moving-coil instrument is proportional to the area of the moving coil.*

A moving-coil voltmeter gives full-scale deflection with a current of 5 mA. The coil has 100 turns, effective depth of 3 cm and width of 2.5 cm. The controlling torque of the spring is 0.5 cm for full-scale deflection. Estimate the flux density in the gap. **(Elect. Meas, Marathwads Univ.)**

Solution. The full-scale deflecting torque is $T_d = NBIA$ N-m

where I is the full-scale deflection current ; $I = 5$ mA $= 0.005$ A

$T_d = 100 \times B \times 0.005 \times (3 \times 2.5 \times 10^{-4}) = 3.75 \times 10^{-4}\, B$ N-m

The controlling torque is

$$T_c = 0.5 \text{ g-cm} = 0.5 \text{ g. wt.cm} = 0.5 \times 10^{-3} \times 10^{-2} \text{ kg wt-m}$$
$$= 0.5 \times 10^{-5} \times 9.8 = 4.9 \times 10^{-5} \text{ N-m}$$

For equilibrium, the two torques are equal and opposite.

$\therefore \quad 4.9 \times 10^{-5} = 3.75 \times 10^{-4}\, B \quad \therefore\ B =$ **0.13 Wb/m²**

Example 10.14. *A moving-coil milliammeter has a resistance of 5 Ω and a full-scale deflection of 20 mA. Determine the resistance of a shunt to be used so that the instrument could measure currents upto 500 mA at 20° C. What is the percentage error in the instrument operating at a temperature of 40°C ? Temperature co-efficient of copper = 0.0039 per °C.*

(Measu. & Instrumentation, Allahabad Univ. 1991)

Solution. Let R_{20} be the shunt resistance at 20°C. When the temperature is 20°C, line current is 500 mA and shunt current is = (500−20) = 480 mA.

$\therefore \quad 5 \times 20 = R_{20} \times 480, \quad R_{20} = 1/4.8\ \Omega$

If R_{40} is the shunt resistance at 40ºC, then

$$R_{40}\ R_{20}\ (1 + 20\,\alpha) = \frac{1}{4.8}\ (1 + 0.0039 \times 20) = \frac{1.078}{4.8}\ \Omega$$

$$\text{Shunt current at } 40^\circ\text{C is} = \frac{5 \times 20}{1.078/4.8} = 445 \text{ mA}$$

Line current = 445 + 20 = 465 mA

Although, line current would be only 465 mA, the instrument will indicate 500 mA.

$\therefore \quad \text{error} = 35/500 = 0.07 \text{ or } 7\%$

Example 10.15. *A moving-coil millivoltmeter has a resistance of 20Ω and full-scale deflection of 120º is reached when a potential difference of 100 mV is applied across its terminals. The moving coil has the effective dimensions of 3.1 cm × 2.6 cm and is wound with 120 turns. The flux density in the gap is 0.15 Wb/m². Determine the control constant of the spring and suitable diameter of copper wire for coil winding if 55% of total instrument resistance is due to coil winding. ρ for copper = 1.73 × 10^{-6} Ω cm.* **(Elect. Inst. and Meas. M.S. Univ. Baroda)**

Solution. Full-scale deflection current is = 100/20 = 5 mA

Deflecting torque for full-scale deflection of 120º is

$$T_d = NBIA = 120 \times 0.15 \times (5 \times 10^{-3}) \times (3.1 \times 2.6 \times 10^{-4}) = 72.5 \times 10^{-6} \text{ N-m}$$

Control constant is defined as the deflecting torque per radian (or degree) or deflection of moving coil. Since this deflecting torque is for 120° deflection.

Control constant = $72.5 \times 10^{-6}/120$ = **6.04 × 10-7 N-m/degree**

Now, resistance of copper wire = 55% of 20 Ω = 11 Ω

Total length of copper wire = 120 × 2 (3.1 + 2.6) = 1368 cm

Now $R = \rho l/A$ ∴ $A = 1.73 \times 10^{-6} \times 1368/11 = 215.2 \times 10^{-6}\ \text{cm}^2$

∴ $\pi d^2/4 = 215.2 \times 10^{-6}$

∴ $d = \sqrt{215 \times 4 \times 10^{-6}/\pi} = 16.55 \times 10^{-3}$ cm = **0.1655 mm**

10.18. Voltmeter Sensitivity

It is defined in terms of resistance per volt (Ω/V). Suppose a meter movement of 1 kΩ internal resistance has s full-scale deflection current of 50 µ A. Obviously, full-scale voltage drop of the meter movement is = 50 µA × 1000 Ω = 50 mV. When used as a voltmeter, its sensitivity would be $1000/50 \times 10^{-3}$ = 20 kΩ/V. It should be clearly understood that a sensitivity of 20 kΩ/V means that the total resistance of the circuit in which the above movement is used should be 20 kΩ for a full-scale deflection of 1 V.

10.19. Multi-range Voltmeter

It is a voltmeter which measures a number of voltage ranges with the help of different series resistances. The resistance required for each range can be easily calculated provided we remember one basic fact that the sensitivity of a meter movement is always the same regardless of the range selected. Moreover, the full-scale deflection current is the same in every range. For any range, the total circuit resistance is found by multiplying the sensitivity by the full-scale voltage for that range.

For example, in the case of the above-mentioned 50µA, 1 kΩ meter movement, total resistance required for 1 V full-scale deflection is 20 kΩ. It means that an additional series resistance of 19 kΩ is required for the purpose as shown in Fig. 10.22 (*a*).

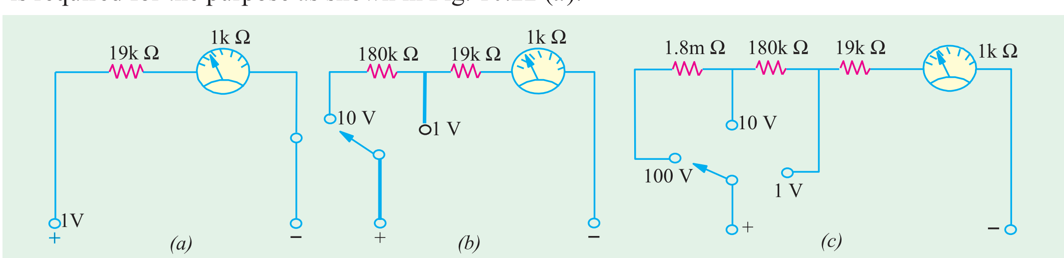


Fig. 10.22

For 10-V range, total circuit resistance must be (20 kΩ/V) (10 V) = 200 kΩ. Since total resistance for 1 V range is 20 kΩ, the series resistance *R* for 10-V range = 200 –20 = 180 kΩ as shown in Fig. 10.22 (*b*).

For the range of 100 V, total resistance required is (20 kΩ/V) (100 V) = 2 MΩ. The additional resistance required can be found by subtracting the existing two-range resistance from the total resistance of 2 MΩ. Its value is

$$= 2\ \text{M}\ \Omega - 180\ \text{k}\Omega - 19\ \text{k}\Omega - 1\ \text{k}\Omega = 1.8\ \text{M}\Omega$$

It is shown in Fig. 10.22 (*c*).

Example 10.16. *A basic 1' Arsonval movement with internal resistance R_m = 100 Ω and full scale deflection current I_f = 1 mA is to be converted into a multirange d.c. voltmeter with voltage ranges of 0–10 V, 0–50 V, 0–250 V and 0–500 V. Draw the necessary circuit arrangement and find the values of suitable multipliers.* **(Instrumentation AMIE Sec. B Winter 1991)**

Solution. Full-scale voltage drop = (1 mA) (100 Ω) = 100 mV. Hence, sensitivity of this movement is $100/100 \times 10^{-3}$ = 1 kΩ/V.

(i) 0–10 V range

Total resistance required = (1 kΩ/V) (10 V) = 10 kΩ. Since meter resistance is 1 kΩ additional series resistance required for this range R_1 = 10 –1 = **9 kΩ**

(ii) 0-50 V range

R_T = (1 kΩ/V) (50 V) = 50 kΩ; R_2 = 50–9–1 = **40 kΩ**

(iii) 0–250 V range

R_T = (1 kΩ/V) (250 V) = 250 kΩ; R_3 = 250 –50 = **200 kΩ**

(iv) 0–500 V range

R_T = (1 kΩ/V) (500 V) = 500 kΩ ; R_4 = 500 –250 = **250 kΩ**

The circuit arrangement is similar to the one shown in Fig. 10.22

Tutorial problem No. 10.2

1. The flux density in the gap of a 1-mA (full scale) moving '-coil' ammeter is 0.1 Wb/m^2. The rectangular moving-coil is 8 mm wide by 1 cm deep and is wound with 50 turns. Calculate the full-scale torque which must be provided by the springs. **[4 × 10–7 N-m] (*App. Elec. London Univ.*)**

2. A moving-coil instrument has 100 turns of wire with a resistance of 10 Ω, an active length in the gap of 3 cm and width of 2 cm. A p.d. of 45 mV produces full-scale deflection. The control spring exerts a torque of 490.5×10^{-7} N-m at full-scale deflection. Calculate the flux density in the gap. **[0.1817 Wb/m2] (*I.E.E. London*)**

3. A moving-coil instrument, which gives full-scale deflection with 0.015 A has a copper coil having a resistance of 1.5 Ω at 15°C and a temperature coefficient of 1/234.5 at 0°C in series with a swamp resistance of 3.5 Ω having a negligible temperature coefficient.

Determine (*a*) the resistance of shunt required for a full-scale deflection of 20 A and (*b*) the resistance required for a full-scale deflection of 250 V.

If the instrument reads correctly at 15°C, determine the percentage error in each case when the temperature is 25°C. **[(a) 0.00376 Ω ; 1.3 % (b) 16,662 Ω, negligible] (*App. Elect. London Univ.*)**

4. A direct current ammeter and leads have a total resistance of 1.5 Ω. The instrument gives a full-scale deflection for a current of 50 mA. Calculate the resistance of the shunts necessary to give full-scale ranges of 2-5, 5.0 and 25.0 amperes **[0.0306 ; l 0.01515 ; 0.00301 Ω] (*I.E.E. London*)**

5. The following data refer to a moving-coil voltmeter : resistance = 10,000 Ω, dimensions of coil = 3 cm × 3 cm ; number of turns on coil = 100, flux density in air-gap = 0.08 Wb/m^2, stiffness of springs = 3×10^{-6} N-m per degree. Find the deflection produced by 110 V. **[48.2°] (*London Univ.*)**

6. A moving-coil instrument has a resistance of 1.0 Ω and gives a full-scale deflection of 150 divisions with a p.d. of 0.15 V. Calculate the extra resistance required and show how it is connected to enable the instrument to be used as a voltmeter reading upto 15 volts. If the moving coil has a negligible temperature coefficient but the added resistance has a temperature coefficient of 0.004 Ω per degree *C*, what reading will a p.d. of 10 V give at 15ºC, assuming that the instrument reads correctly at 0°C. **[99 Ω, 9.45]**

10.20. Electrodynamic or Dynamometer Type Instruments

An electrodynamic instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet but by another fixed coil. This instrument can be used either as an ammeter or a voltmeter but is generally used as a wattmeter.

As shown in Fig. 10.23, the fixed coil is usually arranged in two equal sections *F* and *F* placed close together and parallel to each other. The two fixed coils are air-cored to avoid hysteresis effects when used on a.c. circuits. This has the effect of making the magnetic field in which moves the moving coil M, more uniform. The moving coil is spring-controlled and has a pointer attached to it as shown.

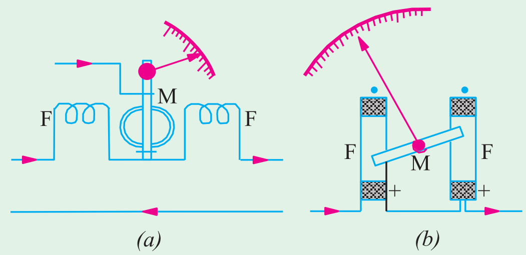


Fig. 10.23

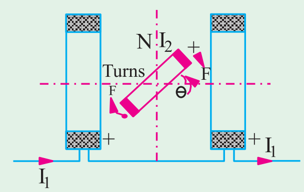


Fig. 10.24

Deflecting Torque*

The production of the deflecting torque can be understood from Fig. 10.24. Let the current passing through the fixed coil be I_1 and that through the moving coil be I_2. Since there is no iron, the field strength and hence the flux density is proportional to I_1.

$\therefore \quad B = KI_1$ where K is a constant

Let us assume for simplicity that the moving coil is rectangular (it can be circular also) and of dimensions $l \times b$. Then, force on each side of the coil having N turns is (NBI_2l) newton.

The turning moment or deflecting torque on the coil is given by

$$T_d = NBI_2lb = NKI_1I_2lb \text{ N-m}$$

Now, putting $NKlb = K_1$, we have $T_d = K_1I_1I_2$ where K_1 is another constant.

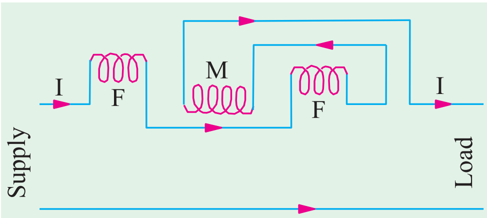


Fig. 10.25

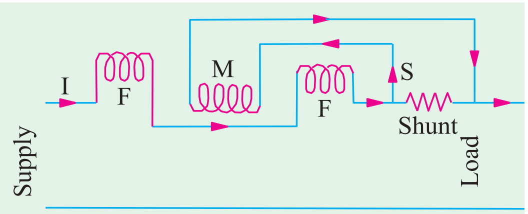


Fig. 10.26

It shows that the deflecting torque is proportional to the product of the currents flowing in the fixed coils and the moving coil. Since the instrument tis spring-controlled, the restoring or control torque is proportional to the angular deflection θ.

i.e. $$T_c \propto = K_2\,\theta \quad \therefore \quad K_1I_1I_2 = K_2\theta \quad \text{or } \theta \propto I_1I_2$$

---

* As shown in Art. 10.12, the value of torque of a moving-coil instrument is

$$T_d = \frac{1}{2} I^2\, dL/d\theta\ N-m$$

The equivalent inductance of the fixed and moving coils of the electrodynamic instrument is

$$L = L_1 + L_2 + 2M$$

where M is the mutual inductance between the two coils and L_1 and L_2 are their individual self-inductances.

Since L_1 and L_2 are fixed and only M varies,

$$\therefore \quad dL/d\theta = 2dM/d\theta \quad \therefore \quad T_d \frac{1}{2} I^2 \times 2.dM/d\theta = I^2.dM/d\theta$$

If the currents in the fixed and moving coils are different, say I_1 and I_2 then

$$T_d = I_1\,.I_2.\ dM/d\theta\ N\text{-}m$$

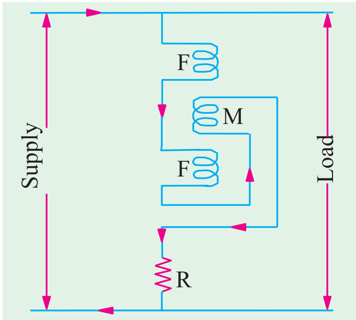


Fig. 10.27

When the instrument is used as an ammeter, the same current passes through both the fixed and the moving coils as shown in Fig. 10.25.

In that case $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 10.24 are used when small currents are to be measured. In the case of heavy currents, a shunt S is used to limit current through the moving coil as shown in Fig. 10.26.

When used as a voltmeter, the fixed and moving coils are joined in series along with a high resistance and connected as shown in Fig. 10.27. Here, again $I_1 = I_2 = I$, where $I = \dfrac{V}{R}$ in d.c. circuits and $I = V/Z$ in a.c. circuits.

$$\therefore \quad \theta \propto V \times V \quad \text{or} \quad \theta \propto V^2 \quad \text{or} \quad V \propto \sqrt{\theta}$$

Hence, it is found that whether the instrument is used as an ammeter or voltmeter, its scale is uneven throughout the whole of its range and is particularly cramped or crowded near the zero.

Damping is pneumatic, since owing to weak operating field, eddy current damping is inadmissible. Such instruments can be used for both a.c. and d.c. measurements. But it is more expensive and inferior to a moving-coil instrument for d.c. measurements.

As mentioned earlier, the most important application of electrodynamic principle is the wattmeter and is discussed in detail in Art. 10.34.

Errors

Since the coils are air-cored, the operating field produced is small. For producing an appreciable deflecting torque, a large number of turns is necessary for the moving coil. The magnitude of the current is also limited because two control springs are used both for leading in and for leading out the current. Both these factors lead to a heavy moving system resulting in frictional losses which are somewhat larger than in other types and so frictional errors tend to be relatively higher. The current in the field coils is limited for the fear of heating the coils which results in the increase of their resistance. A good amount of screening is necessary to avoid the influence of stray fields.

Advantages and Disadvantages

1. Such instruments are free from hysteresis and eddy-current errors.
2. Since (torque/weight) ratio is small, such instruments have low sensitivity.

Example 10.17. *The mutual inductance of a 25-A electrodynamic ammeter changes uniformly at a rate of 0.0035 μH/degree. The torsion constant of the controlling spring is 10^{-6} N-m per degree. Determine the angular deflection for full-scale.*

(Elect. Measurements, Poona Univ.)

Solution. By torsion constant is meant the deflecting torque per degree of deflection. If full-scale deflecting is θ degree, then deflecting torque on full-scale is $10^{-6} \times \theta$ N-m.

Now, $T_d = I^2 dM/d\theta$ Also, $I = 25$ A

$$dM/d\theta = 0.0035 \times 10^{-6} \text{ H/degree} = 0.0035 \times 10^{-6} \times 180/\pi \text{ H/radian}$$

$$10^{-6} \times \theta = 25^2 \times 0.0035 \times 10^{-6} \times 180/\pi \quad \therefore \quad \theta = \mathbf{125.4°}$$

Example 10.18. *The spring constant of a 10-A dynamometer wattmeter is 10.5×10^{-6} N-m per radian. The variation of inductance with angular position of moving system is practically linear over the operating range, the rate of change being 0.078 mH per radian. If the full-scale deflection of the instrument is 83 degrees, calculate the current required in the voltage coil at full scale on d.c. circuit.* **(Elect. Inst. and Means. Nagpur Univ. 1991)**

Solution. As seen from foot-note of Art. 10.20, $T_d = I_1 I_2 \, dM/d\theta$ N-m

Spring constant = 10.5×10^{-6} N/m/rad = $10.5 \times 10^{-6} \times \pi/180$ N-m/degree

T_d = spring constant × deflection = $(10.5 \times 10^{-6} \times \pi/180) \times 83 = 15.2 \times 10^{-6}$ N-m

$\therefore \quad 15.2 \times 10^{-6} = 10 \times I_2 \times 0.078$; I_2 = **19.5 μA.**

10.21. Hot-wire Instruments

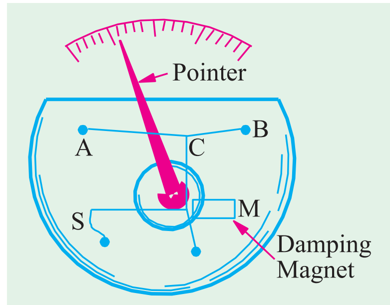


Fig. 10.28

The working parts of the instrument are shown in Fig. 10.28. It is based on the heating effect of current. It consists of platinum-iridium (It can withstand oxidation at high temperatures) wire *AB* stretched between a fixed end *B* and the tension-adjusting screw at *A*. When current is passed through *AB*, it expands according to I^2R formula. This sag in *AB* produces a slack in phosphor-bronze wire *CD* attached to the centre of *AB*. This slack in *CD* is taken up by the silk fibre which after passing round the pulley is attached to a spring *S*. As the silk thread is pulled by *S*, the pulley moves, thereby deflecting the pointer. It would be noted that even a small sag in *AB* is magnified (Art. 10.22) many times and is conveyed to the pointer. Expansion of *AB* is magnified by *CD* which is further magnified by the silk thread.

It will be seen that the deflection of the pointer is proportional to the extension of *AB* which is itself proportional to I^2. Hence, deflection is $\propto I^2$. If spring control is used, then $T_c \propto \theta$.

Hence $\quad \theta \propto I^2$

So, these instruments have a 'squre law' type scale. They read the r.m.s. value of current and their readings are independent of its form and frequency.

Damping

A thin light aluminium disc is attached to the pulley such that its edge moves between the poles of a permanent magnet *M*. Eddy currents produced in this disc give the necessary damping.

These instruments are primarily meant for being used as ammeters but can be adopted as voltmeters by connecting a high resistance in series with them. These instruments are suited both for a.c. and d.c. work.

Advantages of Hot-wire Instruments :

1. As their deflection depends on the r.m.s. value of the alternating current, they can be used on direct current also.

2. Their readings are independent of waveform and frequency.

3. They are unaffected by stray fields.

Disadvantages

1. They are sluggish owing to the time taken by the wire to heat up.

2. They have a high power consumption as compared to moving-coil instruments. Current consumption is 200 mA at full load.

3. Their zero position needs frequent adjustment. **4.** They are fragile.

10.22. Magnification of the Expansion

As shown in Fig. 10.29 (*a*), let *L* be the length of the wire *AB* and *dL* its expansion after steady temperature is reached. The sag *S* produced in the wire as seen from Fig. 10.29 (*a*) is given by

$$S^2 = \left(\frac{L + dL}{2}\right)^2 - \left(\frac{L}{2}\right)^2 = \frac{2L.\,dL + (dL)^2}{4}$$

Neglecting $(dL)^2$, we have $S = \sqrt{L\,.\,dL/2}$

Magnification produced is $= \dfrac{S}{dL} = \dfrac{\sqrt{L\,.\,dL/2}}{dL} = \sqrt{\dfrac{L}{2.dL}}$

As shown in Fig. 10.29 (*b*), in the case of double-sag instruments, this sag is picked up by wire *CD* which is under the constant pull of the spring. Let L_1 be the length of wire *CD* and let it be pulled at its center, so as to take up the slack produced by the sag *S* of the wire *AB*.

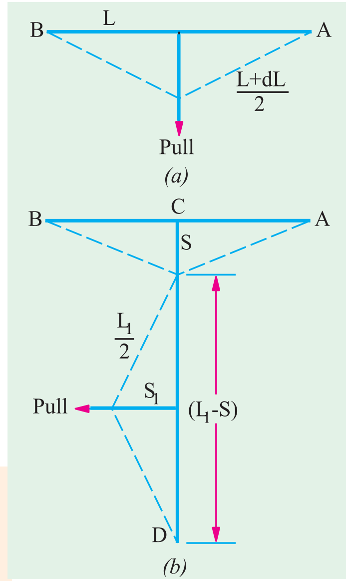


Fig. 10.29

$$S_1^2 = \left(\frac{L_1}{2}\right)^2 - \left(\frac{L_1 - S}{2}\right)^2 = \frac{2L_1 S - S^2}{4}$$

Neglecting S^2 as compared to $2L_1S$, we have $S_1 = \sqrt{L_1 S/2}$

Substituting the value of *S*, we get $S_1 = \sqrt{\frac{L_1}{2}} \sqrt{\frac{L.dL}{2}}$

$$\sqrt{\frac{L_1}{2\sqrt{2}}} \quad \sqrt{L.dL}$$

Example 10.19. *The working wire of a single-sag hot wire instrument is 15 cm long and is made up of platinum-silver with a coefficient of linear expansion of 16 × 10^{-6}. The temperature rise of the wire is 85°C and the sag is taken up at the center. Find the magnification*

(*i*) *with no initial sag and* **(*ii*)** *with an initial sag of 1 mm.*

(Elect. Meas and Meas. Inst., Calcutta Univ.)

Solution. **(*i*)** Length of the wire at room temperature = 15 cm

Length when heated through 85°C is $= 15\,(1 + 16 \times 10^{-6} \times 85) = 15.02$ cm

Increase in length, $dL = 15.02 - 15 = 0.02$ cm

Magnification $\sqrt{\frac{L}{2\,.\,dL}} \quad \sqrt{\frac{15}{0.04}} \quad$ **19.36**

(*ii*) When there is an initial sag of 1 mm, the wire is in the position *ACB* (Fig. 10.30). With rise in temperature, the new position becomes *ADB*. From the right-angled Δ *ADE*,

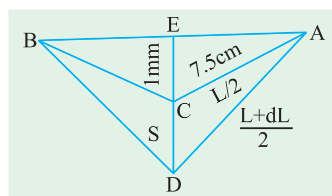


Fig. 10.30

we have $(S + 0.1)^2 = \left(\frac{L + dL}{2}\right)^2 - AE^2$

Now $AE^2 = AC^2 - EC^2 = \left(\frac{L}{2}\right)^2 - 0.1^2 = \frac{L^2}{4} - 0.1^2$

$$\therefore \quad (S + 0.1)^2 = \frac{L^2}{4} \quad \frac{(dL)^2}{4} \quad \frac{L\,.\,dL}{2} \quad \frac{L^2}{4} \quad 0.1^2 \quad \frac{L^2}{4} \quad \frac{(dL)^2}{4} \quad \frac{L\,.\,dL}{2} \quad \frac{L^2}{4} \quad 0.01$$

$$= \frac{L\,.\,dL}{2} + 0.01 \qquad \text{...neglecting } (dL)^2/4$$

$$= \frac{15 \times 0.02}{2} + 0.01 = 0.16 \;\therefore\; (S + 0.1) = 0.4 \;\therefore\; S = 0.3 \text{ cm}$$

Magnification $= S/dL = 0.3/0.02 =$ **15**

10.23. Thermocouple Ammeter

The working principle of this ammeter is based on the Seebeck effect, which was discovered in 1821. A thermocouple, made of two dissimilar metals (usually bismuth and antimony) is used in the construction of this ammeter. The hot junction of the thermocouple is welded to a heater wire *AB*, both of which are kept in vacuum as shown in Fig. 10.31 (*a*). The cold junction of the thermocouple is connected to a moving-coil ammeter.

When the current to be measured is passed through the heater wire *AB*, heat is generated, which raises the temperature of the thermocouple junction *J*. As the junction temperature rises, the generated

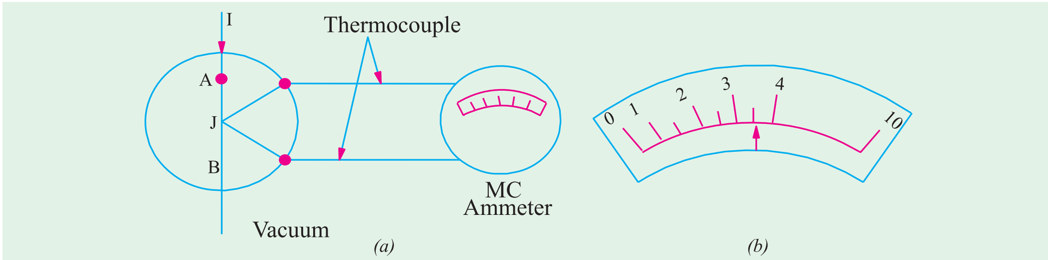


Fig. 10.31

thermoelectric EMF increases and derives a greater current through the moving-coil ammeter. The amount of deflection on the *MC* ammeter scale depend on the heating effect, since the amount of heat produced is directly proportional to the square of the current. The ammeter scale is non-linear so that, it is cramped at the low end and open at the high end as shown in Fig. 10.31 (*b*). This type of "current-squared: ammeter is suitable for reading both direct and alternating currents. It is particularly suitable for measuring radio-frequency currents such as those which occur in antenna system of broadcast transmitters. Once calibrated properly, the calibration of this ammeter remains accurate from dc upto very high frequency currents.

Megger

10.24. Megger

It is a portable instrument used for testing the insulation resistance of a circuit and for measuring resistances of the order of megaohms which are connected across the outside terminals XY in Fig. 10.32 (*b*).

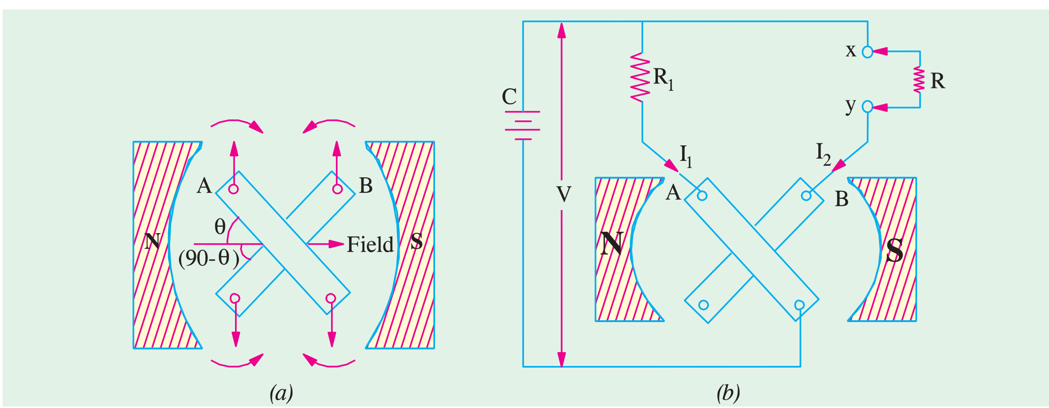


Fig. 10.32

1. Working Principle

The working principle of a 'corss-coil' type megger may be understood from Fig. 10.32 (*a*) which shows two coils *A* and *B* mounted rigidly at right angles to each other on a common axis and free to rotate in a magnetic field. When currents are passed through them, the two coils are acted upon by torques which are in opposite directions. The torque of coil *A* is proportional to $I_1 \cos\theta$ and that of *B* is proportional to $I_2 \cos(90 - \theta)$ or $I_2 \sin\theta$. The two coils come to a position of equilibrium where the two torques are equal and opposite *i.e.* where

$$I_1 \cos\theta = I_2 \sin\theta \quad \text{or} \quad \tan\theta = I_1/I_2$$

In practice, however, by modifying the shape of pole faces and the angle between the two coils, the ratio I_1/I_2 is made proportional to θ instead of tan θ in order to achieve a linear scale.

Suppose the two coils are connected across a common source of voltage *i.e.* battery *C*, as shown in Fig. 10.32 (*b*). Coil *A*, which is connected directly across *V*, is called the voltage (or control) coil. Its current $I_1 = V/R_1$. The coil *B* called current or deflecting coil, carries the current $I_2 = V/R$, where *R* is the external resistance to be measured. This resistance may vary from infinity (for good insulation or open circuit) to zero (for poor insulation or a short-circuit). The two coils are free to rotate in the field of a permanent magnet. The deflection θ of the instrument is proportional to I_1/I_2 which is equal to R/R_1. If R_1 is fixed, then the scale can be calibrated to read *R* directly (in practice, a current-limiting resistance is connected in the circuit of coil *B* but the presence of this resistance can be allowed for in scaling). The value of *V* is immaterial so long as it remains constant and is large enough to give suitable currents with the high resistance to be measured.

2. Construction

The essential parts of a megger are shown in Fig. 10.33. Instead of battery *C* of Fig. 10.32 (*b*), there is a hand-driven d.c. generator. The crank turns the generator armature through a clutch mechanism which is designed to slip at a pre-determined speed. In this way, the generator speed and voltage are kept constant and at their correct values when testing.

The generator voltage is applied across the voltage coil *A* through a fixed resistance R_1 and across deflecting coil *B* through a current-limiting resistance R' and the external resistance is connected across testing terminal *XY*. The two coils, in fact, constitute a moving-coil voltmeter and an ammeter combined into one instrument.

(i) Suppose the terminals *XY* are open-circuited. Now, when crank is operated, the generator voltage so produced is applied across coil *A* and current I_1 flows through it but no current flows through coil *B*. The torque so produced rotates the moving element of the megger until the scale points to 'infinity', thus indicating that the resistance of the external circuit is too large for the instrument to measure.

(ii) When the testing terminals *XY* are closed through a low resistance or are short-circuited, then a large current (limited only by R') passes through the deflecting coil *B*. The deflecting torque produced by coil *B* overcomes the small opposing torque of coil *A* and rotates the moving element until the needle points to 'zero', thus shown that the external resistance is too small for the instrument to measure.

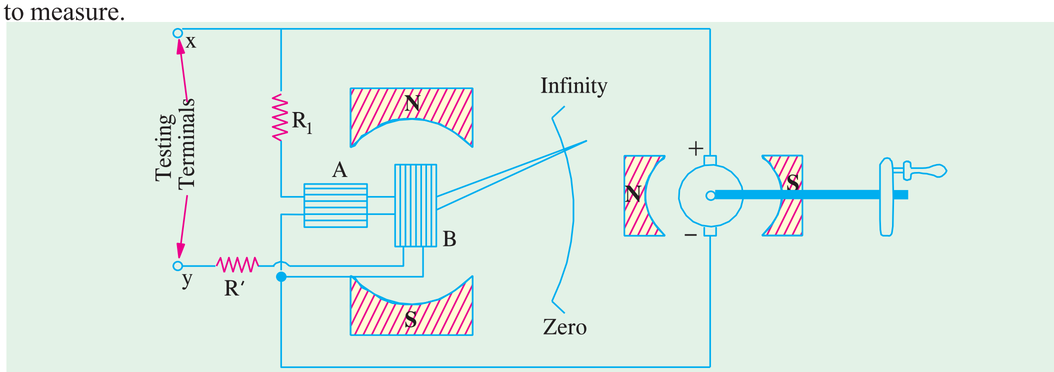


Fig. 10.33

Although, a megger can measure all resistance lying between zero and infinity, essentially it is a high-resistance measuring device. Usually, zero is the first mark and 10 kΩ is the second mark on its scale, so one can appreciate that it is impossible to accurately measure small resistances with the help of a megger.

The instrument described above is simple to operate, portable, very robust and independent of the external supplies.

10.25. Induction type Voltmeters and Ammeters

Induction type instruments are used only for a.c. measurements and can be used either as ammeter, voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter. In such instruments, the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux. Before discussing the two types of most commonly-used induction instruments, we will first discuss the underlying principle of their operation.

Principle

The operation of all induction instruments depends on the production of torque due to the reaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy currents induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depend on the flux producing them, the *instantaneous* value of torque is proportional to the square of current or voltage under measurement and the value of *mean* torque is proportional to the mean square value of this current or voltage.

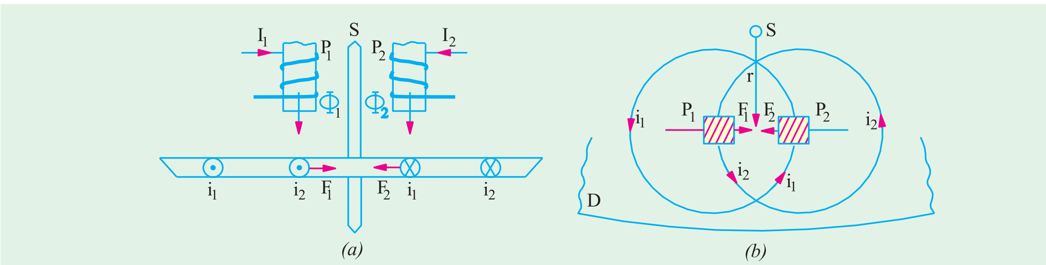


Fig. 10.34

Consider a thin aluminium or Cu disc *D* free to rotate about an axis passing through its centre as shown in Fig. 19.34. Two a.c. magnetic poles P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 respectively which cut this disc. Consider any annular portion of the disc around P_1 with center on the axis of P_1. This portion will be linked by flux Φ_1 and so an alternating e.m.f. e_1 be induced in it. This e.m.f. will circulate an eddy current i_1 which, as shown in Fig. 10.34, will pass under P_2. Similarly, Φ_2 will induce an e.m.f. e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_2. This eddy current i_2 flows under pole P_1.

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law, the directions of the induced currents i_1 and i_2 can be found and are as indicated in Fig. 10.34.

The portion of the disc which is traversed by flux Φ_1 and carries eddy current i_2 experiences a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \propto \Phi_1\, i_2$. Similarly, the portion of the disc lying in flux Φ_2 and carrying eddy current i_1 experiences a force $F_2 \propto \Phi_2\, i_1$.

$\therefore \quad F_1 \propto \Phi_1 i_2 = K\,\Phi_1\, i_2$ and $F_2 \propto \Phi_2\, i_1 = K\,\Phi_2 i_1$.

It is assumed that the constant *K* is the same in both cases due to the symmetrically positions of P_1 and P_2 with respect to the disc.

If *r* is the effective radius at which these forces act, the net instantaneous torque *T* acting on the disc begin equal to the difference of the two torques, is given by

$$T = r\,(K\,\Phi_1 i_2 - K\,\Phi_2 i_1) = K_1\,(\Phi_1 i_2 - \Phi_2 i_1) \qquad ...(i)$$

Let the alternating flux Φ_1 be given by $\Phi_1 = \Phi_{1m} \sin \omega t$. The flux Φ_2 which is assumed to lag Φ_1 by an angle α radian is given by $\Phi_2 = \Phi_{2m} \sin (\omega t - \alpha)$

Induced e.m.f. $$e_1 = \frac{d\,\Phi_1}{dt} = \frac{d}{dt}(\Phi_{1m} \sin \omega t) = \omega\,\Phi_{1m} \cos \omega t$$

Assuming the eddy current path to be purely resistive and of value *R**, the value of eddy current is

* If it has a reactance of *X*, then impedance *Z* should be taken, whose value is given by $Z = \sqrt{R^2 + X^2}$.

$i_1 \quad \frac{e_1}{R} \quad \frac{\quad_{1m}}{R} \cos \quad t$ Similarly $e_2 \quad _{2m}\,(\quad t \quad)$ and $i_2 \quad \frac{\quad_{2m}}{R} \cos(\quad t \quad)$ *

Substituting these values of i_1 and i_2 in Eq. (*i*) above, we get

$$T = \frac{K_1\,\omega}{R}[\Phi_{1m} \sin \omega t.\, \Phi_{2m} \cos(\omega t - \alpha) - \Phi_{2m} \sin(\omega t - \alpha)\, \Phi_{1m} \cos \omega t]$$

$$= \frac{K_1\,\omega}{R}.\Phi_{1m}\,\Phi_{2m}\,[\sin \omega t.\cos(\omega t - \alpha) - \cos \omega t.\sin(\omega t - \alpha)]$$

$$= \frac{K_1\,\omega}{R}.\Phi_{1m}\,\Phi_{2m} \sin \alpha = k_2\,\omega\,\Phi_{1m}\,\Phi_{2m} \sin \alpha \text{ (putting } K_1/R = K_2\text{)}$$

It is obvious that

(*i*) if $\alpha = 0$ *i.e.* if two fluxes are in phase, then *net* torque is zero. If on the other hand, $\alpha = 90°$, the net torque is maximum for given values of Φ_{1m} and Φ_{2m}.

(*ii*) the net torque is in such a direction as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(*iii*) since the expression for torque does not involve '*t*', it is independent of time *i.e.* it has a steady value at all time.

(*iv*) the torque *T* is inversely proportional to *R*-the resistance of the eddy current path. Hence, for large torques, the disc material should have low resistivity. Usually, it is made of Cu or, more often, of aluminium.

10.26. Induction Ammeters

It has been shown in Art.10.22 above that the net torque acting on the disc is

$$T = K_2\,\omega\Phi_{1m}\,\Phi_{2m} \sin \alpha$$

Obviously, if both fluxes are produced by the same alternating current (of maximum value I_m) to be measured, then $T = K_3 \omega I_m^2 \sin \alpha$

Hence, for a given frequency ω and angle α, the torque is proportional to the square of the current. If the disc has spring control, it will take up a steady deflected position where controlling torque becomes equal to the deflecting torque. By attaching a suitable pointer to the disc, the apparatus can be used as an ammeter.

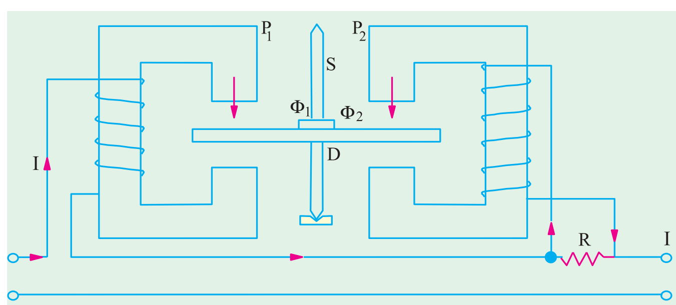


Fig. 10.35

There are three different possible arrangements by which the operational requirements of induction ammeters can be met as discussed below.

(*i*) Disc Instrument with Split-phase Winding

In this arrangement, the windings on the two laminated a.c. magnets P_1 and P_2 are connected in series (Fig. 10.35). But, the winding of P_2 is shunted by a resistance *R* with the result that the current in this winding lags with respect to the total line current. In this way, the necessary phase angle α is produced between two fluxes Φ_1 and Φ_2 produced by P_1 and P_2 respectively. This angle is of the order of 60°. If the hysteresis effects etc. are neglected, then each flux would be proportional to the current to be measured *i.e.* line current *I*

$$T_d \propto \Phi_{1m}\,\Phi_{2m} \sin \alpha$$

or $T_d \propto I^2$ where *I* is the r.m.s. value.

If spring control is used, then $T_c \propto \theta$

* It being assumed that both paths have the same resistance.

In the final deflected position, $T_c = T_d \quad \therefore \; \theta \propto I^2$

Eddy current damping is employed in this instrument. When the disc rotates, it cuts the flux in the air-gap of the magnet and has eddy currents induced in it which provide efficient damping.

(*ii*) Cylindrical type with Split-phase Winding

The operating principle of this instrument is the same as that of the above instrument except that instead of a rotating disc, it employs a hollow aluminium drum as shown in Fig. 10.36. The poles P_1 produce the alternating flux Φ_1 which produces eddy current i_1 in those portions of the drum that lie under poles P_2. Similarly, flux F_2 due to poles P_2 produces eddy current i_2 in those parts of the drum that lie under poles P_1. The force F_1 which is $\propto \Phi_1 \, i_2$ and F_2 which is $\propto \Phi_2 \, i_1$ are tangential to the surface of the drum and the resulting torque tends to rotate the drum about its own axis. Again, the winding of P_2 is shunted by resistance R which helps to introduce the necessary phase difference α between F_1 and F_2.

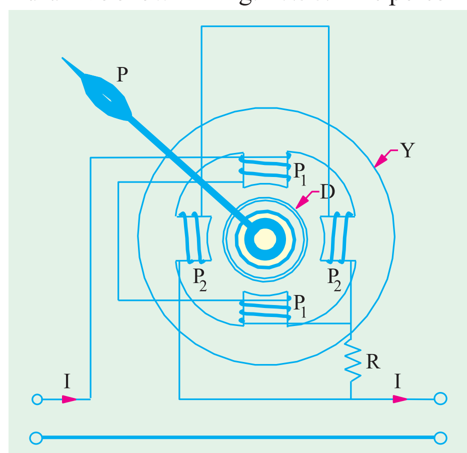


Fig. 10.36

The spiral control springs (not shown in the figure) prevent any continuous rotation of the drum and ultimately bring it to rest at a position where the deflecting torque becomes equal to the controlling torque of the springs. The drum has a pointer attached to it and is itself carried by a spindle whose two ends fit in jewelled bearings. There is a cylindrical laminated core inside the hollow drum whole function is to strengthen the flux cutting the drurm. The poles are laminated and magnetic circuits are completed by the yoke Y and the core.

Damping is by eddy currents induced in a separate aluminium disc (not shown in the figure) carried by the spindle when it moves in the air-gap flux of a horse-shoe magnet (also not shown in the figure).

(*iii*) Shaded-pole Induction Ammeter

In the shaded-pole disc type induction ammeter (Fig. 10.37) only single flux-producing winding is used. The flux F produced by this winding is split up into two fluxes Φ_1 and Φ_2 which are made to have the necessary phase difference of α by the device shown in Fig. 10.37. The portions of the upper and lower poles near the disc D are divided by a slot into two halves one of which carries a closed 'shading' winding or ring. This shading winding or ring acts as a short-circuited secondary and the main winding as a primary. The current induced in the ring by transformer action retards the phase of flux Φ_2 with respect to that of Φ_1 by about 50° or so. The two fluxes Φ_1 and Φ_2 passing through the unshaded and shaded parts respectively, react with eddy currents i_2 and i_1 respectively and so produce the net driving torque whose value is

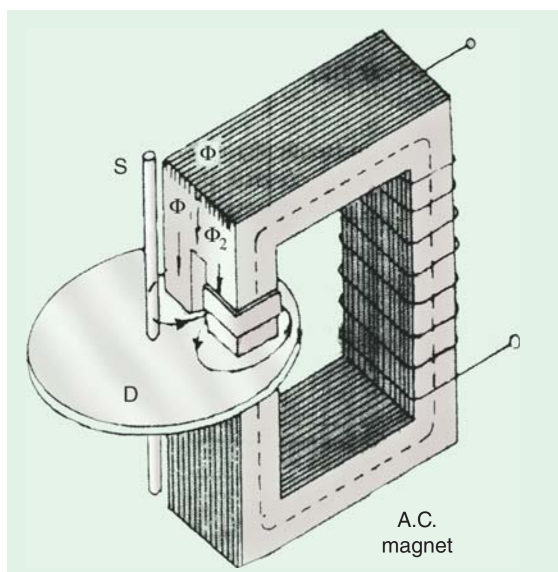


Fig. 10.37

$$T_d \propto \Phi_{1m} \, \Phi_{2m} \sin \alpha$$

Assuming that both Φ_1 and Φ_2 are proportional to the current I, we have

$$T_d \propto I^2$$

This torque is balanced by the controlling torque provided by the spiral springs.

The actual shaded-pole type induction instruments is shown in Fig. 10.38. It consists of a suitably-shaped aluminium or copper disc mounted on a spindly which is supported by jewelled bearings. The spindle carries a pointer and has a control spring attached to it. The edge or periphery of the disc moves in the air-gap of a laminated a.c. electromagnet which is energised either by the current to be measured (as ammeter) or by a current proportional to the voltage to be measured (as a voltmeter). Damping is by eddy currents induced by a permanent magnet embracing another portion of the ***same*** disc. As seen, the disc serves both for damping as well as operating purposes. The main flux is split into two components fluxes by shading one-half of each pole. These two fluxes have a phase difference of 40° to 50° between them and they induce two eddy currents in the disc. Each eddy current has a component in phase with the ***other*** flux, so that two torques are produced which are oppositely directed. The resultant torque is equal to the difference between the two. This torque ***deflects*** the disc-continuous rotation being prevented by the control spring and the deflection produced is proportional to the square of the current or voltage being measured.

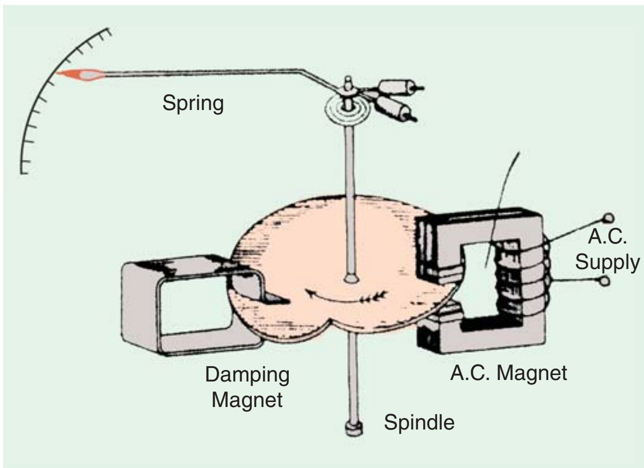


Fig. 10.38

As seen, for a given frequency, $T_d \propto I^2 = KI^2$

For spring control $T_c \propto \theta$ or $T_c = K_1 \theta$

For steady deflection, we have $T_c = T_d$ or $\theta \propto I^2$

Hence, such instruments have uneven scales *i.e.* scales which are cramped at their lower ends. A more even scale can, however, be obtained by using a cam-shaped disc as shown in Fig. 10.38.

10.27. Induction Voltmeter

Its construction is similar to that of an induction ammeter except for the difference that its winding is wound with a large number of turns of fine wire. Since it is connected across the lines and carries very small current (5 –10mA), the number of turns of its wire has to be large in order to produce an adequate amount of m.m.f. Split phase windings are obtained by connecting a high resistance R in series with the winding of one magnet and an inductive coil in series with the winding of the other magnet as shown in Fig. 10.39.

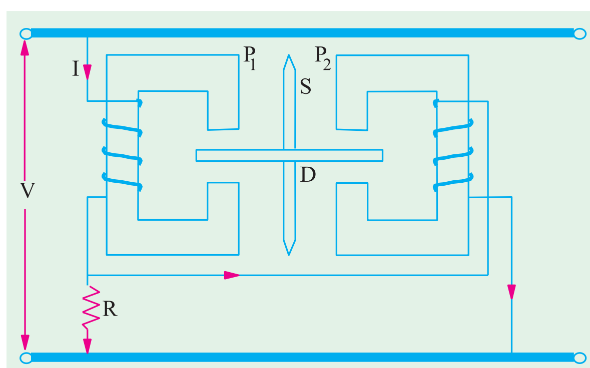


Fig. 10.39

10.28. Errors in Induction Instruments

There are two types of errors ***(i) frequency*** error and ***(ii) temperature*** error.

1. Since deflecting torque depends on frequency, hence unless the alternating current to be measured has same frequency with which the instrument was calibrated, there will be large error in its readings. Frequency errors can be compensated for by the use of a non-inductive shunt in the case of ammeters. In voltmeters, such errors are not large and, to a great extent, are self-compensating.

2. Serious errors may occur due to the variation of temperature because the resistances of eddy current paths depends on the temperature. Such errors can, however, be compensated for by hunting in the case of ammeters and by combination of shunt and swamping resistances in the case of voltmeters.

10.29. Advantages and Disadvantages

1. A full-scale deflection of over 200° can be obtained with such instruments. Hence, they have long open scales.
2. Damping is very efficient.
3. They are not much affected by external stray fields.
4. Their power consumption is fairly large and cost relatively high.
5. They can be used for a.c. measurements only.
6. Unless compensated for frequency and temperature variations, serious errors may be introduced.

10.30. Electrostatic Voltmeters

Electrostatic instruments are almost always used as voltmeters and that too more as a laboratory rather than as industrial instruments. The underlying principle of their operation is the force of attraction between electric charges on neighboring plates between which a p.d. is maintained. This force gives rise to a deflecting torque. Unless the p.d. is sufficiently large, the force is small. Hence, such instruments are used for the measurement of very high voltages.

There are two general types of such instruments :

(i) the quadrant type-used upto 20 kV. ***(ii) the attracted disc type*** – used upto 500 kV.

10.31. Attracted-disc Type Voltmeter

As shown in Fig. 10.40, it consists of two-discs or plates *C* and *D* mounted parallel to each other. Plate *D* is fixed and is earthed while *C* suspended by a coach spring, the support for which carries a micrometer head for adjustment. Plate *C* is connected to the positive end of the supply voltage. When a p.d. (whether direct or alternating) is applied between the two plates, then *C* is attracted towards *D* but may be returned to its original position by the micrometer head. The movement of this head can be made to indicate the force *F* with which *C* is pulled downwards. For this purpose, the instrument can be calibrated by placing known weights in turn on *C* and observing the movement of micrometer head necessary to bring *C* back to its original position. Alternatively, this movement of plate *C* is balanced by a control device which actuates a pointer attached to it that sweeps over a calibrated scale.

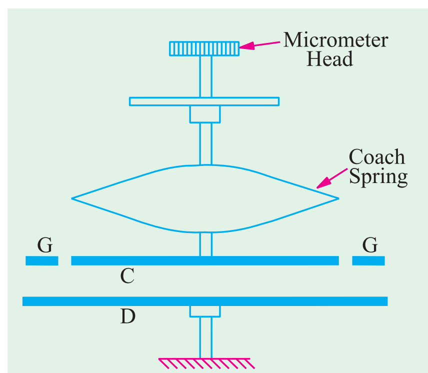


Fig. 10.40

There is a guard ring *G* surrounding the plate *C* and separated from it by a small air-gap. The ring is connected electrically to plate *C* and helps to make the field uniform between the two plates. The effective area of plate *C*, in that case, becomes equal to its actual area plus half the area of the air-gap.

Theory

In Fig. 10.41 are shown two parallel plates separated by a distance of *x* meters. Suppose the lower plate is fixed and carries a charge of $-Q$ coulomb whereas the upper plate is movable and

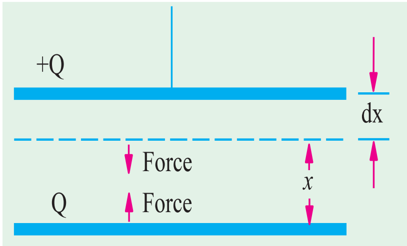


Fig. 10.41

carries a charge of + Q coulomb. Let the mutual force of attraction between the two plates be F newtons. Suppose the upper plates is moved apart by a distance dx. Then mechanical work done during this movement is $F \times dx$ joule. Since charge on the plate is constant, no electrical energy can move into the system from outside. This work is done at the case of the energy stored in the parallel-plate capacitor formed by the two plates.

Before movement, let the capacitance of the capacitor be C farad. Then,

$$\text{Initial energy stored} = \frac{1}{2} \cdot \frac{Q^2}{C}$$

If the capacitance changes to $(C + dC)$ because of the movement of plate, then

$$\text{Final energy stored} = \frac{1}{2} \frac{Q^2}{(C + dC)} = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{1}{\left(1 + \frac{dC}{C}\right)} = \frac{1}{2} \frac{Q^2}{C} \cdot \left(1 + \frac{dC}{C}\right)^{-1}$$

$$= \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) \text{if } dC \leq C$$

$$\text{Change in stored energy} = \frac{1}{2} \frac{Q^2}{C} - \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C}$$

$$\therefore \quad F \times dx = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C} \quad \text{or} \quad F \quad \frac{1}{2} \frac{Q^2}{C^2} \cdot \frac{dC}{dx} \quad \frac{1}{2} V^2 \frac{dC}{dx}$$

$$\text{Now,} \quad C = \frac{_0 A}{x} \quad \frac{dC}{dx} \quad \frac{_0 A}{x^2} \quad F \quad \frac{1}{2} V^2 \frac{_0 A}{x^2} \text{ N}$$

Hence, we find that force is directly proportional to the square of the voltage to be measured. The negative sign merely shows that it is a force of attraction.

10.32. Quadrant Type Voltmeters

The working principle and basic construction of such instruments can be understood from Fig. 10.42. A light aluminium vane C is mounted on a spindle S and is situated partially within a hollow metal quadrant B. Alternatively, the vane be suspended in the quadrant. When the vane and the quadrant are oppositely charged by the voltage under measurement, the vane is further attracted inwards into the quadrant thereby causing the spindle and hence the pointer to rotate. The amount of rotation and hence the deflecting torque is found proportional to V^2. The deflecting torque in the case of arrangement shown in Fig. 10.42 is very small unless V is extremely large.

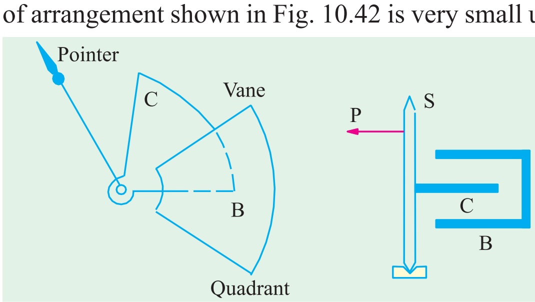


Fig. 10.42

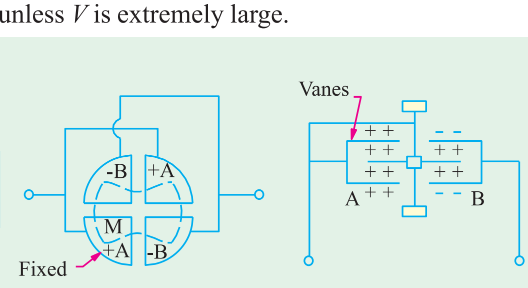


Fig. 10.43

The force on the vane may be increased by using a larger number of quadrants and a double-ended vane. In Fig. 10.43 are shown four fixed metallic double quadrants arranged so as to form a circular box with short air-gaps between the quadrants in which is suspended or pivoted as aluminium vane. Opposite quadrants AA and BB are joined together and each pair is connected to on terminal of the a.c. or d.c. supply and at the same time, one pair is connected to the moving vane *M*. Under these conditions [Fig. 10.43.] the moving vane is recalled by quadrants *AA* and attracted by quadrants *BB*. Hence, a deflecting torque is produced which is proportional to $(\text{p.d.})^2$. Therefore, such voltmeters have an uneven scale. Controlling torque is produced by torsion of the suspension spring or by the spring (used in pivoted type voltmeters). Damping is by a disc or vane immersed in oil in the case of suspended type or by air friction in the case of pivoted type instruments.

Theory

With reference to Fig 10.42, suppose the quadrant and vane are connected across a source of V volts and let the resulting deflection be θ. If C is a capacitance between the quadrant and vane in the deflected position, then the charge on the instrument will be CV coulomb. Suppose that the voltage is charged from V to $(V + dV)$, then as a result, let θ, C and Q charge to $(\theta + d\theta)$, $(C + dC)$ and $(Q + dQ)$ respectively. Then, the energy stored in the electrostatic field is increased by

$$dE \;=\; d\;\frac{1}{2}CV^2 \qquad \frac{1}{2}V^2.\,dC \qquad C.V\;dV \text{ joule}$$

If T is the value of controlling torque corresponding to a deflection of θ, then the additional energy stored in the control will be $T \times d\theta$ joule.

Total increase in stored energy $= \; T \times d\theta + \frac{1}{2}V^2\;dC + CVdV$ joule

It is seen that during this charge, the source supplies a change dQ at potential V. Hence, the value of energy supplied is

$$= \; V \times dQ = V \times d\,(CV) = V^2 \times dC + CV.\,dV$$

Since the energy supplied by the source must be equal to the extra energy stored in the field and the control

$\therefore$
$$T \times d\theta + \frac{1}{2}V^2\;dC + CV.\,dV = V^2.\,dC + CVdV$$

or
$$T \quad d \quad \frac{1}{2}V^2.\,dC \qquad T \quad \frac{}{2}V^2\,\frac{dC}{d}\;\text{N-m}$$

The torque is found to be proportional to the square of the voltage to be measured whether that voltage is alternating or direct. However, in alternating circuits the scale will read r.m.s. values.

10.33. Kelvin's Multicellular Voltmeter

As shown in Fig. 10.44, it is essentially a quadrant type instrument, as described above, but with the difference that instead of four quadrants and one vane, it has a large number of fixed quadrants and vanes mounted on the same spindle. In this way, the deflecting torque for a given voltage is increased many times. Such voltmeters can be used to measure voltages as low as 30 V. As said above, this reduction in the minimum limit of voltage is due to the increasing operating force in proportion to the number of component units. Such an instrument has a torsion head for zero adjustment and a coach spring for protection against accidental fracture of suspension due to vibration etc. There is a pointer and scale of edgewise pattern and damping is by a vane immersed in an oil dashpot.

10.34. Advantages and Limitation of Electrostatic Voltmeters

Some of the main advantages and use of electrostatic voltmeters are as follows :

1. They can be manufactured with first grade accuracy.
2. They give correct reading both on d.c. and a.c. circuits. On a.c. circuits, the scale will, however, read r.m.s. values whatever the wave-form.
3. Since no iron is used in their construction, such instruments are free from hysteresis and eddy current losses and temperature errors.
4. They do not draw any continuous current on d.c. circuits and that drawn on a.c. circuits (due to the capacitance of the instrument) is extremely small. Hence, such voltmeters do not cause any disturbance to the circuits to which they are connected.

5. Their power loss is negligibly small.
6. They are unaffected by stray magnetic fields although they have to be guarded against any stray electrostatic field.
7. They can be used upto 1000 kHz without any serious loss of accuracy.

However, their main limitations are :

1. Low-voltage voltmeters (like Kelvin's Multicellular voltmeter) are liable to friction errors.
2. Since torque is proportional to the square of the voltage, their scales are not uniform although some uniformity can be obtained by suitably shaping the quadrants of the voltmeters.
3. They are expensive and cannot be made robust.

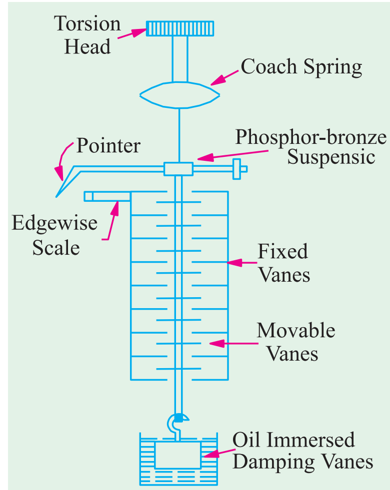


Fig. 10.44

10.35. Range Extension of Electrostatic Voltmeters

The range of such voltmeters can be extended by the use of multipliers which are in the form of a resistance potential divider or capacitance potential divider. The former method can be used both for direct and alternating voltages whereas the latter method is useful only for alternating voltages.

(*i*) Resistance Potential Divider

This divider consists of a high non-inductive resistance across a small portion which is attached to the electrostatic voltmeter as shown in Fig. 10.45. Let R be the resistance of the whole of the potential divider across which is applied the voltage V under measurement. Suppose V is the maximum value of the voltage which the voltmeter can measure without the multiplier. If r is the resistance of the portion of the divider across which voltmeter is connected, then the multiplying factor is given by

$$\frac{V}{v} = \frac{R}{r}$$

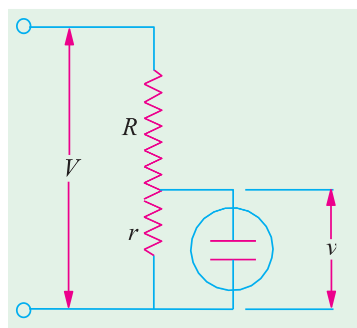


Fig. 10.45

The above expression is true for d.c. circuits but for a.c. circuits, the capacitance of the voltmeter (which is in parallel with r) has to be taken into account. Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

(*ii*) Capacitance Potential Divider

In this method, the voltmeter may be connected in series with a single capacity C and put across the voltage V which is to be measured [Fig. 10.46 (*a*)] or a number of capacitors may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. 10.46 (*b*).

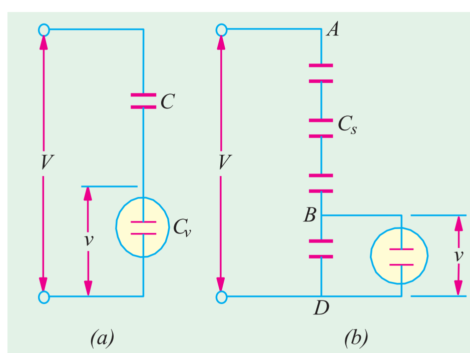


Fig. 10.46

Consider the connection shown in Fig. 10.46 (*a*). It is seen that the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{reactance of total circuit}}{\text{reactance of voltmeter}}$$

Now, capacitance of the total circuit is $\dfrac{CC_v}{C + C_v}$ and its reactance is

$$= \frac{1}{\omega \times \text{capacitance}} = \frac{C + C_v}{\omega C C_v}$$

Reactance of the voltmeter $= \dfrac{1}{\omega C_v}$

$$\therefore \quad \frac{V}{v} \quad \frac{(C \quad C_v)/\quad C C_v}{1/\quad C_v} \quad \frac{C \quad C_v}{C} \quad \text{Multiplying factor} \quad \frac{C \quad C_v}{C} \quad 1 \quad \frac{C_v}{C}.^{*}$$

Example 10.20. *The reading '100' of a 120-V electrosatatic voltmeter is to represent 10,000 volts when its range is extended by the use of a capacitor in series. If the capacitance of the voltmeter at the above reading is 70 μμF, find the capacitance of the capacitor multiplier required.*

Solution. Multiplying factor $= \dfrac{V}{v} = 1 + \dfrac{C_v}{C}$

Here, V = 10,000 volt, v = 100 volts ; C_v = capacitance of the voltmeter = 70μμ F

C = capacitance of the multiplier $\therefore \dfrac{10,000}{100} = 1 + \dfrac{70}{C}$

or $70/C = 99 \quad \therefore \; C = 70/99$ μμ F = **0.707 μμ F (approx)**

Example 10.21 (*a*). *An electrostatic voltmeter is constructed with 6 parallel, semicircular fixed plates equal-spaced at 4 mm intervals and 5 interleaved semi-circular movable plates that move in planes midway between the fixed plates, in air. The movement of the movable plates is about an axis through the center of the circles of the plates system, perpendicular to the planes of the plates. The instrument is spring-controlled. If the radius of the movable plates is 4 cm, calculate the spring constant if 10 kV corresponds to a full-scale deflection of 100°. Neglect fringing, edge effects and plate thickness.* **(Elect. Measurements, Bombay Univ.)**

Solution. Total number of plates (both fixed and movable) is 11, hence there are 10 parallel plate capacitors.

Suppose, the movable plates are rotated into the fixed plates by an angle of θ radian. Then, overlap area between one fixed and one movable semi-circular plate is

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 0.04^2 \times \theta = 8 \times 10^{-4}\ \theta\ \text{m}^2;\ d = 4/2 = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$$

Capacitance of each of ten parallel-plate capacitors is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 8 \times 10^{-4}\ \theta}{2 \times 10^{-3}} = 3.54 \times 10^{-12}\ \theta\ \text{F}$$

Total capacitance $C = 10 \times 3.54 \times 10^{-12}\ \theta = 35.4 \times 10^{-12}\ \theta$ F $\therefore dC/d\theta = 35.4 \times 10^{-12}$ farad/radian

$$\text{Deflecting torque} = \frac{1}{2} V^2 \frac{dC}{d\theta}\ \text{N-m} = \frac{1}{2} \times (10,000)^2 \times 35.4 \times 10^{-12} = 17.7 \times 10^{-4}\ \text{N-m}$$

If S is spring constant *i.e.* torque per radian and θ is the plate deflection, then control torque is

$T_c = S\theta$

Here, $\theta = 100° = 100 \times \pi/180 = 5\pi/9$ radian

$\therefore \quad S \times 5\pi/9 = 17.7 \times 10^{-4} \quad \therefore \; S$ = **10.1 × 10–4 N-m/rad.**

Example 10.21 (*b*). *A capacitance transducer of two parallel plates of overlapping area of* $5 \times 10^{-4}\ m^2$ *is immersed in water. The capacitance 'C' has been found to be 9.50 pF. Calculate the separation 'd' between the plates and the sensitivity, S = ∂C/∂d, of this transducer, given :* ε_r *water = 81 ;* ε_0 *= 8.854 pF/m.* **(Elect. Measuer. A.M.I.E. Sec. B, 1992)**

Solution. Since $C = \varepsilon_0 \varepsilon_r A/d$, $d = 3\ \varepsilon_0 \varepsilon_r$ A/C.

Substituting the given values we get, $d = 37.7 \times 10^{-3}$ m

* It is helpful to compare it with a similar expression in Art. 10.17 for permanent magnet moving-coil instruments.

Sensitivity $\frac{\partial C}{\partial d} = \frac{\partial}{d}\left(\frac{\varepsilon_0 \varepsilon_r A}{d}\right) = -\frac{8.854 \times 10^{-12} \times 81 \times 5 \times 10^{-4}}{(37.7 \times 10^{-3})^2} = -0.025 \times 10^{-8}$ F/m

10.36. Wattmeters

We will discuss the two main types of wattmeters in general use, that is, **(*i*)** the dynamometer or electrodynamic type and **(*ii*)** the induction type.

10.37. Dynamometer Wattmeter

The basic principle of dynamometer instrument has already been explained in detail in Art. 10.20. The connections of a dynamometer type wattmeter are shown in Fig. 10.47. The fixed

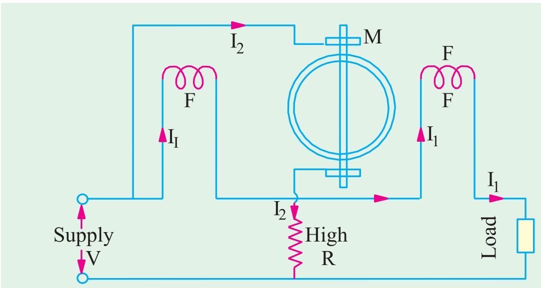


Fig. 10.47

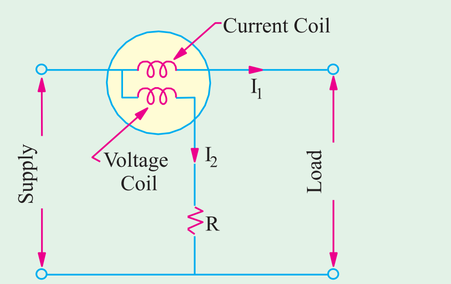


Fig. 10.48

circular coil which carries the main circuit current I_1 is wound in two halves positioned parallel to each other. The distance between the two halves can be adjusted to give a uniform magnetic field. The moving coil which is pivoted centrally carries a current I_2 which is proportional to the voltage V. Current I_2 is led into the moving coil by two springs which also supply the necessary controlling torque. The equivalent diagrammatic view is shown in Fig. 10.48.

Deflecting Torque

Since coils are air-cored, the flux density produced is directly proportional to the current I_1.

$\therefore \qquad B \propto I_1$ or $B = K_1 I_1$; current $I_2 \propto V$ or $I_2 = K_2 V$

Now $\qquad T_d \propto B I_2 \propto I_1 V \qquad \therefore T_d = KV I_1 = K \times$ power

In d.c. circuits, power is given by the product of voltage and current in amperes, hence torque is directly proportional to the power.

Let us see how this instrument indicates true power on a.c. circuits.

For a.c. supply, the value of instantaneous torque is given by $T_{inst} \propto vi = K\, vi$

where $\qquad v$ = instantaneous value of voltage across the moving coil

$\qquad i$ = instantaneous value of current through the *fixed* coils.

However, owing to the large inertia of the moving system, the instrument indicates the mean or average power.

$\therefore$ Mean deflecting torque $T_m \propto$ average value of $\upsilon\, i$

Let $v = V_{max} \sin\theta$ and $i = I_{max} \sin(\theta - \phi)$ $\quad \therefore T_m \propto \frac{1}{2\pi}\int_0^{2\pi} V_{max} \sin\theta \times I_{max} \sin(\theta - \phi)\, d\theta$

$$\propto \frac{V_{max} I_{max}}{2} \;{}^{2}_{0}\; \sin \;\; \sin(\quad)\, d \qquad \frac{V_{max} I_{max}}{2} \;{}^{2}_{0}\; \frac{\cos \quad \cos(2 \qquad)}{2}\, d$$

$$\propto \frac{V_{max} I_{max}}{4\pi}\left[\theta\cos\phi - \frac{\sin(2\theta - \phi)}{2}\right]_0^{2\pi} \propto \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cdot \cos\phi \propto V I \cos\phi$$

where V and I are the r.m.s. values. $\quad \therefore T_m \propto VI\cos\phi \propto$ true power.

Hence, we find that in the case of a.c. supply also, the deflection is proportional to the true power in the circuit.

Scales of dynamometer wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection. Hence $\theta \propto$ power. Damping is pneumatic with the help of a piston moving in an air chamber as shown in Fig. 10.49.

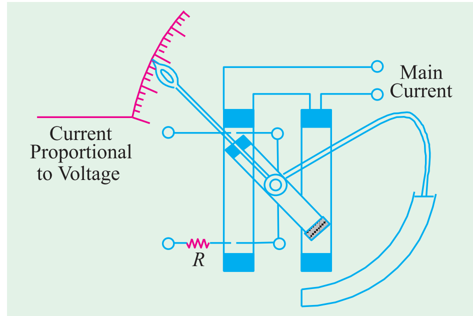


Fig. 10.49

Errors

The inductance of the moving or voltage coil is liable to cause error but the high non-inductive resistance connected in series with the coil swamps, to a great extent, the phasing effect of the voltage-coil inductance.

Another possible error in the indicated power may be due to ***(i)*** some voltage drop in the circuit or ***(ii)*** the current taken by the voltage coil. In standard wattmeters, this defect is overcome by having an additional compensating winding which is connected in series with the voltage coil but is so placed that is produces a field in opposite direction to that of the fixed or current coils.

Advantages and Disadvantages

By careful design, such instruments can be built to give a very high degree of accuracy. Hence they are used as a standard for calibration purposes. They are equally accurate on d.c. as well as a.c. circuits.

However, at low power factors, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect [Art. 10.38 (*ii*)].

10.38. Wattmeter Errors

(*i*) Error Due to Different Connections

Two possible ways of connecting a wattmeter in a single-phase a.c. circuit are shown in Fig. 10.50 along with their phasor diagrams. In Fig. 10.50 (*a*), the pressure or voltage-coil current does not pass through the current coil of the wattmeter whereas in the connection of Fig. 10.50 (*b*) is passes. A wattmeter is supposed to indicate the power consumed by the load but its actual reading is slightly higher due to power losses in the instrument circuits. The amount of error introduced depends on the connection.

(a) Consider the connection of Fig. 10.50 (*a*). If $\cos \phi$ is the power factor of the load, then power in the load is $VI \cos \theta$.

Now, voltage across the pressure-coil of the wattmeter is V_1 which is the phasor sum of the load voltage V and p.d. across current-coil of the instrument *i.e.* V' (= Ir where r is the resistance of the current coil).

Hence, power reading as indicated by the wattmeter is = $V_1 I \cos \theta$

where θ = phase difference between V_1 and I as shown in the phasor diagram of Fig. 10.50 (*a*).

As seen from the phasor diagram, $V_1 \cos \theta = (V \cos \phi + V')$

$$\therefore \text{ wattmeter reading } = V_1 \cos \theta \,.\, I = (V \cos \phi + V')\, I$$

$$= VI \cos \phi + V' I = VI \cos \phi + I^2 r = \text{power in load} + \text{power in current coil.}$$

(b) Next, consider the connection of Fig. 10.50 (*b*). The current through the current-coil of the wattmeter is the phasor sum of load current I and voltage-coil current $I' = V/R$. The power reading indicated by the wattmeter is = $VI_1 \cos \theta$.

As seen from the phasor diagram of Fig. 10.50 (*b*), $I_1 \cos \theta = (I \cos \phi + I')$

$$\therefore \quad \text{wattmeter reading} = V(I \cos \phi + I') = VI \cos \phi + VI' = VI \cos \phi + V^2/R$$

$$= \text{power in load} + \text{power in pressure-coil circuit}$$

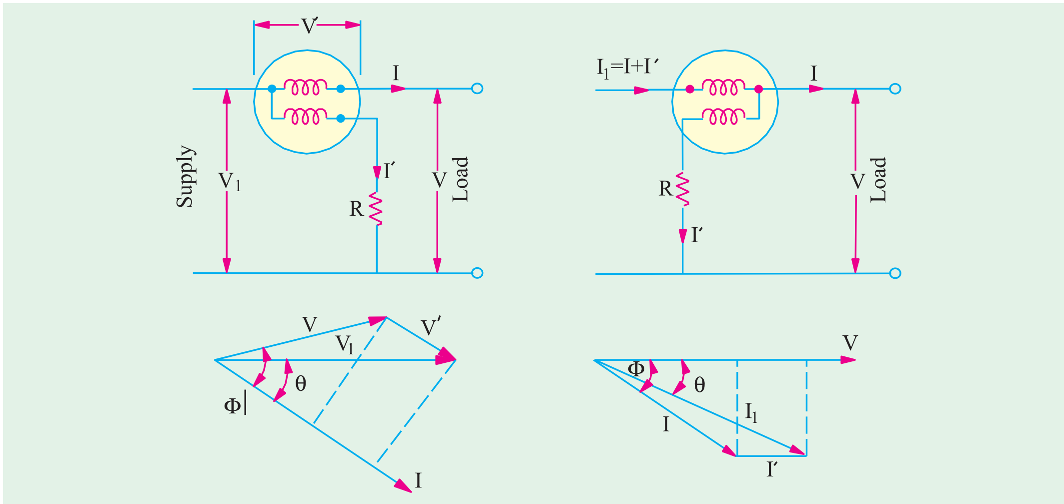


Fig. 10.50

(*ii*) Error Due to Voltage-coil Inductance

While developing the theory of electrodynamic instruments, it was assumed that pressure-coil does not posses any inductance (and hence reactance) so that current drawn by it was = V/R. The wattmeter reading is proportional to the mean deflecting torque, which is itself proportional to I_1I_2 cos θ, where θ is the angle between two currents (Fig. 10.52).

In case the inductance of the voltage-coil is neglected.

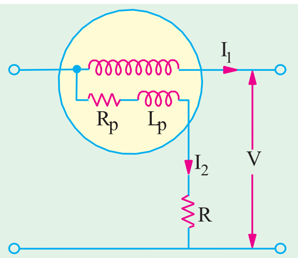


Fig. 10.51

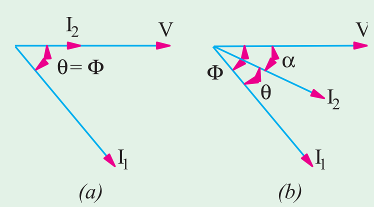


Fig. 10.52

$$I_2 = V/(R + R_p) = V/R \text{ approximately}$$

and $\theta = \phi$ as shown in the phasor diagram of 10.52 (*a*)

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1V}{R}\cos\phi \qquad \text{...(}i\text{)}$$

In case, inductance of the voltage coil is taken into consideration, then

$$I_2 = \frac{V}{\sqrt{(R_p+R)^2+X_L^2}} = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{V}{Z_p}$$

It lags behind V by an angle α [Fig. 10.52 (*b*)] such that

$$\tan\alpha = X_L/(R_p + R) = X_L/R \text{ (approx.)} = \omega L_p/R$$

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1V}{Z_p}\cos\theta \propto \frac{I_1V}{Z_p}\cos(\phi-\theta)$$

Now

$$\cos\alpha = \frac{R_p+R}{Z_p} = \frac{R}{Z_p} \qquad \therefore \; Z_p = \frac{R}{\cos\alpha}$$

$\therefore$ wattmeter reading in this case is $\propto I_1 \dfrac{V}{R} \cos\alpha \cos(\phi - \alpha)$ ...***(ii)***

Eq. ***(i)*** above, gives wattmeter reading when inductance of the voltage coil is neglected and Eq. ***(ii)*** gives the reading when it is taken into account.

The correction factor which is given by the ratio of the true reading (W_1) and the actual or indicated reading (W_a) of the wattmeter is

$$\frac{W_t}{W_a} = \frac{\dfrac{VI_1}{R_1}\cos\ \ }{\dfrac{VI_1}{R_1}\cos\ \ \cos(\ \ \ )} \qquad \frac{\cos\ \ }{\cos\ \ \cos(\ \ \ )}$$

Since, in practice, α is very small, $\cos\alpha = 1$. Hence the correction factor becomes $= \dfrac{\cos\phi}{\cos(\phi - \alpha)}$

$\therefore$ True reading $= \dfrac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times$ actual reading $\cong \dfrac{\cos\phi}{\cos(\phi-\alpha)} \times$ actual reading

The error in terms of the actual wattmeter reading can be found as follows :

Actual reading–true reading

$$= \text{actual reading} - \frac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times \text{actual reading}$$

$$= 1 \quad \frac{\cos\ \ }{\cos(\ \ \ )} \quad \text{actual reading} \quad 1 \quad \frac{\cos\ \ }{\cos\ \ \ \sin\ \ \sin\ \ } \quad \text{actual reading}$$

$$= \frac{\sin\ \ .\sin\ \ }{\cos\ \ \ \sin\ \ .\sin\ \ } \quad \text{actual reading} \quad \frac{\sin\ \ }{\cot\ \ \ \sin\ \ } \quad \text{actual reading}$$

The error, expressed as a fraction of the actual reading, is $= \dfrac{\sin\ \ }{\cot\ \ \ \sin\ \ }$

Percentage error $= \dfrac{\sin\ \ }{\cot\ \ \ \sin\ \ } \times 100$

(iii) Error Due to Capacitance in Voltage-coil Circuit

There is always present a small amount of capacitance in the voltage-coil circuit, particularly in the series resistor. Its effect is to reduce angle α and thus reduce error due to the inductance of the voltage coil circuit. In fact, in some wattmeters, a small capacitor is purposely connected in parallel with the series resistor for obtaining practically non-inductive voltage-coil circuit. Obviously, over-compensation will make resultant reactance capacitive thus making α negative in the above expressions.

(iv) Error Due to Stray Fields

Since operating field of such an instrument is small, it is very liable to stray field errors. Hence, it should be kept as far away as possible from stray fields. However, errors due to stray fields are, in general, negligible in a properly-constructed instrument.

(v) Error Due to Eddy Currents

The eddy current produced in the solid metallic parts of the instrument by the alternating field of the current coil changes the magnitude and strength of this operating field thus producing an error in the reading of the wattmeter. This error is not easily calculable although it can be serious if care is not taken to remove away solid masses of metal from the proximity of the current coil.

Example 10.22. *A dynamometer type wattmeter with its voltage coil connected across the load side of the instrument reads 250 W. If the load voltage be 200 V, what power is being taken by load? The voltage coil branch has a resistance of 2,000 Ω*

(Elect. Engineering, Madras Univ.)

Solution. Since voltage coil is connected across the load side of the wattmeter (Fig. 10.53), the power consumed by it is also included in the meter reading.

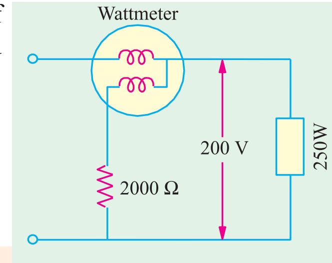


Fig. 10.53

Power consumed by voltage coil is

$= V^2/R = 200^2/2{,}000 = 20$ W

∴ Power being taken by load = 250 −20

= 230 W

Example 10.23. *A 250-V, 10-A dynamometer type wattmeter has resistance of current and potential coils of 0.5 and 12,500 ohms respectively. Find the percentage error due to each of the two methods of connection when unity p.f. loads at 250 volts are of* **(a)** *4A* **(b)** *12 A.*

Neglect the error due to the inductance of pressure coil.

(Elect. Measurements, Pune. Univ.)

Solution. **(*a*) When I = 4 A**

(*i*) Consider the type of connection shown in Fig. 10.50 (*a*)

Power loss in current coil of wattmeter = $I^2r = 4^2 \times 0.5 =$ **8 W**

Load power = 250 × 4 × 1 = 1000 W ; Wattmeter reading = 1008 W

∴ percentage error = (8/1008) × 100 = **0.794%**

(*ii*) Power loss in pressure coil resistance = $V^2/R = 250^2/12{,}500 = 5$ W

∴ Percentage error = 5 × 100/1005 = **0.497 %**

(*b*) When I = 12 A

(*i*) Power loss in current coil = $12^2 \times 0.5 = 72$ W

Load power = 250 × 12 × 1 = 3000 W ; wattmeter reading = 3072 W

∴ percentage error = 72 × 100/3072 = **2.34 %**

(*ii*) Power loss in the resistance of pressure coil is $250^2/12{,}500 = 5$ W

∴ percentage error = 5 × 100/3005 = **0.166 %**

Example 10.24. *An electrodynamic wattmeter has a voltage circuit of resistance of 8000 Ω and inductance of 63.6 mH which is connected directly across a load carrying a current of 8A at a 50-Hz voltage of 240-V and p.f. of 0.1 lagging. Estimate the percentage error in the wattmeter reading caused by the loading and inductance of the voltage circuit.*

(Elect & Electronic Measu. & Instru. Nagpur, Univ. 1992)

Solution. The circuit connections are shown in Fig. 10.54.

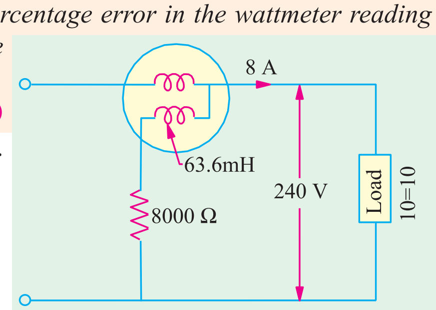


Fig. 10.54

Load power = 240 × 8 × 0.1 = 192 W

$\cos\phi = 0.1,\ \phi = \cos^{-1}(0.1) = 84^\circ\ 16'$

Power loss in voltage coil circuit is = V^2/R

$= 240^2/8000 = 7.2$ W

Neglecting the inductance of the voltage coil, the wattmeter reading would be

= 192 + 7.2 = **199.2 W**

Now, $X_p = 2\pi \times 50 \times 63.3 \times 10^{-3} = 20\ \Omega$

$\alpha = \tan^{-1}(20/8000) = \tan^{-1}(0.0025) = 0^\circ\ 9'$

Error factor due to inductance of the voltage coil = $\dfrac{\cos(\phi - \alpha)}{\cos\phi} = \dfrac{\cos 84^\circ 7'}{\cos 84^\circ 16'} = 1.026$

Wattmeter reading = 1.026 × 199.2 = 204.4 W

$$\text{Percentage error} = \left(\frac{204.4 - 199.2}{199.2}\right) \times 100 = \mathbf{2.6\ \%}$$

Example 10.25. *The inductive reactance of the pressure-coil circuit of a dynamometer wattmeter is 0.4 % of its resistance at normal frequency and the capacitance is negligible.*

*Calculate the percentage error and correction factor due to reactance for load at **(i)** 0.707 p.f. lagging and **(ii)** 0.5 p.f. lagging.* **(Elect. Measurement. Bombay Univ.)**

Solution. It is given that $X_p/R = 0.4\ R = 0.004$
$\tan\alpha = X_p/R = 0.004 \quad \therefore\ \alpha = 0^\circ\ 14'$ and $\sin\alpha = 0.004$

(*i*) When p.f. = 0.707 (*i.e.* $\phi = 45^\circ$)

$$\text{Correction factor} = \frac{\cos}{\cos(\qquad)} \quad \frac{\cos 45^\circ}{\cos 44^\circ\ 46} \quad \mathbf{0.996}$$

$$\text{Percentage error} = \frac{\sin\alpha}{\cot\phi + \sin\alpha} \times 100 = \frac{\sin 0^\circ\ 14'}{\cot 45^\circ + \sin 0^\circ\ 14'} \times 100$$

$$= \frac{0.004 \times 100}{1 + 0.004} = \frac{0.4}{1.004} = \mathbf{0.4\ (approx)}$$

(*ii*) When p.f. = 0.5 (*i.e.,* $\phi = 60^\circ$)

$$\text{Correction factor} = \frac{\cos 60^\circ}{\cos 59^\circ\ 46} \quad \mathbf{0.993}$$

$$\text{Percentage error} = \frac{\sin 0^\circ\ 14}{\cot 60^\circ \quad \sin 0^\circ\ 14} \quad 100 \quad \frac{0.004 \quad 100}{0.577 \quad 0.004} \quad \frac{0.4}{0.581} \quad \mathbf{0.7}$$

Example. 10.26. *The current coil of wattmeter is connected in series with an ammeter and an inductive load. A voltmeter and the voltage circuit of the wattmeter are connected across a 400-Hz supply. The ammeter reading is 4.5 A and voltmeter and wattmeter readings are respectively 240 V and 29 W. The inductance of the voltage circuit is 5 mH and its resistance is 4 kΩ. If the voltage drops across the ammeter and current coil are negligible, what is the percentage error in wattmeter readings ?*

Solution. The reactance of the voltage-coil circuit is $X_p = 2\pi \times 400 \times 5 \times 10^{-3}\ \pi$ ohm

$\tan\alpha = X_p/R = 4\pi/4000 = 0.00314^2$
$\therefore\ \alpha = 0.003142$ radian ($\because$ angle is very small)
$= 0.18^\circ$ or $0^\circ 11'$

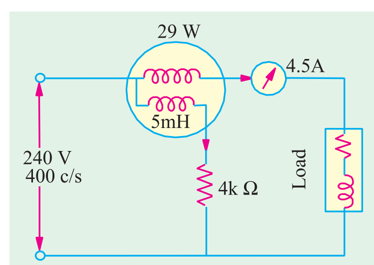


Fig. 10.55

Now, $\text{true reading} = \dfrac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times \text{actual reading}$

or $VI\cos\phi = \dfrac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times \text{actual reading}$

or $VI = \dfrac{\text{actual reading}}{\cos(\phi-\alpha)}$

taking $(\cos\alpha = 1)$
$\therefore\ \cos(\phi-\alpha) = 29/240 \times 4.5 = 0.02685$
$\therefore\ \phi - \alpha = 88^\circ 28'$ or $\phi = 88^\circ\ 39'$

$$\therefore \text{Percentage error} = \frac{\sin\alpha}{\cot\phi + \sin\alpha} \times 100 = \frac{\sin 11'}{\cot 88^\circ\ 39' + \sin 11'} \times 100$$

$$= \frac{0.0032}{0.235 \quad 0.0032} \quad \mathbf{100 = 12\ \%}$$

10.39. Induction Wattmeters

Principle of induction wattmeters is the same as that of induction ammeters and voltmeters. They can be used on a.c. supply only in constant with dynamometer wattmeters, which can be used both on d.c. and a.c. supply. Induction wattmeters are useful only when the frequency and supply voltage are constant.

Since, both a current and a pressure element are required in such instrument, it is not essential to use the shaded-pole principle. Instead of this, two separate a.c. magnets are used, which produce two fluxes, which have the required phase difference.

Construction

The wattmeter has two laminated electromagnets, one of which is excited by the current in the main circuit-exciting winding being joined in series with the circuit, hence it is also called a ***series*** magnet. The other is excited by current which is proportional to the voltage of the circuit. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as ***shunt*** magnet.

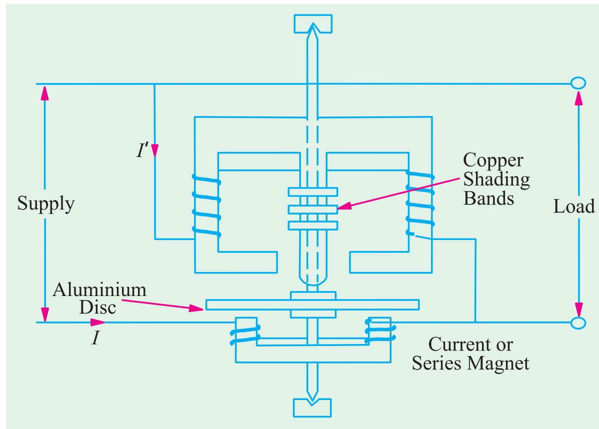


Fig. 10.56

A thin aluminium disc is so mounted that it cuts the fluxes of both magnets. Hence, two eddy currents are produced in the disc. The deflection torque is produced due to the interaction of these eddy current and the inducing fluxes. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by 90º.

Two most common forms of the electromagnets are shown in Fig. 10.56 and 10.57. It is seen that in both cases, one magnet is placed above and the other below the disc. The magnets are so positioned and shaped that their fluxes are cut by the disc.

In Fig. 10.56, the two pressure coils are joined in series and are so wound that both send the flux through the central limb in the same direction. The series magnet carries two coils joined in series and so wound that they magnetise their respective cores in the same direction. Correct phase displacement between the shunt and series magnet fluxes can be obtained by adjusting the position of the copper shading bands as shown.

In the type of instrument shown in Fig. 10.57, there is only one pressure winding and one current winding. The two projecting poles of the shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with respect to the voltage.

Both types of induction wattmeters shown above, are spring-controlled, the spring being fitted to the spindle of the moving system which also carries the pointer. The scale is uniformly even and extends over 300º.

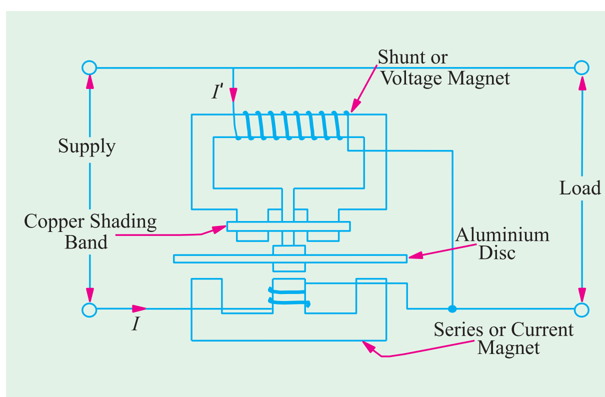


Fig. 10.57

Currents upto 100 A can be handled by such wattmeters directly but for currents greater than this value, they are used in conjunction with current transformers. The pressure coil is purposely made as much inductive as possible in order that the flux through it should lag behind the voltage by 90º.

Theory

The winding of one magnet carries line current I so that $\Phi_1 \propto I$ and is in phase with I (Fig. 10.58).

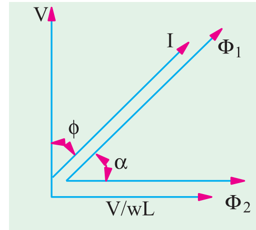


Fig. 10.58

The other coil *i.e.*, pressure or voltage coil is made highly inductive having an inductance of L and and negligible resistance. This is connected across the supply voltage V. The current in the pressure coil is therefore, equal to $V/\omega L$. Hence, $\Phi_2 \propto V/\omega L$ and lags behind the voltage by 90º. Let the load current I lag behind V by ϕ *i.e.*, let the load power factor angle be ϕ. As shown in Fig. 10.56, the phase angle between Φ_1 and Φ_2 is $\alpha = (90 - \phi)$.

The value of the torque acting on the disc is given by

$$T = k\omega \Phi_{1m}\, \Phi_{2m} \sin \alpha \qquad \text{– Art. 10.25}$$

or $$T \propto 2\, \omega I . \frac{V}{\omega L} . \sin (90 - \phi) \propto VI \cos \phi \propto \text{power}$$

Hence, the torque is proportional to the power in the load circuit. For spring control, the controlling torque $T_c \propto \theta$. $\therefore$ $\theta \propto$ power. Hence, the scale is even.

10.40. Advantage and Limitations of Induction Wattmeters

These wattmeters possess the advantages of fairly long scales (extending over 300º), are free from the effects of stray fields and have good damping. They are practically free from frequency errors. However, they are subject to (sometimes) serious temperature errors because the main effect of temperature is on the resistance of the eddy current paths.

10.41. Energy Meters

Energy meters are integrating instruments, used to measure quantity of electric energy supplied to a circuit in a given time. They give no direct indication of power *i.e.*, as to the rate at which energy is being supplied because their registrations are independent of the rate at which a given quantity of electric energy is being consumed. Supply or energy meters are generally of the following types :

(i) ***Electrolytic meters*** - their operation depends on electrolytic action.

(ii) ***Motor meters*** - they are really small electric motors.

(iii) ***Clock meters*** - they function as clock mechanisms.

10.42. Electrolytic Meter

It is used on d.c. circuits* only and is essentially an ampere-hour meter and not a true watt-hour meter. However, its registrations are converted into watt-hour by multiplying them by the voltage (assumed constant) of the circuits in which it is used. Such instruments are usually calibrated to read kWh directly at the declared voltage. Their readings would obviously be incorrect when used on any other voltage. ***Because of the question of power factor, such instrument cannot be used on a.c. circuits.***

The advantages of simplicity, cheapness and of low power consumption of ampere-hour meters are, to a large extent, discounted by the fact that variations in supply voltage are not taken into account by them. As an example suppose that the voltage of a supply whose nominal value is 220 V, has an average value of 216 volts in one hour during which a consumer draws a current of 100 A. Quantity of electricity as measured by the instrument which is calibrated on 220 V, is $220 \times 100/1000 = 22$ kWh. Actually, the energy consumed by the customer is only $216 \times 100/1000 = 21.6$ kWh. Obviously, the consumer is being overcharged to the extent of the cost of $22 - 21.6 = 0.4$ kWh of energy per hour. A true watthour-meter would have taken into account the decrease in the supply voltage and would have, therefore, resulted in a saving to the consumer. If the supply voltage would

* Recently such instruments have been marketed for measurement of kilovoltampere-hours on a.c. supply, using a small rectifier unit, which consists of a current transformer and full-wave copper oxide rectifier.

have been higher by that amount, then the supply company would have been the loser (Ex. 10.27).

In this instrument, the operating current is passed through a suitable electrolyte contained in a voltmeter. Due to electrolysis, a deposit of mercury is given or a gas is liberated (depending on the type of meter) in proportion to the quantity of electricity passed (Faraday's Laws of Electrolysis). The quantity of electricity passed is indicated by the level of mercury in a graduated tube. Hence, such instruments are calibrated in amp-hour or if constancy of supply voltage is assumed, are calibrated in watt-hour or kWh.

Such instruments are cheap, simple and are accurate even at very small loads. They are not affected by stray magnetic fields and due to the absence of any moving parts are free from friction errors.

10.43. Motor Meters

Most commonly-used instruments of this type are :

(i) ***Mercury motor*** meters ***(ii) Commutator motor*** meters and ***(ii) Induction motor*** meters.

Of these, mercury motor meter is normally used on d.c. circuits whereas the induction type instrument is used only on a.c. circuits. However, the commutator type meter can be used both for d.c. as well as a.c. work.

Instruments used for d.c. work can be either in the form of a amp-hour meters or watt-hour meters. In both cases, the moving system is allowed to revolve continuously instead of being merely allowed to deflect or rotate through a fraction of a revolution as in indicating instruments. The speed of rotation is directly proportional to the current in the case of amp-hour meter and to power in the case of watt-hour meter. Hence the number of revolutions made in a given time is proportional, in the case of amp-hour meter, to the quantity of electricity ($Q = i \times t$) and in the case of Wh meter, to the quantity of energy supplied to the circuit. The number of revolutions made are registered by a counting mechanism consisting of a train of gear wheels and dials.

The control of speed of the rotating system is brought about by a permanent magnet (known as braking magnet) which is so placed as to set up eddy currents in some parts of the rotating system. These eddy currents produce a retarding torque which is proportional to their magnitude-their magnitude itself depending on the speed of rotation of the rotating system. The rotating system attains a *steady speed* when the braking torque exactly balances the driving torque which is produced either by the current or power in the circuit.

The essential parts of motor meters are :

1. An operating system which produces an operating torque proportional to the current or power in the circuit and which causes the rotation of the rotating system.

2. A retarding or braking device, usually a permanent magnet, which produces a braking torque is proportional to the speed of rotation. Steady speed of rotation is achieved when braking torque becomes equal to the operating torque.

3. A registering mechanism for the revolutions of the rotating system. Usually, it consists of a train of wheels driven by the spindle of the rotating system. A worm which is cut on the spindle engages a pinion and so driven a wheel-train.

10.44. Errors in Motor Meters

The two main errors in such instruments are : ***(i)*** friction error and ***(ii)*** braking error. Friction error is of much more importance in their case than the corresponding error in indicating instruments because ***(a)*** it operates continuously and ***(b)*** it affects the speed of the rotor. The braking action in such meters corresponding to damping in indicating instruments. The braking torque directly affects the speed for a given driving torque and also the number of revolution made in a given time.

Friction torque can be compensated for providing a small constant driving torque which is applied to the moving system independent of the load.

As said earlier, steady speed of such instruments is reached when driving torque is euqal to the braking torque. The braking torque is proportional to the flux of the braking magnet and the eddy current induced in the moving system due to its rotation in the field of the braking magnet

$$\therefore \qquad T_B \propto \Phi_i$$

where Φ is the flux of the braking magnet and i the induced current. Now $i = e/R$ where e is the induced e.m.f. and R the resistance of the eddy current path. Also $e \propto \Phi n$ where n is the speed of the moving part of the instrument.

$$\therefore \qquad T_b \;\alpha\; \frac{n}{R} \quad \frac{^2 n}{R}$$

The torque T_B' at the steady speed of N is given $T_B' \propto \Phi^2 N/R$

Now $\qquad T_B' = T_D \qquad$ – the driving torque

$$\therefore \qquad T_D \propto \Phi^2 N/R \quad \text{or} \quad N \propto T_D R/\Phi^2$$

Hence for a given driving torque, the steady speed is direcly proportional to the resistance of the eddy current of path and inversely to the square of the flux.

Obviously, it is very important that the strength of the field of the brake magnet should be constant throughout the time the meter is in service. The constancy of field strengths can be assured by careful design treatment during the manufacturing of the brake magnet. Variations in temperature will affect the braking torque since the resistance of the eddy current path will change. This error is difficult to fully compensate for.

10.45. Quantity or ampere-hour Meters

The use of such meters is mostly confined to d.c. circuits. Their operation depends on the production of two torques ***(i)*** a driving torque which is proportional to the current I in the circuit and ***(ii)*** a braking torque which is proportional to the speed n of the spindle. This speed attains a steady value N when these two torques become numerically equal. In that case, speed becomes proportional to current *i.e.*, $N \propto I$. Over a certain period of time, the total number of revolutions $\int Ndt$ will be proportional to the quantity of electricity $\int Idt$ passing through the meter. A worm cut in the spindle as its top engages gear wheels of the recording mechanism which has suitably marked dials reading directly in ampere hours. Since electric supply charges are based on watt-hour rather than ampere-hours, the dials of ampere-hour meters are frequently marked in corresponding watt-hour at the normal supply voltage. Hence, their indications of watt-hours are correct only when the supply voltages remains constant, otherwise reading will be wrong.

10.46. Ampere-hour Mercury Motor Meter

It is one of the best and most popular forms of mercury Ah meter used for d.c. work.

Construction

It consists of a thin Cu disc D mounted at the base of a spindle, working in jewelled cup bearings and revolving between a pair of ***permanent*** magnets M_1 and M_2. One of the two magnets *i.e.*, M_2 is used for driving purposes whereas M_1 is used for braking. In between the poles of M_1 and M_2 is a hollow circular box B in which rotates the Cu disc and the rest of the space is filled up with mercury which exerts considerable upward thrust on the disc, thereby reducing the pressure on the bearings. The spindle is so weighted that it just sinks in the mercury bath. A worm cut in the spindle at its top engages the gear wheels of the recording mechanism as shown in Fig. 10.59.

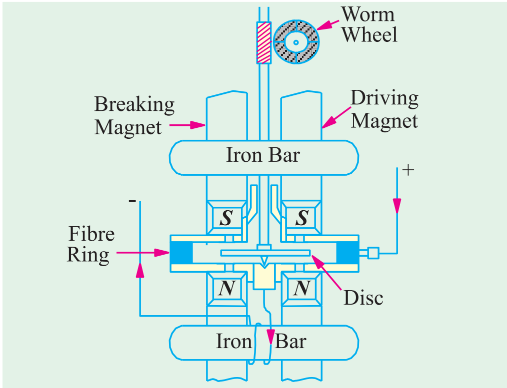


Fig. 10.59

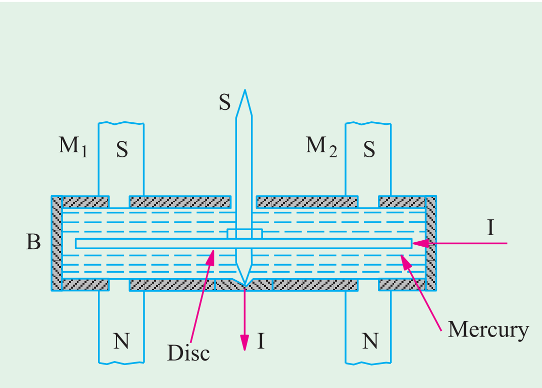


Fig. 10.60

Principle of Action

Its principle of action can be understood from Fig. 10.61 which shows a separate line drawing of the motor element.

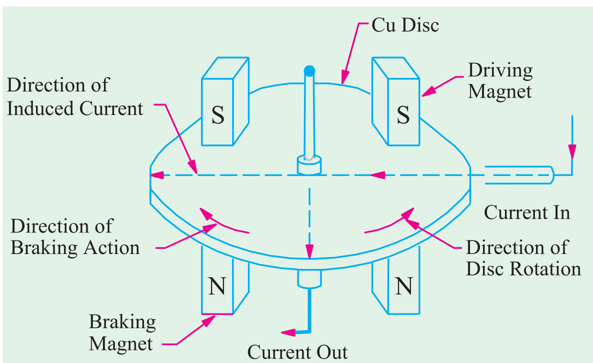


Fig. 10.61

The current to be measured is led into the disc through the mercury at a point at its circumference on the right-hand side. As shown by arrows, it flows radially to the centre of the disc where it passes out to the external circuit through the spindle and its bearings. It is worth noting ***that current flows takes place only under the right-hand side magnet M_2*** and not under the left-hand side magnet M_1. The field of M_2 will, therefore, exert a force on the right-side portion of the disc which carries the current (motor action). The direction of the force, as found by Fleming's Left-hand rule, is as shown by the arrow. The magnitude of the force depends on the flux density and current ($\because F = BIl$). The driving or motoring torque T_d so produced is given by the product of the force and the distance from the spindle at which this force acts. When the disc rotates under the influence of this torque, it cuts through the field of left-hand side magnet M_1 and hence eddy currents are produced in it which results in the production of braking torque. The magnitude of the retarding or braking torque is proportional to the speed of rotation of the disc.

Theory

Driving torque $T_d \propto$ force on the disc $\times$ $B\,I$

If the flux density of M_2 remains constant, then $T_d \propto I$.

The braking torque T_B is proportional to the flux Φ of braking magnet M_1 and eddy current i induced in the disc due to its rotation in the field of M_1.

$\therefore$ $$T_B \propto \Phi_i$$

Now $i = e/R$ where e is the induced e.m.f. and R the resistance of eddy current path.

Also $e \propto \Phi_n$ - where n is the speed of the disc $\therefore T_B \propto \Phi \times \frac{n}{R}$ $\frac{{}^2n}{R}$

The speed of the disc will attain a steady value N when the driving and braking torques becomes equal. In that case, $T_B \propto \Phi^2 N/R$.

If Φ and R are constant, then $I \propto N$

The total number of revolution in any given time t *i.e.*, $\int_0^t N.dt$ will become proportional to

$\int_0^t I.dt$ *i.e.*, to the total quantity of electricity passed through the meter.

10.47. Friction Compensation

There are two types of frictions in this ampere-hour meter.

(*i*) **Bearing Friction.** The effect of this friction is normally negligible because the disc and spindle float in mercury. Due to the upward thrust, the pressure on bearings is considerably reduced which results in freedom from wear as well as freat reduction in the bearing friction.

(*ii*) **Mercury Friction.** Since the disc revolves in mercury, there is friction between mercury and the disc, which gives rise to a torque, approximately proportional to the square of the speed of rotation. Hence, this friction causes the meter to run shown on heavy loads. It can be compensated for in the following two ways :

(*a*) a coil of few turns is wound on one of the poles of the driving magnet M_2 and the meter current is passed through it in a suitable direction so as to increase the strength of M_2. The additional driving torque so produced can be made just sufficient to compensate for the mercury friction.

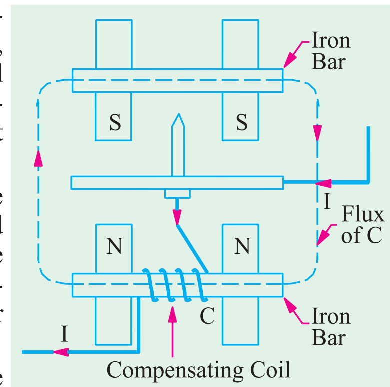


Fig. 10.62

(*b*) in the other method, two iron bars are placed across the permanent magnets, one above and other one below the mercury chamber as shown in Fig. 10.62. The lower bar carries a small compensating coil through which is passed the load current. The local magnetic field set up by this coil strengthens the field of driving magnet M_2 and weakens that of the braking magnet M_1, thereby compensating for mercury friction.

10.48. Mercury Meter Modified as Watt-hour meter

If the permanent magnet M_2 of the amp-hour meter, used for producing the driving torque, is replaced by a wound electromagnet connected across the supply, the result is a watt-hour meter. The exciting current of this electromagnet is proportional to the voltage of the supply. The driving torque is exerted on the aluminium disc immersed in the mercury chamber below which is placed this eleccromagnet. The aluminimum disc has radial slots cut in it for ensuring the radial flow of current through it the current being led into and out of this disc through mercury contacts situated at diameterically opposite points. These radial slots, moreover, prevent the same disc being used for braking purposes. Braking is by a separate aluminium disc mounted on the same spindle and revolving in the air-gap of a separate braking magnet.

10.49. Commutator Motor Meters

These meters may be either ampere-hour or true watt-hour meters. In Fig. 10.63 is shown the principle of a common type of watt-hour meter known as Elihu-Thomson meter. It is based on the dynamometer principle (Art. 10.20) and is essentially an ironless motor with a wound armature having a commutator.

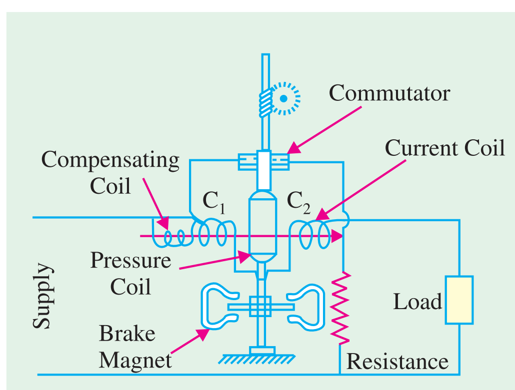


Fig. 10.63

Construction

There are two fixed coils C_1 and C_2 each consisting of a few turns of heavy copper strip and joined in series with each other and with the supply circuit so that they carry the main current in the circuit (a shunt is used if the current is too heavy). The field produced by them is proportional to the current to be measured. In this field rotates an armature carrying a number of coils which are connected to the segments of a small commutator. The armature coils are wound on a former made of non-magnetic material and are connected through the brushes and in series with a large resistance across the supply lines. The commutator is made of silver and the brushes are silver tipped in order to reduce friction. Obviously, the current passing through the armature is proportional to the supply voltage.

The operating torque is produced due to the reaction of the field produced by the fixed coils and the armature coils. The magnitude of this torque is proportional to the product of the two currents *i.e.*,

$$T_d \propto \phi I_1 \quad \text{or} \quad T_d \propto I_1 \times I \qquad (\because \Phi \propto I)$$

where I = main circuit current

I_1 = current in armature coils.

since $I_1 \propto V \quad \therefore \; T_d \propto V\, I$ -power

Brake torque is due to the eddy currents induced in an aluminium disc mounted on the *same* spindle and running in the air-gaps of two permanent magnets. As shown in Art, 10.44, this braking torque is proportional to the speed of the disc if the flux of the braking magnet and the resistance of the eddy current paths are assumed constant.

When steady speed of rotation is reached, then

$$T_B = T_d \quad \therefore \; N \propto VI \propto \text{power } W$$

Hence, steady number of revolutions in a given time is proportional to $\int Wt$ = the energy in the circuit.

The friction effect is compensated for by means of a small compensating coil placed coaxially with the two currents coils and connected in series with the armature such that it strengthens the field of current coil. But its position is so adjusted that with zero line current the armature just fails to rotate. Such meters are now employed mainly for switchboard use, house service meters being invariably of the mercury ampere-hour type.

10.50. Induction Type Single-phase Watthour Meter

Induction type meters are, by far, the most common form of a.c. meters met with in every day domestic and industrial installations. These meters measure electric energy in kilo-watthours. The principle of these meters is practically the same as that of the induction wattmeters. Constructionally, the two are similar that the control spring and the pointer of the watt-meter are replaced, in the case of watthour meter, by a brake magnet and by a spindle of the meter.

The brake magnet induces eddy currents in the disc which revolves continuously instead of rotating through only a fraction of a revolution as in the case of wattmeters.

Construction

The meter consists of two a.c. electromagnets as shown in Fig. 10.64. (*a*), one of which *i.e.*, M_1 is excited by the line current and is known as ***series*** magnet. The alternating flux Φ_1 produced by it is proportional to and in phase with the line current (provided effects of

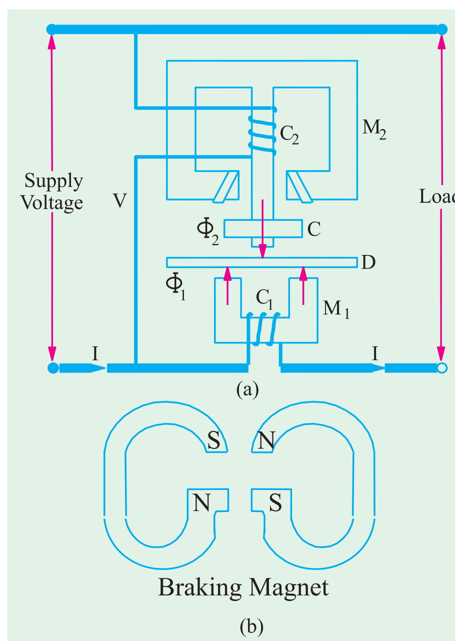


Fig. 10.64

hysteresis and iron saturation are neglected). The winding of the other magnet M_2 called ***shunt*** magnet, is connected across the supply line and carries current proportional to the supply voltage V. The flux Φ_2 produced by it is proportional to supply voltage V and lags behind it by 90º. This phase displacement of exact 90º is achieved by adjustment of the copper shading band C (also known as power factor compensator) on the shunt magnet M_2. Major portion of Φ_2 crosses the narrow gap between the centre and side limbs of M_2 but a small amount, which is the useful flux, passes through the disc D. The two fluxes Φ_1 and Φ_2 induce e.m.f.s in the disc which further produce the circulatory eddy currents. The reaction between these fluxes and eddy currents produces the driving torque on the disc in a manner similar to that explained in Art. 10.39. The braking torque is produced by a pair of magnets [Fig. 10.64 (*b*)] which are mounted diametrically opposite to the magnets M_1 and M_2. The arrangements minimizes the interaction between the fluxes of M_1 and M_2. This arrangements minimizes the interaction between the fluxes of M_1 and M_2 and that of the braking magnet. When the peripheral portion of the rotating disc passes through the air-gap of the braking magnet, the eddy currents are induced in it which give rise to the necessary torque. The braking torque $T_B \propto \Phi^2 N/R$ where Φ is the flux of braking magnet, N the speed of the rotating disc and R the resistance of the eddy current path. If Φ and R are constant, then $T_B \propto N$.

The register mechanism is either of pointer type of cyclometer type. In the former type, the pinion on the rotor shaft drives, with the help of a suitable train of reduction gears, a series of five or six pointers rotating on dials marked with ten equal divisions. The gearing between different pointers is such that each pointer advances by 1/10th of a revolution for a complete revolution of the adjacent pointer on the main rotor disc in the train of gearing as shown in Fig. 10.65.

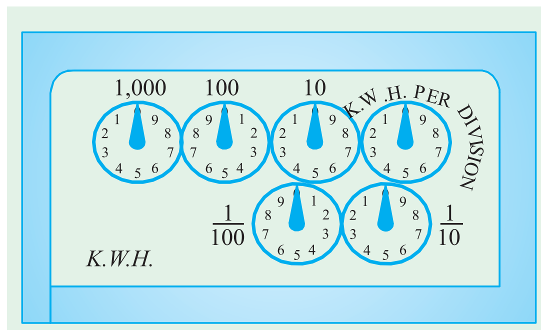


Fig. 10.65

Theory

As shown in Art. 10.39. and with reference to Fig. 10.58, the driving torque is given by $T_d \propto \omega\Phi_{1m}$ and Φ_{2m} where sin $\propto$, Φ_{1m} and Φ_{2m} are the maximum fluxes produced by magnets M_1 and α the angle between these fluxes. Assuming that fluxes are proportional to the current, we have

Current through the windings of M_1 = I –the line current

Current through the winding of M_2 = $V/\omega L$

$$\alpha = 90 - \phi \text{ where } \phi \text{ is the load p.f. angle}$$

$$T_d \propto \; . \frac{V}{L} . I \cos (90 \quad) \quad VI \cos \quad \text{power}$$

Also, $T_b \propto N$

The disc achieves a steady speed N when the two torques are equal *i.e.*, when

$$T_d = T_B \quad \therefore \; N \propto \text{power } W$$

Hence, in a given period of time, the total number of revolution $\int_0^t N.dt$ is proportional to $\int_0^t W.dt$ *i.e.*, the electric energy consumed.

10.51. Errors in Induction Watthour Meters

1. Phase and speed errors

Because ordinary the flux due to shunt magnet does not lag behind the supply voltage by exactly 90º owing to the fact that the coil has some resistance, the torque is not zero power factor. This is compensated for by means of an adjustable shading ring placed over the central limb of the sunt magnet. That is why this shading ring is known as ***power factor compensator.***

An error in the speed of the meter, when tested on a non-inductive load, can be eliminated by correctly adjusting the position of the brake magnet. Movement of the poles of the braking magnet towards the centre of the disc reduces the braking torque and ***vice-versa***.

The supply voltage, the full load current and the correct number of revolutions per kilowatthour are indicated on the name plate of the meter.

2. Friction compensation and creeping error

Frictional forces at the rotor bearings and in the register mechanism gives rise to unwanted braking torque on the disc rotor. This can be reduced to an unimportant level by making the ratio of the shunt magnet flux Φ_2 and series magnet flux Φ_1 large with the help of two shading bands. These bands embrace the flux contained in the two outer limbs of the shunt magnet and so eddy currents are induced in them which cause phase displacement between the enclosed flux and the main-gap flux. As a result of this, a small driving torque is exerted on the disc rotor solely by the pressure coil and independent of the main driving torque. The amount of this corrective torque is adjusted by the variation of the position of the two bands, so as to exactly compensate for firctional torque in the instrument. Correctness of friction compensation is achieved when the rotor does not run on no-load ***with only the supply voltage connected.***

By **'*creeping*'** is meant the slow but continuous rotation of the rotor when only the pressure coils are excited but with no current flowing in the circuit. It may be caused due to various factors like incorrect friction compensation, to vibration, to stray magnetic fields or due to the voltage supply being is excess of the normal. In order to prevent creeping on no-load, two holes are drilled in the disc on a diameter *i.e.*, on the opposite sides of the spindle.

This causes sufficient distortion of the field to prevent rotation when one of the holes comes under one of the poles of the sunt magnet.

3. Errors due to temperature variations

The errors due to temperature variations of the instruments are usually small, because the various effects produced tend to neutralise one another.

Example 10.27. *An ampere-hour meter, calibrated at 210 V, is used on 230 V circuit and indicates a consumption of 730 units in a certain period. What is the actual energy supplied ?*

If this period is reckoned as 200 hours, what is the average value of the current ?

(Elect. Technology, Utkal Univ.)

Solution. As explained in Art. 10.42, ampere-hour meters are calibrated to read directly in mWh at the *declared voltage*. Obviously, their readings would be incorrect when used on any other voltage.

Reading on 210 volt = 730 kWh

Reading on 230 volt = 730 × 230/210 = **800 kWh (approx.)**

Average current = 800,000/230 × 200 = **17.4 A**

Example 10.28. *In a test run of 30 min. duration with a constant current of 5 A, mercury-motor amp-hour meter, was found to register 0.51 kWh. If the meter is to be used in a 200-V circuit, find its error and state whether it is running fast or slow. How can the instrument be adjusted to read correctly ?* **(Elect. Mean Inst. and Mean., Jadavpur Univ.)**

Solution. Ah passed in 30 minutes = 5 × 1/2 = 2.5

Assumed voltage = 0.51 × 1000/2.5 = 204 V

When used on 200-V supply, it would obviously show higher values because actual voltage is less than the assumed voltage. It would be fast by 4 × 100/200 = **2%**

Example 10.29. *An amp-hour meter is calibrated to read kWh on a 220-V supply. In one part of the gear train from the rotor to the first counting dial, there is a pinion driving a 75-tooth wheel. Calculate the number of teeth on a wheel which is required to replace 75-tooth wheel, in order to render the meter suitable for operation on 250-V supply.*

Solution. An amp-hour meter, which is calibrated on 220-V supply would run fast when operated on 250-V supply in the ratio 250/220 or 25/22. Hence, to neutralize the effect of increased voltage, the number of teeth in the wheel should be reduced by the same ratio.

∴ Teeth on the new wheel = 75 × 22/25 = **66**

Example 10.30. *A meter, whose constant is 600 revolutions per kWh, makes five revolution in 20 seconds. Calculate the load in kW.* **(Elect. Meas. and Meas. Inst. Gujarat Univ.)**

Solution. Time taken to make 600 revolution is = 600 × 20/5 = 2,400 second

During this time, the load consumes 1 kWh of energy. If W is load in kW, then

$W \times 2400(60) \times 60 = 1$ or $W =$ **1.5 kW**

Example 10.31. *A current of 6 A flows for 20 minutes through a 220-V ampere hour meter. If during a test the initial and final readings on the meter are 3.53 and 4.00 kWh respectively, calculate the meter error as a percentage of the meter readings.*

If during the test, the spindle makes 480 revolutions, calculate the testing constant in coulomb/rev and rev/kWh.

Solution. Energy actually consumed = $6 \times \left(\frac{20}{60}\right) \times \frac{200}{1000} = 0.44$ kWh

Energy as registered by meter = 4.00 –3.53 = 0.47 kWh

Error = 0.47 –0.44 = 0.03 kWh ; % error = 0.03 × 100/0.47 = 6.38 %

No. of coulombs passed through in 20 minutes = 6 × 20 × 7,2000 coulomb

Testing for 480 revolutions, only 0.44 kWh are consumed, hence testing constant = 480/0.44 = **1091 rev/kWh.**

Example 10.32. *A 230-V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter are 1472 when operating at 230 V and 5 V for 4 hours.*

(Elect, Measure, A.M.I.E. Sec B, 1991)

Solution. Energy consumption in 6 hr = 230 × 4 × 1 × 6 = 5520 W = 5.52 kW

Meter constant = 2208/5.52 = 400 rev/kWh.

Now, 1472 revolution represents energy consumption of 1472/400 = 3.68 kWh

∴ $VI \cos \phi \times$ hours $= 3.68 \times 10^{-10}$ or $230 \times 5 \times \cos \phi \times 4 = 3680$, ∴ $\cos \phi =$ **0.8**

Example 10.33. *A 230 V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the motor disc makes 2208 revolution during this period, what is the constant in rev kWh ? Calculate the power factor of the load if the No. of rev made by the meter are 1472 when operating at 230 V and 5 A for 4 hours.*

(Elect. Measuring. AMIE Sec. Winter 1991)

Solution. Energy supplied at unity p.f. = 230 × 4 × 6 × 1/1000 = 5.52

∴ $230 \times 5 \times 4 \times \cos \phi/2000 = 3.68$ ∴ $\cos \phi =$ **0.8.**

Example 10.34. *The testing constant of a supply meter of the amp-hour type is given as 60 coulomb/revolution. It is found that with a steady current of 50 A, the spindle makes 153 revolutions in 3 minutes. Calculate the factor by which dial indications of the meter must be multiplied to give the consumption.* **(City and Guilds, London)**

Solution. Coulombs supplied in 3 min. = 50 × 3 × 60

At the rate of 60 C/rev., the correct of revolution should have been

= 50 × 3 × 60/60 = 150

Registered No. of revolutions = 153

Obviously, the meter is fact. The registered readings should be multiplied by 150/153 = **0.9804** for correction.

Example 10.35. *A single phase kWhr meter makes 500 revolutions per kWh. It is found on testing as making 40 revolutions in 58.1 seconds at 5 kW full load. Find out the percentage error.*

(Elect. Measurement & Measuring Instrument Nagpur Univ. 1993)

Solution. The number of revolutions the meter will make in one hour on testing = 40 × 3600/58.1 = 2478.5

These revolutions correspond to an energy of 5 × 5 kWh

∴ No. of revolutions kWh = 2478.5/5 = 495.7

Percentage error = (500 –495.7) × 100/500 = **0.86 %**

Example 10.36. *An energy meter is designed to make 100 revolution of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 40 A at 230-V and 0.4 power factor for an hour. If it actually makes 360 revolutions, find the percentage error.* **(Elect. Engg - I Nagpur Univ. 1993)**

Solution. Energy consumed in one hour = 230 × 40 × 0.4 × 1/1000 = 3.68 kWh

No. of revolutions the meter should make if it is correct = 3.68 × 100 = 368

No. of revolutions actually made = 360

∴ Percentage error = (369 –360) × 100/368 = **2.17 %**

Example 10.37. *The constant of a 25-ampere, 220-V meter is 500 rev/kWh. During a test at full load of 4.400 watt, the disc makes a 50 revolutions in 83 seconds. Calculate the meter error.*

Solution. In one hour, at full-load the meter should make (4400 × 1) × 500/1000 = 2200 revolutions. This corresponds to a speed of 2200/60 = 36.7 r.p.m.

Correct time for 50 rev. = (50 × 60)/36.7 = 81.7s

Hence, meter is slow by 83 –81.7 = 1.3 s

∴ Percentage error = 1.3 × 100/81.7 = **1.59 %**

Example 10.38. *A 16-A amp-hour meter with a dial marked in kWh, has an error of + 2.5 % when used on 250-V circuit. Find the percentage error in the registration of the meter if it is connected for an hour in series with a load taking 3.2 kW at 200.*

Solution. In one hour, the reading given by the meter is = 16 × 250/1000 = 4 kWh

Correct reading = 4 + 2.5 % of 4 = 4.1 kWh

Meter current on a 3.2 kW load at 200 V = 3200/200 = 16 A

Since on the given load, meter current is the same as the normal current of the meter, hence in one hour it would give a corrected reading of 4.1 kWh.

But actual load is 3.2 kWh. ∴ Error = 4.1 –3.2 = 0.9 kWh

% error = 0.9 × 100/3.2 = **28.12 %**

Example 10.39. *The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.s if the load is on for 12 hours, how many units are recorded as error ?* **(Measurs, Instru. Allahabad Univ. 1992)**

Solution. Since load power is one kWh, energy actually consumed is

= 1 × 12 = 12 kWh

Total number of revolutions made by the disc during the period of 12 hours

= 10.2 × 60 × 12 = 7,344

since 600 revolutions record one kWh, energy recorded by the meter is

= 7,344/600 = 12.24 kWh

Hence, **0.24** unit is recorded extra.

Example 10.40. *A d.c. ampere-hour meter is rated at 5-A, 250-V. The declared constant is 5 A-s/rev. Express this constant in rev/kWh. Also calculate the full-load speed of the meter.*

(Elect. Meas. Inst. and Meas., Jadavpur Univ.)

Solution. Meter constant = 5 A-s/rev

Now, 1 kWh = 10^3 Wh = $10^3 \times 3600$ volt-second

= 36×10^5 volt × amp × second

Hence, on a 250-V circuit, this corresponds to $36 \times 10^5/250 = 14{,}400$ A-s

Since for every 5 A-s, there is one revolution, the number of revolution is one kWh is

$= 14{,}400/5 = 2{,}880$ revolutions

∴ Meter constant = **2,800 rev/kWh**

Since full-load meter current is 5 A and its constant is 5 A-s/rev, it is obvious that it makes one revolution every second.

∴ full load speed = 60 r.p.m.

Example 10.41. *The declared constant of a 5-A, 200-V amp-hour meter is 5 coulomb per revolution. Express the constant in rev/kWh and calculate the full-load speed of the meter.*

In a rest at half load, the meter disc completed 60 revolutions in 119-5 seconds. Calculate the meter error.

Solution. Meter constant = 5 C/rev, or 5 A-s/rev.

1 kWh 1000 Wh = 1000×3600 watt-second

$= 1000 \times 3600$ volt × amp × second

Hence, on a 200-V circuit, this corresponds to

$= 100 \times 3600/200 = 18{,}000$ A-s

Since for every 5 A-s, there is one revolution, hence number of revolution is one kWh = 18,000/5 = 3600 revolution ∴ Meter constant = **3600 rev/kWh**

Since full-load meter current is 5 A and its constant 5 A-s/rev, it is obvious that it makes one revolution every one second.

∴ its full-load speed = **60 r.p.m.**

At half-load

Quantity passed in 60 revolutions = 119.5×2.5 A-s or 298.75 C

Correct No. of revolutions = 298.75/5 = 69.75

Obviously the meter is running fast because instead of making 59.75 revolutions, it is making 60 revolutions.

Error = 60 –59.75 = 0.25 ∴% error = $0.25 \times 100/59.75$ = **0.418 %**

Example 10.42. (*a*). *A single-phase energy meter of the induction type is rated 230-V; 10-A, 50-Hz and has a meter constant of 600 rev/kWh when correctly adjusted. If 'quadrature' adjustment is slightly disturbed so that the lag is 85º, calculate the percentage error at full-load 0.8 p.f. lag.*

(Measu & Instru. Nagpur Univ 1991)

Solution. As seen from Art. 10.50, the driving torque. T_d depends among other factors, on $\sin \alpha$ where α is the angle between the two alternating fluxes. ∴ $T_d \propto \sin \alpha$

If the voltage flux-lagging adjustment is disturbed so that the phase angle between the voltage flux and the voltage is less than 90º (instead of being exactly 90º) the error is introduced.

Now $\cos \phi = 0.8, \quad \phi = \cos^{-1}(0.8) = 36º52'$

∴ $\alpha = 85º - 36º52' = 48º8'$ where it should be $= 90 - 36º52' = 53º8'$

∴ $$\text{error} = \frac{\sin 53º8 \quad \sin 48º8}{\sin 53º8} \quad 100 \quad \mathbf{7\%}$$

Example 10.42 (*b*). *A 50-A, 230-V energy meter is a full-load test makes 61 revolutions in 37 seconds if the normal speed of the disc is 520 revolutions/kWh, compute the percentage error.*

[Nagpur University November 1999]

Solution. Unity power-factor is assumed.

Energy consumed, in kWh, in 37 seconds

$$= \frac{50 \times 230}{1000} \times \frac{37}{3600} = 0.1182 \text{ kWh}$$

Number of revolutions corresponding to this energy = 520 × 0.1182 = 61.464

The meter makes 61 revolutions

$$\therefore \qquad \% \text{ Error} = \frac{61 - 61.464}{61.464} \times 100\,\% = \mathbf{-0.755\ \%}$$

10.52. Ballistic Galvanometer

Ballistic Galvanometer

It is used principally for measuring small electric charges such as those obtained in magnetic flux measurements. Constructionally, it is similar to a moving-coil galvanometer except that ***(i)*** it has extremely small electromagnetic damping and ***(ii)*** has long period of undamped oscillation (several seconds). These conditions are necessary if the galvanometer is to measure electric charge. In fact, the moment of inertia of the coil is made so large that whole of the charge passes through the galvanometer before its coil has had time to move sufficiently. In that case, the first swing of the coil is proportional to the charge passing through the galvanometer. After this swing has been observed, the oscillating coil may be rapidly brought to rest by using eddy-current damping.

As explained above, the coil moves after the charge to be measured has passed through it. Obviously, during the movement of the coil, there is no current flowing through it. Hence, the equation of its motion is

$$J\frac{d^2\phi}{dt^2} + D\frac{d\theta}{dt} + C\theta = 0$$

where J is the moment of inertia, D is damping constant and C is restoring constant.

Since damping is extremely small, the approximate solution of the above equation is

$$0 = U_e^{-(D/2J)t} \sin(\omega_0 t + \phi)$$

At the start of motion, where $r = 0$, $\theta = 0$, hence $\phi = 0$ $\quad \therefore \ \theta = Ue^{-(D/2J)t} \sin \omega_0 t$ ...***(i)***

During the passage of charge, at any instant, there will be a deflecting torque of Gi acting on the coil. If t is the time taken by the whole charge to pass through, the torque impulse due to this charge is

$$\int_0^t Gidt \text{ . Now } \int_0^t i\,dt = Q$$

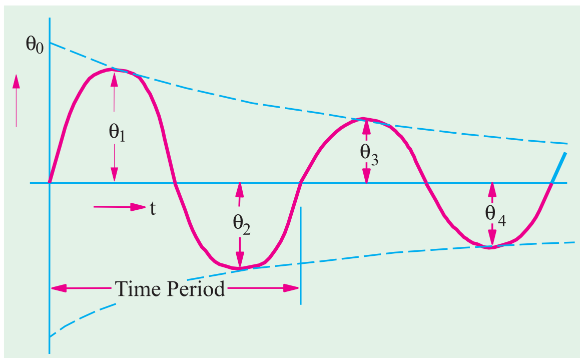


Fig. 10.66

Hence, torque impulse = GQ. This must be equal to the change of angular momentum produced *i.e.*, $j\alpha$ where α is the angular velocity of the coil at the end of the impulse period.

$$\therefore \qquad GQ = J\alpha$$

$$\text{or} \qquad \alpha = GQ/J$$

Differentiating Eq. ***(i)*** above, we get

$$\frac{d\theta}{dt} = U\left(e^{-(D/2J)'}\ \omega_0 t - \frac{D}{2}J\ e^{-(D/2J)r} \sin \omega_0 t\right)$$

Since duration of the passage of charge is very small, at the end of the passage, $t \cong 0$, so that from above, $dq/dt = U\omega_0$.

However, at this time, $\frac{d\theta}{dt} = \alpha = \frac{GQ}{J} \quad \therefore \quad \frac{GQ}{J} = U\,\omega_0 \quad \text{or} \quad Q = \frac{J\omega_0}{G}U$

Now, U being the amplitude which the oscillations would have if the damping were zero, it may be called undamped swing θ_0.

$$\therefore \qquad Q = \frac{J\omega_0}{G}\theta_0 \quad \text{or} \quad Q \propto \theta_0 \qquad \text{...(ii)}$$

However, in practice, due to the presence of small amount of damping, the successive oscillations diminish exponentially (Fig. 10.66). Even the first swing θ_1 is much less than θ_0. Hence, it becomes necessary to obtain the value of θ_0 from the observed value of first maximum swing θ_1.

As seen from Fig. 10.66, the successive peak value θ_1, θ_2, θ_3 etc. are ϕ radian apart or ϕ/ω_0 second apart. The ratio of the amplitude of any two successive peaks is

$$= \frac{\theta_0 e^{-(D/2J)^t} \sin \omega_0 t}{\theta_0 e^{-(D/2J)t + (\pi/\omega_0)} \sin(\omega_0 t + \pi)} = \frac{e^{-(D/2J)^t} \sin \omega_0 t}{e^{-(D/2J)^t}\, e^{-(D/2J)(\pi/\omega_0)} (-\sin \omega_0 t)} = -e^{(D/2J)(\pi/\omega_0)}$$

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots = e^{(D/2J)(\pi/\theta_0)}$$

Let $e^{(D/2J)(\pi/\omega_0)} = \Delta^2$ where Δ is called the damping factor*.

The time period of oscillation $T_0 = 2\pi/\omega_0$. If damping is very small $\theta_0 = \theta_1$, $t = T_0/4 = \pi/2\omega_0$ as a very close approximation.

Hence, from Eq. (*i*) above, putting $t = \pi/2\omega_0$, we have

$$\theta = \theta_0^{(D/2J)(\pi/2\omega_0)} \sin \pi/2 = \theta_0 \Delta - 1 \quad \therefore \quad \theta_0 = \Delta\,\theta_1 \qquad \text{...(iii)}$$

or undamped swing = damping factor × 1st swing

Suppose, a steady current of I_s flowing through the galvanometer produces a steady deflection ϕ_s, then $\qquad C\theta_s = GI_s \quad \text{or} \quad G = C\theta_s/I_s$

Since damping is small, $\omega_0 = \sqrt{C/J} \quad \therefore C = J\omega_0^2 = J \times \left(\frac{2\pi}{T_0}\right)^2 = 4\pi^2 \frac{J^2}{T_0^2} \quad \therefore \quad G = \frac{4\pi^2 J\theta s}{T_0^2 I_s}$

Substituting this value of G in Eq. (*ii*), we get

$$Q = \frac{J\omega_0\,\theta_0}{4\pi^2 J\theta_s / T_0^2 I_s^2} \quad \text{or} \quad Q_0 = \frac{T_0}{2\pi}\cdot\frac{I_s}{\theta_s}\cdot\theta_0 = \frac{T_0}{2\pi}\cdot\frac{I_s}{2\pi}\cdot\Delta.\theta_1 \ \text{...(iv)}$$

Alternatively, let quantity $(D/2J)(\pi/\omega_0)$ be called the logarithmic decrement λ. Since, $\Delta^2 = e^{\lambda}$, we have

$$\Delta = e^{\lambda/2} = 1 + (\lambda/2) + \frac{(\lambda/2)^2}{2!} + \ldots \cong \left(1 + \frac{\lambda}{2}\right) \text{ when } \lambda \text{ is small**}$$

Hence, from Eq. (**iv**) above, we have $Q = \frac{T_0}{2\pi}, \frac{I_s}{\theta_s}\left(1 + \frac{\lambda}{2}\right)\theta_1 \qquad \text{...(v)}$

In general, Eq. (**iv**) may be put as $Q = k\theta_1$

---

* Now, $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_{n-1}}{\theta_n} = \Delta^2 \quad \therefore \quad \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \times \ldots \times \frac{\theta_{n-1}}{\theta_n} = (\Delta^2)^{n-1} \therefore \frac{\theta_1}{\theta_2} = (\Delta^2)^{n-1} \quad \text{or } \Delta = \left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)}$

Hence, Δ may be obtained by observing the first and nth swing.

** Since $\Delta^2 = e^{\lambda}$, taking logs, we have $2 \log_c \Delta = \lambda \log_e e = \lambda$

$$\therefore \quad \frac{\lambda}{2} = \log_c \Delta = \log_e \left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)} = \frac{1}{2(n-1)} \log_e \theta_1/\theta_n \ \therefore \ \lambda = \frac{1}{(n-1)} \log_e \frac{\theta_1}{\theta_n}$$

Example. 10.43. *A ballistic galvanometer has a free period of 10 seconds and gives a steady deflection of 200 divisions with a steady current of 0.1 μA. A charge of 121 μC is instantaneously discharged through the galvanometer giving rise to a first maximum deflection of 100 divisions . Calculate the 'decrement' of the resulting oscillations.*

(Electrical Measurements, Bombay Univ.)

Solution. From Eq. (*v*) of Art 10.52, we have $Q = \frac{T_0}{2} \cdot \frac{I_s}{s}\ 1 \quad \overline{2} \quad 1$

Here, $Q = 121\ \mu C = 121 \times 10^{-6}\ C$; $T_0 = 10s$; $I_s = 0.1$ mA $= 10^{-4}$ A ; $\theta_s = 200$; $\theta_1 = 100$

$$\therefore \quad 121 \times 10^{-6} = \frac{10}{2\pi} \times \frac{10^{-4}}{200}\left(1 + \frac{\lambda}{2}\right) \times 100; \qquad \therefore \ \lambda = \mathbf{1.04}$$

10.53. Vibration Galvanometer

Such galvanometers are widely used as null-point detectors in a.c. bridges.

Construction

As shown in Fig. 10.67. (*a*), it consists of a moving coil suspended between poles of a strong permanent magnet. The natural frequency of oscillation of the coil is very high, this being achieved by the use of a large value of control constant and a moving system of very small inertia. The suspension (which provides control) is either a phosphor-bronze strip or is a bifilar suspension in which case the two suspension wires carry the coil and a small piece of mirror (or in some cases the two suspension wires themselves from the coil.)

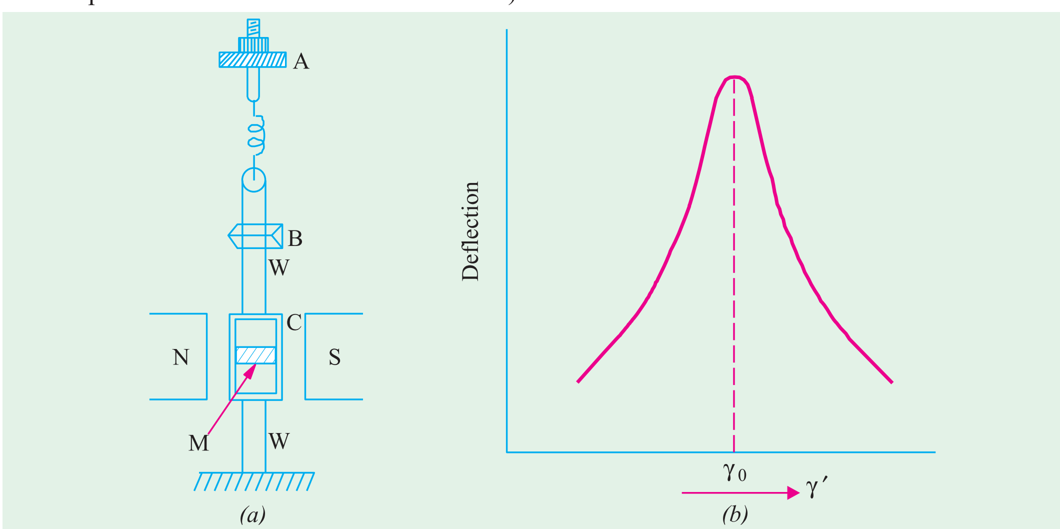


Fig. 10.67

As seen, *W* is the suspension, *C* is the moving coil and *M* the mirror on which as cast a beam of light. From mirror *M*, this beam is deflected on to a scale. When alternating current is passed through *C*, an alternating torque is applied to it so that the reflected spot of light on the scale is drawn out in the form of a band of light. The length of this band of light is maximum if the natural frequency of oscillation of *C* coincides with the the supply frequency due to resonance. The turning of *C* may be done in the following two ways :

(*i*) by changing the length of suspension *W*. This is achieved by raising or lowering bridge piece *B* against which the bifilar loop presses.

(*ii*) by adjusting tension in the suspension. This is achieved by turning the knurled knob *A*.

By making the damping very small, the resonance curve of the galvanometer can be made sharply-peaked [Fig. 10.67 (*b*)]. In that case, the instrument discriminates sharply against frequencies other than its own natural frequency. In other words, its deflection becomes very small even when the frequency of the applied current differs by a very small amount from its resonance frequency.

Theory

If the equation of the current passing through the galvanometer is $i = I_m \sin \omega_t$, then the equation of motion of the coil is :

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gi = GI_m \sin \omega t \quad \text{...(i)}$$

where *J*, *D* and *C* have the usual meaning and *G* is the deflection constant.

The complementary function of the solution represents the transient motion, which in the case of vibration galvanometers, is of no practical importance. The particular integral is of the form

$$\theta = A \sin (\omega t - \phi)$$

where *A* and ϕ are constant.

Now, $d\theta/dt = \omega A \cos(\omega t - \phi)$ and $d^2\theta/dt^2 = -\omega^2 A \sin(\omega t - \theta)$. Substituting these values in Eq. (*i*) above, we get

$$\omega^2 JA \sin(\omega t - \phi) + \omega DA \cos(\omega t - \phi) + CA \sin(\omega t - \phi) = GI_m \sin \omega t$$

It must be true for all values or *i*

When $\omega t = \phi$ $\quad DA\,\omega = G\,I_m \sin\phi$ ...(*ii*)

When $(wt - \phi) = \pi/2$ $\quad \omega^2 JA + CA = G\,I_m \cos\phi$ ...(*iii*)

Since the phase angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (*ii*) and (*iii*).

Since the phase angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. (*ii*) and (*iii*).

$$\therefore \quad \omega^2 D^2 A^2 + A^2 (C - \omega^2 J)^2 = G^2 I_m^2 \quad \text{or} \quad A = \frac{G\,I_m}{\sqrt{[D^2\omega^2 + (C - \omega^2 J)^2]}} \quad \text{...(iv)}$$

This represents the amplitude *A* of the resulting oscillation for a sinusoidally alternating current of peak value I_m flowing through the moving coil of the galvanometer.

10.54. The Vibrating-reed Frequency Meter

1. Working Principle

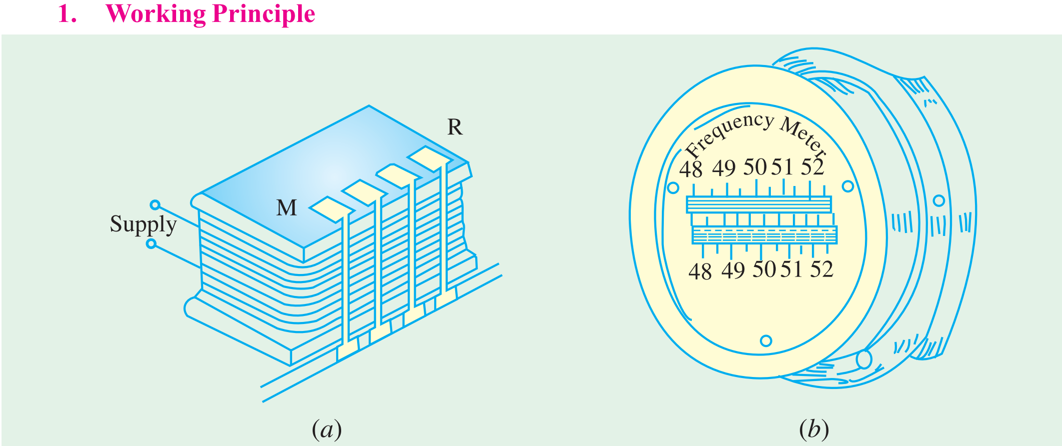


Fig. 10.68

The meter depends for its indication on the mechanical resonance of thin flat steel reeds arranged alongside and, close to, an electromagnet as shown in Fig. 10.68.

2. Construction

The electromagnet has a laminated armature and its winding, in series with a resistance, is connected across a.c. supply whose frequency is required. In that respect, the external connection of this meter is the same as that of a voltmeter.

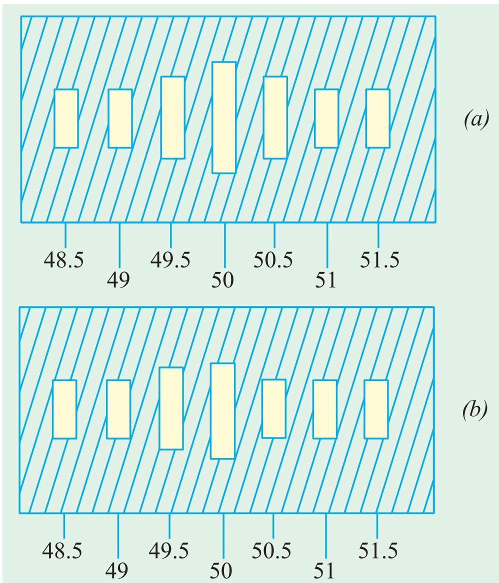


Fig. 10.69

The metallic reeds (about 4 mm wide and 0.5 mm thick) are arranged in a row and are mounted side by side on a common and slightly flexible base which also carries the armature of the electromagnet. The upper free ends of the reeds are bent over a right angles so as to serves as flags or targets and enamelled white for better visibility. The successive reeds are not exactly similar, their natural frequencies of vibration differing by $\frac{1}{2}$ cycle. The reeds are arranged in ascending order of natural frequency.

3. Working

When the electromagnet is connected across the supply whose frequency is to be measured, its magnetism alternates with the same frequency. Hence the electromagnet exerts attracting force on each reed once every half cycle. All reeds tend to vibrate but only that whose natural frequency is exactly double the supply frequency vibrates with maximum amplitude due to mechanical resonance [Fig. 10.69 (*a*)]. The supply frequency is read directly by noting the scale mark opposite the white painted flag which is vibrating the most (f = 50 Hz). The vibrations of other reeds would be so small as to be almost unobservable. For a frequency exactly midway between the natural frequencies of the two reeds (f = 49.75 Hz), both will vibrate with amplitudes which are equal but much less than when the supply frequency exactly coincides with that of the reeds.

4. Range

Such meters have a small range usually from 47 to 53 Hz or from 57 to 63 Hz etc.

The frequency range of a given set of reeds may be doubled by polarising the electromagnets as explained below. As seen from above description, each reeds is attracted twice per cycle of the supply *i.e.*, once every half-cycle and the reeds whose natural frequency is twice that of the current is of the one which responds most. Suppose the electromagnet carries an additional winding carrying direct current whose steady flux is equal in magnitude to the alternating flux of the a.c. winding. The resultant flux would be zero in one half-cycle and double in the other half-cycle when the two fluxes reinforce each other so that the reeds would receive one impulse per cycle. Obviously, a reed will indicate the frequency of the supply if the electromagnet is polarised and half the supply frequency if it is unpolarised. The polarisation may be achieved by using an extra d.c. winding on the electromagnet or by using a permanent tangent which is then wound with an a.c. winding.

5. Advantages

One great advantage of this reed-type meter is that its indications are independent of the waveform of the applied voltage and of the magnitude of the voltage, except that the voltage should be high enough to provide sufficient amplitude for reed vibration so as to make its readings reliable.

However, its limitations are :

(*a*) it cannot read closer than half the frequency difference between adjacent reeds.

(*b*) its error is dependent upon the accuracy with which reeds can be turned to a given frequency.

10.55. Electrodynamic Frequency Meter

It is also referred to as moving-coil frequency meter and is a ratiometer type of instrument.

1. Working Principle

The working principle may be understood from Fig. 10.70 which shows two moving coils rigidly fixed together with their planes at right angles to each other and mounted on the shaft or spindle situated in the field of a permanent magnet. There is no mechanical control torque acting on the two coils. If G_1 and G_2 are displacement constants of the two coils and I_1 and I_2 are the two currents, then their respective torques are $T_1 = G_1 I_1 \cos\theta$, $T_2 = G_2 I_2 \sin\theta$. These torques act in the opposite directions.

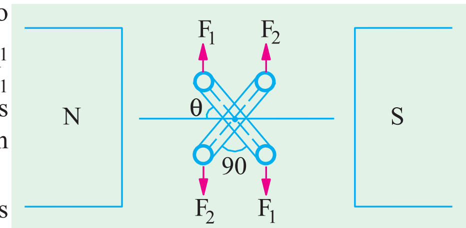


Fig. 10.70

Obviously, T_1 decreases with θ where as T_2 increases but an equilibrium position is possible for same angle θ for which

$$G_1 I_1 \cos\theta = G_2 I_2 \sin\theta \text{ or } \tan\theta = \frac{G_1}{G_2} \cdot \frac{I_1}{I_2}$$

By modifying the shape of pole faces and the angle between the planes of the two coils, the ratio I_1/I_2 is made proportional to angle θ instead of tan θ. In that case, for equilibrium $\theta \propto I_1/I_2$

2. Construction

The circuit connections are shown in Fig. 10.71. The two ratiometer coils *X* and *Y* are connected across the supply lines through their respective bridge rectifiers. The direct current I_1 through

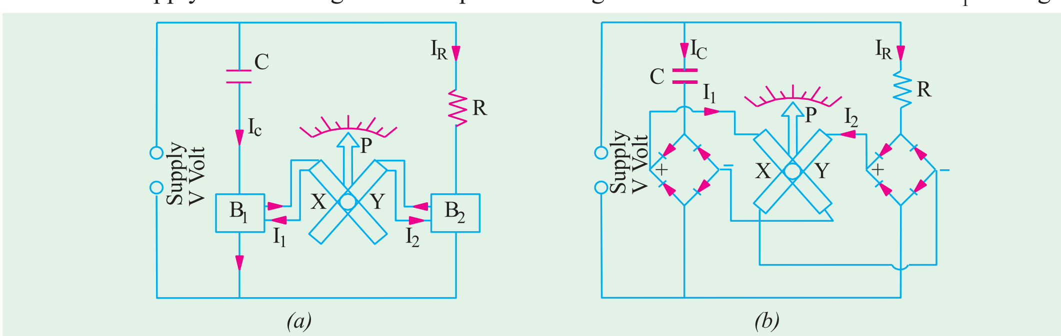


Fig. 10.71

coil *X* represents the R.M.S. value of capacitor current I_C as rectified by B_1. Similarly, direct current I_2 flowing through *Y* is the rectified current I_R passing through series resistance *R*.

3. Working

When the meter is connected across supply lines, rectified currents I_1 and I_2 pass through coils *X* and *Y* they come to rest at an angular position where their torques are equal but opposite. This angular position is dependent on the supply frequency which is read by a pointer attached to the coil.

As proved above, $\theta \propto I_1/I_2$

Assuming sinusoidal waveform, mean values of I_1 and I_2 are proportional to the R.M.S. values of I_C and I_R respectively.

$$\therefore \quad \theta \propto \frac{I_1}{I_2} \propto \frac{I_C}{I_R} \quad \text{Also } I_C \propto V_m\, \omega C \text{ and } I_R \propto V_m/R$$

where V_m is the maximum value of the supply voltage whose equation is assumed as $v = V_m \sin \omega t$.

$$\therefore \quad \theta \propto \frac{V_m\, \omega C}{V_m / R} \propto \omega CR \propto \omega \quad \therefore\ \theta \propto f \qquad (\because\ \omega = 2\pi f)$$

Obviously, such meters have linear frequency scales. Moreover, since their readings are independent of voltage, they can be used over a fairly wide range of voltage although at too low voltages, the distortions introduced by rectifier prevent an accurate indication of frequency.

It will be seen that the range of frequency covered by the meter depends on the value of *R* and *C* and these may be chosen to get ranges of 40–60 Hz, 1200–2000 Hz or 8000–12,000 Hz.

10.56. Moving-iron Frequency Meter

1. Working Principle

The action of this meter depends on the variation in current drawn by two parallel circuits – one inductive and the other non-inductive—when the frequency changes.

2. Construction

The construction and internal connections are shown in Fig. 10.72. The two coils *A* and *B* are so fixed that their magnetic axes are perpendicular to each other. At their centres is pivoted a long and thin soft-iron needle which aligns itself along the resultant magnetic field of the two coils. There is no control device used in the instrument.

It will be noted that the various circuit elements constitute a Wheatstone bridge which becomes balanced at the supply frequency. Coil *A* has a resistance R_A in series with it and a reactance L_A in parallel. Similarly R_B is in series with coil *B* and L_B is in parallel. The series inductance *L* helps to suppress higher harmonics in the current waveform and hence, tends to minimize the waveform errors in the indication of the instrument.

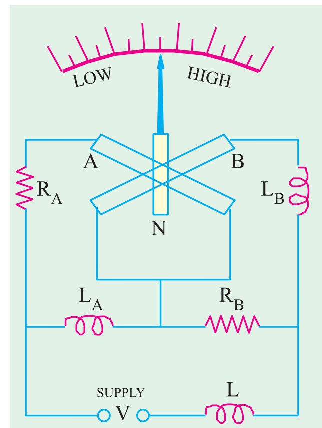


Fig. 10.72

3. Working

On connecting the instrument across the supply, currents pass through coils *A* and *B* and produce opposing torques. When supply frequency is high, currents through coil *A* is more whereas that through coil *B* is less due to the increase in the reactance offered by L_B. Hence, magnetic field of coil *A* is stronger than that of coil *B*. Consequently, the iron needle lies more nearly to the magnetic axis of coil *A* than that of *B*. For low frequencies, coil *B* draws more current than coil *A* and, hence, the needle lies more nearly parallel to the magnetic axis of *B* than to that of coil *A*. The variations of frequency are followed by the needle as explained above.

The instrument can be designed to cover a broad or narrow range of frequencies determined by the parameters of the circuit.

10.57. Electrodynamic Power Factor Meter

1. Working Principle

The instrument is based on the dynamometer principle with spring control removed.

2. Construction

As shown in Fig. 10.73 and 10.74, the instrument has a stationary coil which is divided into two sections F_1 and F_2. Being connected in series with the supply line, it carries the load current. Obviously, the uniform field produced by F_1 and F_2 is proportional to the line current. In this field are situated two moving coils C_1 and C_2 rigidly attached to each other and mounted on the same shaft or spindle. The two moving coils are 'voltage' coils but C_1 has a series resistance *R* whereas C_2 has

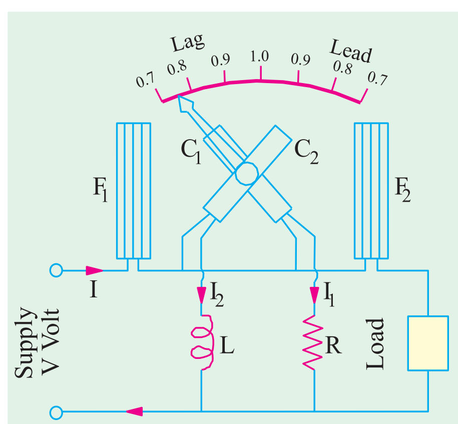


Fig. 10.73

a series inductance L. The values of R and L as well as turns on C_1 and C_2 are so adjusted that the ampere-turns of C_1 and C_2 are exactly equal. However, I_1 is in phase with the supply voltage V whereas I_2 lags behind V by nearly 90º. As mentioned earlier, there is no control torque acting on C_1 and C_2 – the currents being led into them by fine ligaments which exerts no control torque.

3. Working

Consider the case when load power factor is unity *i.e.* I is in phase with V. Then I_1 is in phase with I whereas I_2 lags behind by 90°. Consequently, a torque will act on C_1 which will set its plane perpendicular to the common magnetic axis of coils F_1 and F_2 *i.e.* corresponding to the pointer position of unity p.f. However, there will be no torque acting on coil C_2.

Now, consider the case when load power factor is zero *i.e.* I lags behind V by 90° (like current I_2). In that case, I_2 will be in phase with I where as I_1 will be 90° out of phase. As a result, there will be no toruqe on C_1 but that acting on C_2 will bring its plane perpendicular to the common magnetic axis of F_1 and F_2. For intermediate values of power factor, the deflection of the pointer corresponds to the load power factor angle ϕ or to cos ϕ, if the instrument has been calibrated to read to power factor direcly.

For reliable readings, the instrument has to be calibrated at the frequency of the supply on which it is to be used. At any other frequency (or when harmonics are present), the reactance of L will change so that the magnitude and phase of current through C_2 will be incorrect and that will lead to serious errors in the instrument readings.

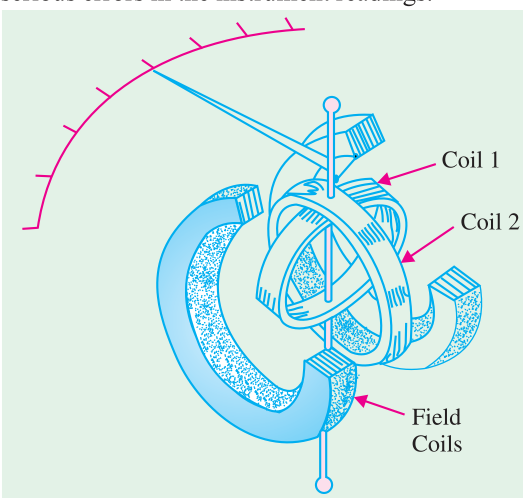


Fig. 10.74

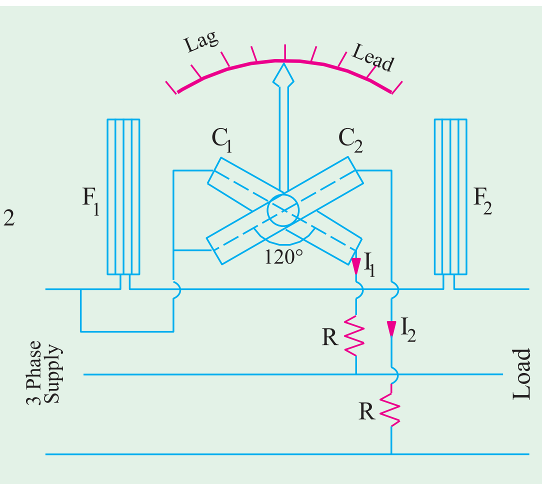


Fig. 10.75

For use on balanced 3-phase load, the instrument is modified, so as to have C_1 and C_2 at 120° to each other, instead of 90°, as in 1-phase supply. As shown in Fig. 10.75, C_1 and C_2 are connected across two different phase of the supply circuit, the stationary coils F_1 and F_2 being connected in series with the third phase (so that it carries the line current). Since there is no need of phase splitting between the currents of C_1 and C_2, I_1 and I_2 are not determined by the phase-splitting circuit and consequently, the instrument is not affected by variations in frequency or waveform.

10.58. Moving-iron Power Factor Meter

1. Construction

One type of power factor meter suitable for 3-phase balanced circuits is shown in Fig. 10.76. It consists of three fixed coils R, Y and B with axes mutually at 120° and intersecting on the centre line of the instrument. These coils are connected respectively in R, Y and B lines of the 3-phase supply through current transformers. When so energised, the three coils produce a synchronously rotating flux.

There is a fixed coil B at the centre of three fixed coils and is connected in series with a high resistance across one of the pair of lines, say, across R and Y lines as shown. Coil B is threaded by the instrument spindle which carries an iron cylinder C [Fig. 10.76 (*b*)] to which are fixed sector-shaped iron vanes V_1 and V_2. The same spindle also carries damping vanes and pointer (not shown in the figure) but ***there are no control springs.*** The moving system is shown separately in Fig. 10.76 (*b*).

2. Working

The alternating flux produced by coil B interacts with the fluxes produced by the three current coils and causes the moving system to take up a position determined by the power factor angle of the load. However, the instrument is calibrated to read the power factor $\cos \phi$ directly instead of ϕ. In other words, the angular deflection ϕ of the iron vanes from the line $M\,N$ in Fig. 10.76 (*a*) is equal to the phase angle ϕ.

Because of the rotating field produced by coils R, Y and B, there is a slight induction-motor action which tends to continuously turn the moving iron in the direction of the rotating flux. Hence, it becomes essential to design the moving iron as to make this torque negligibly small *i.e.* by using high-resistance metal for the moving iron in order to reduce eddy currents in it.

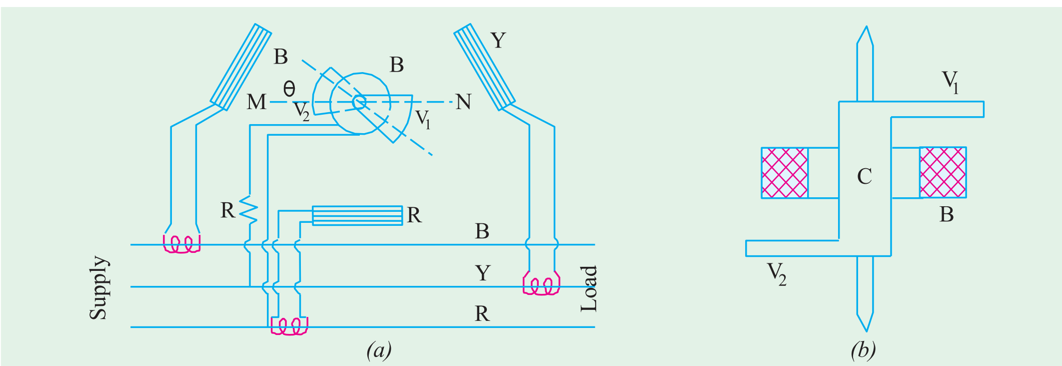


Fig. 10.76

3. Merits and Demerits

Moving iron p.f. meters are more commonly used as compared to the electrodynamic type because

(*i*) they are robust and comparatively cheap **(*ii*)** they have scales upto 360° and

(*ii*) in their case, all coils being fixed, there are no electrical connections to the moving parts.

On the other hand, they are not as accurate as the electrodynamic type of instruments and, moreover, suffer from erros introduced by the hysteresis and eddy-current losses in the iron parts–these losses varying with load and frequency.

10.59. Nalder-Lipman Moving-iron Power Factor Meter

1. Construction

The moving system of this instrument (Fig. 10.77) consists of three iron elements similar to the one shown in Fig. 10.76 (*b*). They are all mounted on a common shaft, one above the other, and are separated from one another by non-magnetic distance pieces D_1 and D_2. The three pairs of sectors are displaced in space by 120° relative to each other. Each iron vane is magnetised by one of the three voltage coils B_1, B_2 and B_3 which are connected (in series with a high resistance R) in star across the supply lines. The whole system is free to move in the space between two parallel halves F_1 and F_2 of a single current coil connected in one line of the supply. The common spindle also carries the damping vanes (not shown) and the pointer P.

2. Working

The angular position of the moving system is determined by the phase angle ϕ between the line current and the respective phase voltage. In other words, deflection θ is equal to ϕ, although, in practice, the instrument is calibrated to read the power factor directly.

3. Advantages

(i) Since no ***rotating*** magnetic field is produced, there is no tendency for the moving system to be dragged around continuously in one direction.

(ii) This instrument is not much affected by the type of variations of frequency, voltage and waveform as might be expected in an ordinary supply.

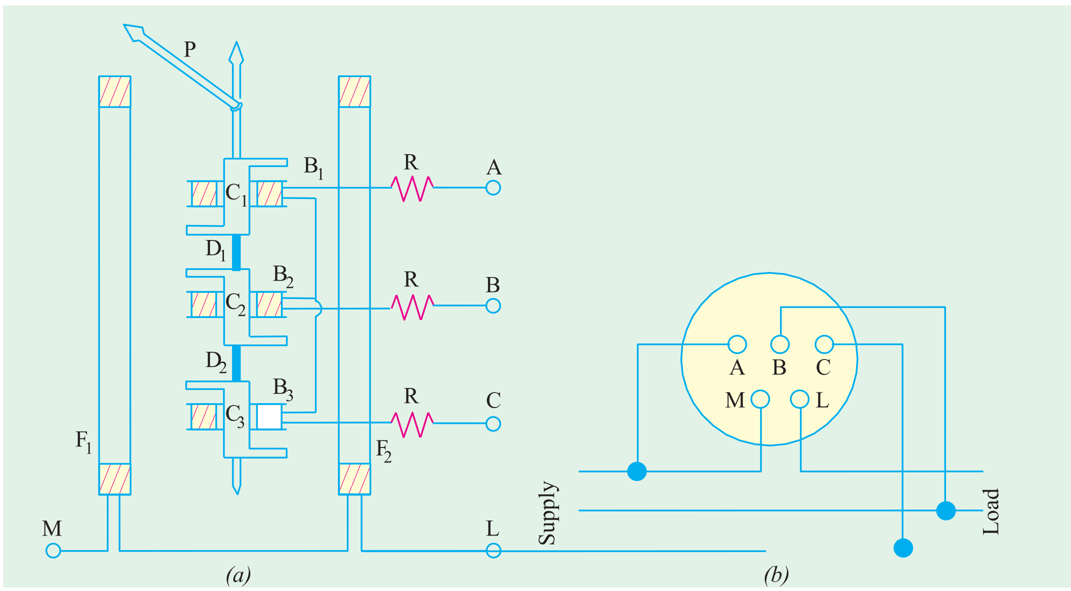


Fig. 10.77

10.60. D.C. Potentiometer

DC potentiometer

A potentiometer is used for measuring and comparing the e.m.fs. of different cells and for calibrating and standardizing voltmeters, ammeters etc. In its simplest from, it consists of a German silver or manganin wire usually one meter long and stretched between two terminals as shown in Fig. 10.78.

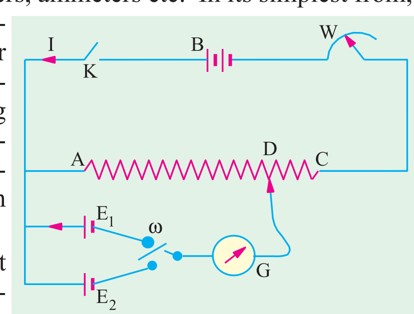


Fig. 10.78

This wire is connected in series with a suitable rheostat and battery *B* which sends a steady current through the resistance wire *AC*. As the wire is of uniform cross-section throughout, the fall in potential across it is uniform and the drop between any two points is proportional to the distance between them. As seen, the battery voltage is spread over the rheostat and the resistance wire *AC*. As we go along *AC,* there is a progressive fall of potential. If ρ is the resistance/cm of this wire, *L* its length, then for a current of *I* amperes, the fall of potential over the whole length of the wire is $\rho\ LI$ volts.

The two cells whose e.m.fs are to be compared are joined as shown in Fig. 10.78, always remembering that ***positive terminals of the cells and the battery must be joined together.*** The cells can be joined with the galvanometer in turn through a two-way key. The other end of the galvanometer is connected to a movable contact on *AC*. By this movable contact, a point like *D* is found when there is no current in and hence no deflection of *G*. Then, it means that the e.m.f. of the cell just balances the potential fall on *AD* due to the battery current passing through it.

Suppose that the balance or null point for first cell of e.m.f. E_1 occurs at a length L_1 as measured from point *A*. The $E_1 = \rho\ L_1 I$.

Similarly, if the balance point is at L_2 for th other cell, then $E_2 = \rho\ L_2 I$.

Dividing one equation by the other, we have $\dfrac{E_1}{E_2} = \dfrac{\rho L_1 I}{\rho L_2 I} = \dfrac{L_1}{L_2}$

If one of the cells is a standard cell, the e.m.f. of the other cell can be found.

10.61. Direct-reading Potentiometer

The simple potentiometer described above is used for educational purposes only. But in it commercial form, it is so calibrated that the readings of the potentiometer give the voltage directly, thereby eliminating tedious arithmetical calculations and so saving appreciable time.

Such a direct-reading potentiometer is shown in Fig. 10.79. The resistance *R* consists of 14 equal resistances joined in series, the resistance of each unit being equal to that of the whole slide were *S* (which is divided into 100 equal parts). The battery current is controlled by slide wire resistance *W*.

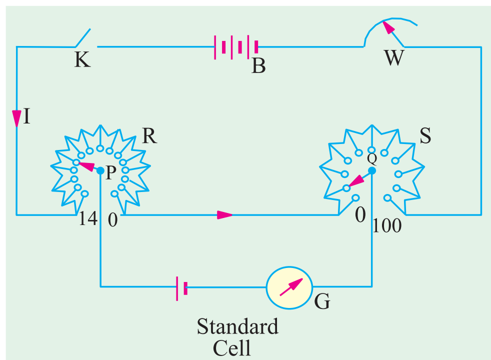


Fig. 10.79

10.62. Standardizing the Potentiometer

A standard cell *i.e.* Weston cadmium cell of e.m.f. 1.0183 V is connected to sliding contacts *P* and *Q* through a sensitive galvanometer *G*. First, *P* is put on stud No. 10 and *Q* on 18.3 division on *S* and then *W* is adjusted for zero deflection on *G*. In that case, potential difference between *P* and *Q* is equal to cell voltage *i.e.* 1.0183 V so that potential drop on each resistance of *R* is 1/10 = 0.1 V and every division of *S* represents 0.1/100 = 0.001 V. After standardizing this way, the ***position of W is not to be changed in any case*** otherwise the whole adjustment would go wrong. After this, the instrument becomes direct reading. Suppose in a subsequent experiment, for balance, *P* is moved to stud No. 7 and *Q* to 84 division, then voltage would be

$= (7 \times 0.1) + (84 \times 0.001) = 0.784$ V.

It should be noted that since most potentiometers have fourteen steps on *R*, it is usually not possible to measure p.ds. exceeding 1.5 V. For measuring higher voltges, it is necessary to use a volt box.

10.63. Calibration of Ammeters

The ammeter to be calibrated is connected in series with a variable resistance and a standard resistance *F*, say, of 0.1 Ω across battery B_1 of ample current capacity as shown in Fig. 10.80. Obviously, the resistance of *F* should be such that with maximum current flowing through the ammeter *A*, the potential drop across *F* should not exceed 1.5 V. Some convenient current, say

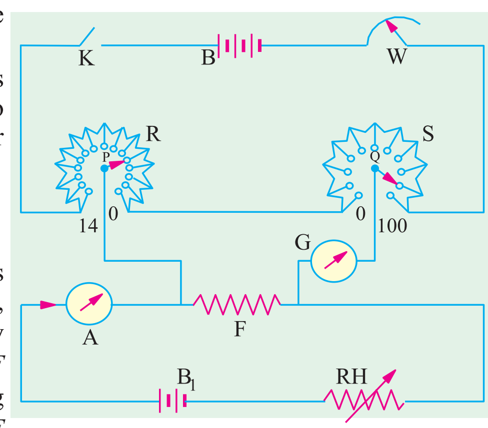


Fig. 10.80

6 amperes (as indicated by *A*) is passed through the circuit by adjusting the rheostat *RH*.

The potential drop across *F* is applied between *P* and *Q* as shown. Next the sliding contacts *P* and *Q* are adjusted for zero deflection on *G*. Suppose *P* reads 5 and *Q* reads 86.7. Then it means that p.d. across *F* is 0.5867 V and since *F* is of 0.1 Ω, hence ***true*** value of current through *F* is 0.5867/0.1 = 5.867 amperes. Hence, the ammeter reads high by (6 – 5.867) = 0.133 A. The test is repeated for various values of current over the entire ranges of the ammeter.

10.64. Calibration of Voltmeters

As pointed out in Art. 10.62, a voltage higher than 1.5 cannot be measured by the potentiometer directly, the limit being set by the standard cell and the type of the potentiometer (since it has only 14 resistances on *R* as in Fig. 10.79). However, with the help of a volt-box which is nothing else but a voltage reducer, measurements of voltage up to 150 V or 300 V can be made, the upper limit of voltage depending on the design of the volt-box.

The diagram of connections for calibration of voltmeters is shown in Fig. 10.81. By calibration is meant the determination of the extent of error in the reading of the voltmeter throughout its range. A high value resistor *AB* is connected across the supply terminals of high voltage battery B_1 so that it acts as a voltage divider. The volt-box consists of a high resistance *CD* with tapings at accurately determined points like *E* and *F* etc. The resistance *CD* is usually 15,000 to 300 Ω The two tappings *E* and *F* are such that the resistances of portions *CE* and *CF* are 1/100th and 1/10th the resistance *CD*. Obviously, whatever the potential drop across *CD*, the corresponding potential drop across *CE* is 1/100th and that across *CF*, 1/10th of that across *CD*.

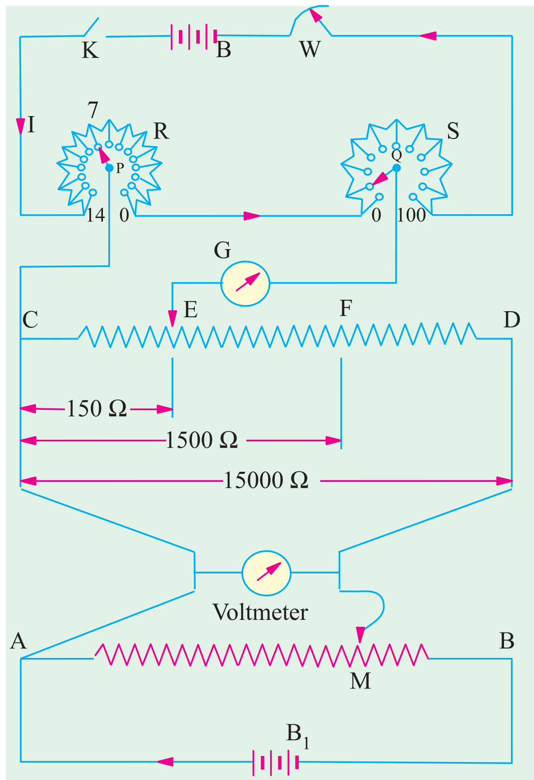


Fig. 10.81

If supply voltage is 150 V, then p.d. across *AB* is also 150 V and if *M* coincides with *B*, then p.d. across *CD* is also 150 V, so across *CF* is 15 volts and across *CE* is 1.5 V. Then p.d. across *CE* can be balanced over the potentiometer as shown in Fig. 10.81. Various voltages can be applied across the voltmeter by moving the contact point *M* on the resistance *AB*.

Suppose that *M* is so placed by voltmeter *V* reads 70 *V* and p.d. across *CE* is balanced by adjusting *P* and *Q*. If the readings on *P* and *Q* to give balance are 7 and 8.4 respectively, then p.d. across *CE* is 0.7084 V.

Hence, the true p.d. across *AM* or *CD* or voltmeter is 0.7048 × 100 = 70.84 V (because resistance of *CD* is 100 times greater than that of *CE*). In other words, the reading of the voltmeter is low by 0.84 V.

By shifting the position of *M* and then balancing the p.d. across *CE* on the potentiometer, the voltmeter can be calibrated throughout its range. By plotting the errors on a graph, a calibration curve of the instrument can also be drawn.

10.65. A.C. Potentiometer

An A.C. potentiometer basically works on the same principle as a d.c. potentiometer. However, there is one very important difference between the two. In d.c. potentiometer, only the ***magnitudes*** of the unknown e.m.f. and slide-wire voltage drop are made equal for obtaining balance. But in an a.c. potentiometer, not only the magnitudes but *phases* as well have to be equal for obtaining balance. Moreover, to avoid frequency and waveform errors, the a.c. supply for slide-wire must be taken from same source as the voltage or current to be measured.

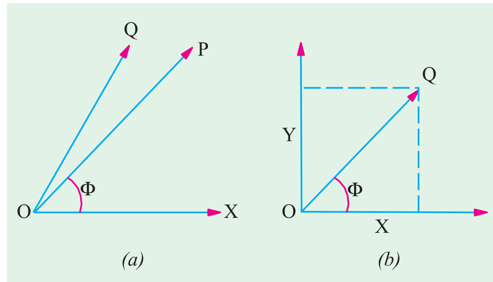


Fig. 10.82

A.C. potentiometers are of two general types differing in the manner in which the value of the unknown voltage is presented by the instrument dials or scales. The two types are :

(i) Polar potentiometers in which the unknown voltage is mesured in polar form *i.e.* in terms of magnitude and relative phase.

(ii) Co-ordinate potentiometers which measure the rectangular co-ordinates of the voltage under test.

The two products are illustrated in Fig. 10.82. In Fig. 10.82 (*a*), vector *OQ* denotes the test voltage whose magnitude and phase are to be imitated. In polar potentiometer, the length *r* of the vector *OP* can be varied with the help of a sliding contact on the slidewire while its phase ϕ is varied independently with the help of a phase-shifter. Drysdale potentiometer is of this type.

In co-ordinate type potentiometers, the unknown voltage vector *OQ* is copied by the adjustment of 'in phase' and 'quadrature' components *X* and *Y*. Their values are read from two scales of the potentiometer. The magnitude of the required vector is $= \sqrt{X^2 + Y^2}$ and its phase is given by $\phi = \tan^{-1}(X/Y)$. Examples of this type are ***(i)*** Gall potentiometer and ***(ii)*** Campbell-Larsen potentiometer.

10.66. Drysdale Polar Potentiometer

As shown in Fig. 10.83 for a.c. measurements, the slide-wire *MN* is supplied from a phase shifting circuit so arranged that magnitude of the voltage supplied by it remains constant while its

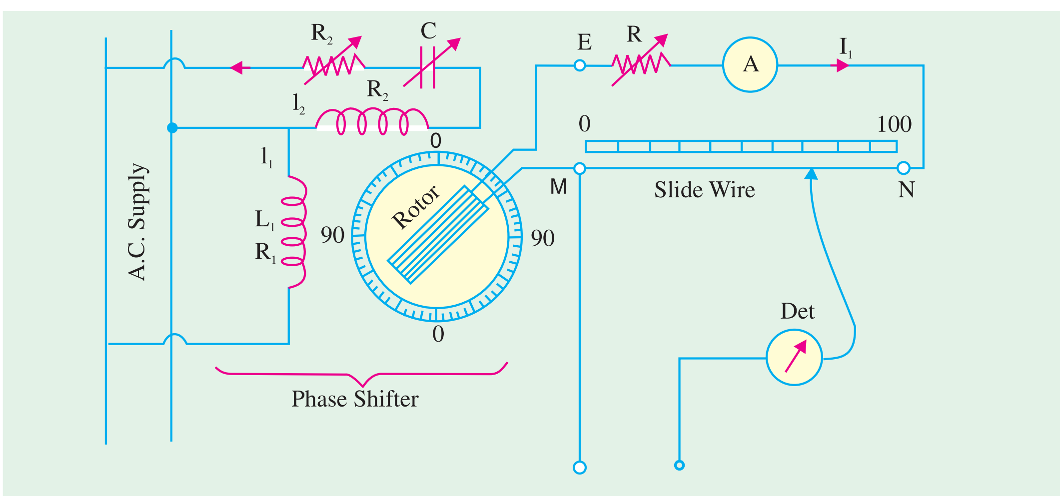


Fig. 10.83

phase can be varied through 360°. Consequently, slide-wire current *I* can be maintained constant in magnitude but varied in phase. The phase-shifting circuit consists of :

(*i*) Two stator coils supplied from the same source in parallel. Their currents I_1 and I_2 are made to differ by 90° by using well-known phase-splitting technique.

(*ii*) The two windings produce a rotating flux which induces a secondary e.m.f. in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position either manually or otherwise. The phase of the rotor e.m.f. is read from the circular graduated dial provided for the purpose.

The ammeter *A* in the slide-wire circuit is of electrodynamic of thermal type. Before using it for a.c. measurements, the potentiometer is first calibrated by using d.c. supply for slidwire and a standard cell for test terminals T_1 and T_2.

The unknown alternating voltage to be measured is applied across test terminals I_2 and T_2 balance is effected by the alternate adjustment of the slide-wire contact and the position of phase-shifting rotor. The slide-wire reading represents the magnitude of the test voltage phase-shifter reading gives its phase with reference to an arbitrary reference vector.

10.67. Gall Co-ordinate Potentiometer

This potentiometer uses two slide-wires *CD* and *MN* with their currents I_1 and I_2 (Fig. 10.84) having a mutual phase difference of 90°. The two currents are obtained from the single phase supply through isolating transformers, the circuit for 'quadrature' slidewire *MN* incorporating a phase shifting arrangement.

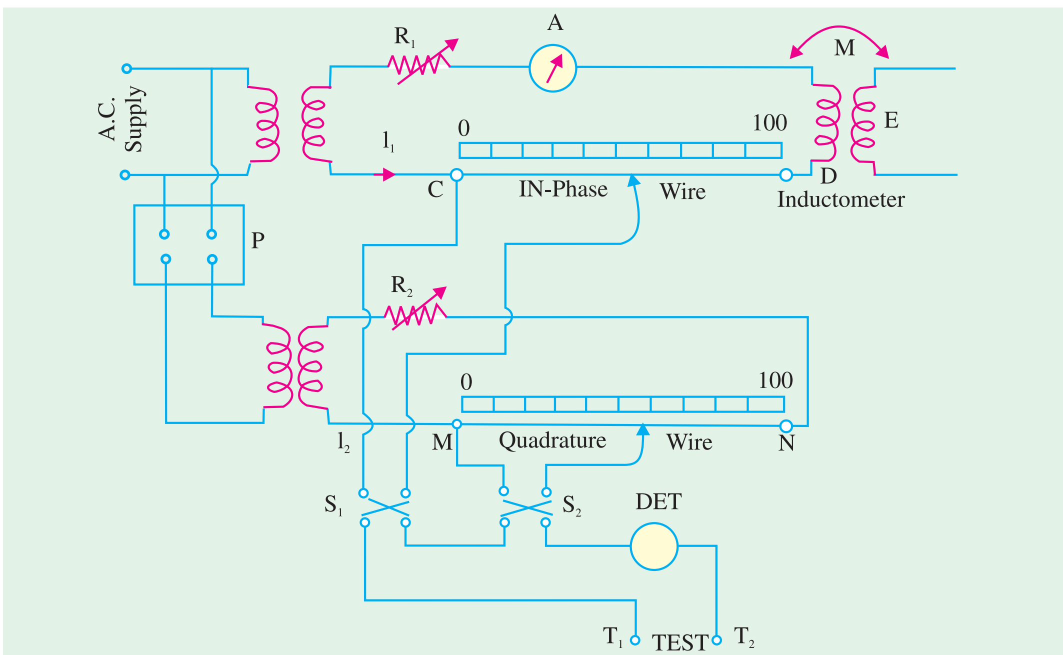


Fig. 10.84

Before use, then current I_1 is first standardised as described for Drysdale potentiometer (Art. 10.66). Next, current I_2 is standardised with the help of the mutually induced e.m.f. *E* in inductometer secondary. This e.m.f. $E = \omega M I_1$ and is in quadrature phase with I_1. Now, *E* is balanced against the voltage drop on slide-wire *MN*. This balance will be obtained only when I_2 is of correct magnitude and is in exact quadrature with I_1. Balance is achieved with the help of the phase-shifter and rheostat R_2.

The unknown voltage is applied across the test terminals T_1 and T_2. Slide-wire *MN* measures that component of the unknown voltage which is in phase with I_2. Similarly, slide-wire *CD* measures that component of the unknown voltage which is in phase with I_1. Since I_1 and I_2 are in quadrature, the two measured values are quadrature components of the unknown voltage. If V_1 and V_2 are these values, then

$$V = \sqrt{V_1^2 + V_2^2} \quad \text{and} \quad \phi = \tan^{-1}(V_2/V_1) \quad \text{—with respect to } I_1$$

Reversing switches S_1 and S_2 are used for measuring both positive and negative in-phase and quadrature components of the unknown-voltage.

10.68. Instrument Transformers

The d.c. circuits when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with high resistances connected in series with them. But it is neither convenient nor practical to use this method with alternating current and voltage instruments. For this purpose, specially constructed accurate-ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. Their purpose is to reduce the line current or supply voltage to a value small enough to be easily measured with meters of moderates size and capacity. In other words, they are used for extending the range of a.c. ammeters and voltmeters. Instruments transformers are of two types :

(*i*) current transformers (*CT*) —for measuring large alternating currents.

(*ii*) potential transformers (*VT*) —for measuring high alternating voltages.

Advantages of using instrument transformers for range extension of a.c. meters are as follows :

(1) the instrument is insulated from the line voltage, hence it can be grounded. **(2)** the cost of the instrument (or meter) together with the instrument transformer is less than that of the instrument alone if it were to be insulated for high voltages. **(3)** it is possible to achieve standardisation of instruments and meters at secondary ratings of 100–120 volts and 5 or 1 amperes **(4)** if necessary, several instruments can be operated from a single transformer and 5 power consumed in the measuring circuits is low.

In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage). These ratios give us the multiplying factor for finding the primary values from the instrument readings on the secondary side.

However, for energy or power measurements, it is essential to know not only the transformation ratio but also the phase angle between the primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

For range extension on a.c. circuits, instrument transformers are more desirable than shunts (for current) and multipliers (for voltage measurements) for the following reasons :

1. time constant of the shunt must closely match the time constant of the instrument. Hence, a different shunt is needed for each instrument.

2. range extension is limited by the current-carrying capacity of the shunt *i.e.* upto a few hundred amperes at the most.

3. if current is at high voltage, instrument insulation becomes a very difficult problem.

4. use of multipliers above 1000 becomes almost impracticable.

5. insulation of multipliers against leakage current and reduction of their distributed capacitance becomes not only more difficult but expensive above a few thousand volts.

10.69. Ratio and Phase-angle Errors

For satisfactory and accurate performance, it is necessary that the ratio of transformation of the instrument transformer should be constant within close limits. However, in practice, it is found that neither current transformation ratio I_1/I_2 (in the case of current transformers) nor voltage transformation ratio V_1/V_2 (in the case of potential transformers) remains constant. The transformation ratio is found to depend on the exciting current as well as the current and the power factor of the secondary circuit. This fact leads to an error called ratio error of the transformer which depends on the working component of primary.

It is seen from Fig. 1.85 (*a*) that the phase angle between the primary and secondary currents is not exactly 180° but slightly less than this value. This difference angle β may be found by reversing

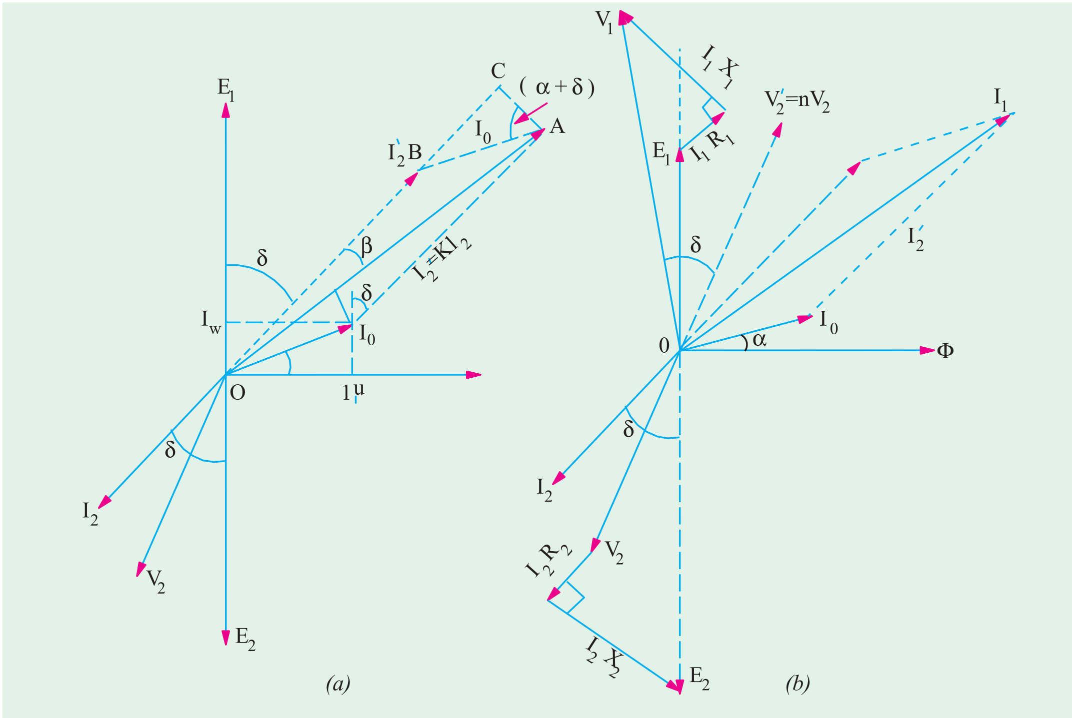


Fig. 10.85

vector I_2. The angular displacement between I_1 and I_2 reversed is called the phase angle error of the current transformer. This angle is reckoned positive if the reversed secondary current ***leads*** the primary current. However, on very low power factors, the phase angle may be negative. Similarly, there is an angle of γ between the primary voltage V_1 and secondary voltage reversed-this angle represents the phase angle error of a voltage transformer. In either case, the phase error depends on the magnetising component $I\mu$ of the primary current. It may be noted that ratio error is primarily due to the reason that the ***terminal*** voltage transformation ratio of a transformer is not exactly equal to its turn ratio. The divergence between the two depends on the resistance and reactance of the transformer windings as well as upon the value of the exciting current of the transformer. Accuracy of voltage ratio is of utmost importance in a voltage transformer although phase angle error does not matter if it is to be merely connected *to* ***a voltmeter.*** Phase-angle error becomes important only when voltage transformer supplies the voltage coil of a wattmeter *i.e.* in power measurement. In that case, phase angle error causes the wattmeter to indicate on a wrong power factor.

In the case of current transformers, constancy of current ratio is of paramount importance. Again, phase angle error is of no significance if the current transformer is merely feeding an ammeter but it assumes importance when feeding the current coil of a wattmeter. While discussing errors, it is worthwhile to define the following terms :

(i) Nominal transformation ratio (k_n). It is the ratio of the ***rated*** primary to the ***rated*** secondary current (or voltage).

$$k_n = \frac{\text{rated primary current } (I_1)}{\text{rated secondary current } (I_2)} \quad \text{—for } CT$$

$$= \frac{\text{rated primary voltage } (V_1)}{\text{rated secondary voltage } (V_2)} \quad \text{—for } VT$$

In the case of current transformers, it may be stated either as a fraction such as 500/5 or 100/1 or simply as the number representing the numerator of the reduced fraction *i.e.* 100. It is also known as ***marked*** ratio.

(ii) Actual transformation ratio (k). The actual transformation ratio or just ratio under any given *condition of loading is*

$$k = \frac{\text{primary current } (I_1)}{\text{corresponding secondary current } (I_2)}$$

In general, k differs from k_n except in the case of an ideal or perfect transformer when $k = k_n$ for all conditions of loading.

(iii) Ratio Error (σ). In most measurements it may be assumed that $I_1 = k_n I_2$ but for very accurate work, it is necessary to correct for the difference between k and k_n. It can be done with the help of ratio error which is defined as

$$\sigma = \frac{k_n - k}{k} = \frac{\text{nominal ratio-actual ratio}}{\text{actual ratio}}$$

Also, $$\sigma = \frac{k_n \cdot I_2 - kI_2}{k \cdot I_2} = \frac{k_n \cdot I_2 - I_1}{I_1}$$

Accordingly, ratio error may be defined as the difference ***between the primary current reading (assuming the nominal ratio) and the true primary current divided by the true primary current.***

(iv) Ratio Correction Factor (R.C.F.). It is given by

$$R.C.F. = \frac{\text{actual ratio}}{\text{nominal ratio}} \quad \frac{k}{k_n}$$

10.70. Current Transformer

A current transformer takes the place of shunt in d.c. measurements and enables heavy alternating current to be measured with the help of a standard 5-A range a.c. ammeter.

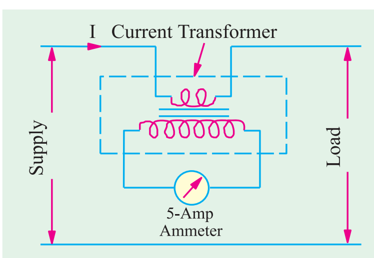


Fig. 10.86

As shown in Fig. 10.86, the current - or series-transformer has a primary winding of one or more turns of thick wire connected in series with the line carrying the current to be measured. The secondary consist of a large number of turns of fine wire and feeds a standard 5-A ammeter (Fig. 10.86) or the current coil of a watt-meter or watthour-meter (Fig. 10.87).

For example, a 1,000/5A current transformer with in single-turn primary will have 200 secondary turns. Obviously, it steps down the current in the 200 : 1 ratio whereas it steps up the voltage drop across the single-turn primary (an extremely small quantity) in the ratio 1 : 200. Hence if we know the current ratio of the transformer and the reading of the a.c. ammeter, the line current can be calculated.

It is worth noting that ammeter resistance being extremely low, a current transformer operates with its secondary under nearly short-circuit conditions. Should it be necessary to remove the ammeter of the current coils of the wattmeter or a relay, the seondary winding must, first of all, be short-circuited *before* the instrument is disconnected.

If it is not done then due to the absence of counter ampere-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating of and damage of the transformer insulation and a high voltage across the secondary

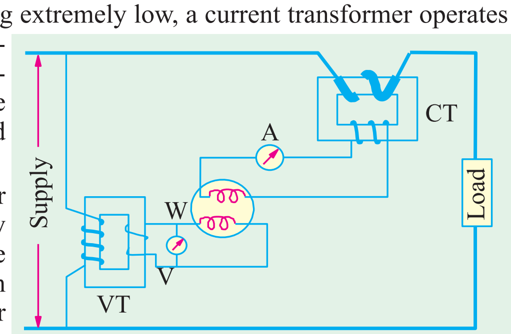


Fig. 10.87

terminals. This is not the case with the ordinary constant-potential transformers because their primary current is determined by the load on their secondary whereas in a current transformer, primary current is determined entirely by the load on the system and not by the load on its own secondary. Hence, the secondary of a current transformer should ***never be left open under any circumstances.***

10.71. Theory of Current Transformer

Fig. 10.85 (*b*) represents the general phase diagram for a current transformer. Current I_0 has been exaggerated for clarity.

(a) Ratio Error. For obtaining an expression for the ratio error, it will be assumed that the turn ratio n (= secondary turns, N_2/primary turns N_1) is made equal to the nominal current ratio *i.e.* $n = k_n$. In other words, it will be assumed that $I_1/I_2 = n$ although actually $n = I_1/I_2'$. As seen from Art. 10.63.

$$\sigma = \frac{n I_2 - I_1}{I_1} = \frac{I_2' - I_1}{I_1} = \frac{OB - OA}{OA} \qquad \text{— } [\because n = k_n]$$

$$\equiv \frac{OB - OC}{OA} \qquad (\because \beta \text{ is very small angle})$$

$$= -\frac{BC}{OA} = -\frac{AB \sin(\alpha + \delta)}{OA} = -\frac{I_0 \sin(\alpha + \delta)}{I_1} = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

For most instrument transformers, the power factor of the secondary burden is nearly unity so that δ is very small. Hence, very approximately.

$$\sigma = \frac{I_0 \sin \alpha}{I_1} - \frac{I_\omega}{I_1}$$

where I_ω is the iron-loss or working or wattful component of the exciting current I_0

Note. The transformation ratio R may be found from Fig. 10.85 (*a*) as under :

$I_1 = OA = OB + BC = nI_2 \cos \beta + I_0 \cos [90 - (\delta + \beta + \alpha)] = nI_2 \cos \beta + I_0 \sin (\delta + \beta + \alpha)$

Now $\beta = (\alpha + \delta)$ hence $I_1 = nI_2 + I_0 \sin (\alpha + \delta)$ where n is the turn ratio of the transformer.

$$\therefore \quad \text{ratio } R = \frac{I_1}{I_2} = \frac{nI_2 + I_0 \sin(\alpha + \delta)}{I_2} \text{ or } R = n + \frac{I_0 \sin(\alpha + \delta)}{I_2} \qquad \text{...}(i)$$

If δ is negligible small, then $R = n + \dfrac{I_0 \sin \alpha}{I_2} = n + \dfrac{I_\omega}{I_2}$

It is obvious from (*i*) above that ratio error can be eliminated if secondary turn are reduced by a number

$$= I_0 \sin(\alpha + \delta)/I_2$$

(*b*) Phase angle (β)

Again from Fig. 10.85 (*a*), we fine that

$$\beta \cong \sin \beta = \frac{AC}{OA} = \frac{AB \cos(\alpha + \delta)}{OA} = \frac{I_0 \cos(\alpha + \delta)}{I_1} = \frac{I_0 \cos(\alpha + \delta)}{nI_2}$$

Again, if the secondary power factor is nearly unity, then δ is very small, hence

$$\beta \cong \frac{I_0 \cos \alpha}{I_1} = \frac{I_\mu}{I_1} \text{ or } \frac{I_\mu}{nI_2}$$

where I_μ is the magnetising component of the exciting current I_0.

$$\therefore \quad \beta = \frac{I_\mu}{I_1} \text{ —in radian; } = \frac{180}{\pi} \times \frac{I_\mu}{I_1} \qquad \text{—in degrees}$$

Note. As found above, $\beta = \dfrac{I_0 \cos(\quad)}{I_1} \quad \dfrac{I_0 (\cos \quad \cos \quad \sin \quad \sin \quad)}{I_1}$

$$= \frac{I_\mu \cos\delta - I_\omega \sin\delta}{I_1} = \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ radian}$$

$$\therefore \quad \beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ degrees.}$$

Dependence of ratio error on working component of I_0 and that of phase angle on the magnetising component is obvious. If R is to come closer to k and β is to become negligible small, then I_μ and I_ω and hence I_0 should be very small.

10.72. Clip-on Type Current Transformer

It has a laminated core which is so arranged that it can be opened out at a hinged section by merely pressing a trigger-like projection (Fig. 10.88). When the core is thus opened, it permits the admission of very heavy current-carrying bus-bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current-carrying conductor of feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle itself.

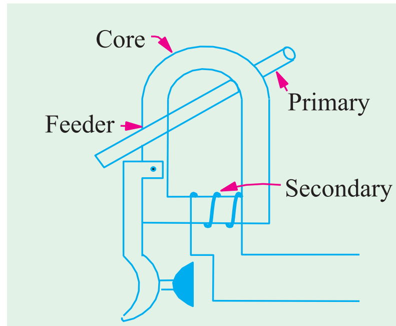


Fig. 10.88

10.73. Potential Transformers

These transformers are extremely accurate-ratio stepdown transformers and are used in conjunction with standard low-range voltmeters (100-120 V) whose deflection when divided by transformation ratio, gives the true voltage on the primary or high-voltage side. In general, they are of the shell type and do not differ much from the ordinary two-winding transformers except that their power rating is extremely small. Sine their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100W. For safety, the secondary is completely insulated from the high voltage primary and is, in addition, grounded for affording protection to the operator. Fig. 10.89 shows the connection of such a transformer.

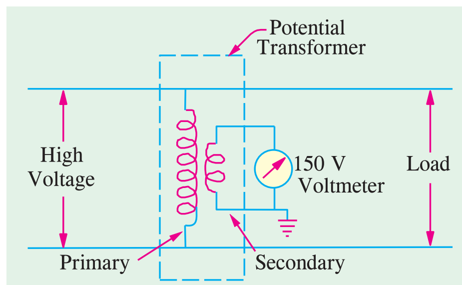


Fig. 10.89


Potential transformer

10.74. Ratio and Phase-angle Errors

In the case of a potential transformer, we are interested in the ratio of the primary to the secondary terminal voltage and in the phase angle γ between the primary and reversed secondary terminal voltage V_2'.

The general theory of voltage transformer is the same as for the power transformers except that, as the current in the secondary burden is very small, the total primary current I_1 is not much greater than I_0.

In the phasor diagram of Fig. 10.90, vectors AB, BC, CD and DE represent small voltage drops due to resistances and reactances of the transformer winding (they have been exaggerated for the sake of clarity). Since the drops as well as the phase angle γ are small, the top portion of diagram 10.90 (*a*) can be drawn with negligible loss of accuracy as in Fig. 10.90 (*b*) where V_2' vector has been drawn parallel to the vector for V_1.

In these diagrams, V_2' is the secondary terminal voltage as referred to primary assuming transformation without voltage drops. All actual voltage drops have been referred to the primary. Vector *AB* represents total resistive drop as referred to primary *i.e.* $I_2' \ R_{01}$. Similarly, *BC* represents total reactive drop as referred to primary *i.e.* $I_2' \ X_{01}$.

In a voltage transformer, the relatively large no-load current produces appreciable resistive drops which have been represented by vectors *CD* and *DE* respectively. Their values are I_0R_1 and I_0X_1 respectively.

(*a*) Ratio Error

In the following theory, *n* would be taken to represent the ratio of ***primary turns to secondary turns*** (Art. 10.69). Further, it would be assumed, as before, that *n* equals the nominal transformation ratio *i.e.* $n = k_n$.

In other words, it would be assumed that $V_1/V_2 = n$, although, actually, $V_1/V_2' = n$.

Then $$\sigma = \frac{k_n - k}{k} = \frac{k_n . V_2 - k V_2}{kV_2} = \frac{V_2' - V_1}{V_1} = -\frac{EN}{OE} \quad \text{...Fig. 10.90 } (a)$$

$$= -\frac{AG + FC + LD + EM}{OE} \quad \text{...Fig. 10.90 } (b)$$

$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_0R_1 \sin\alpha + I_0X_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_\mu R_1 + I_\mu X_1}{V_1}$$

where I_ω and I_μ are the iron-loss and magnetising components of the no-load primary current I_0.

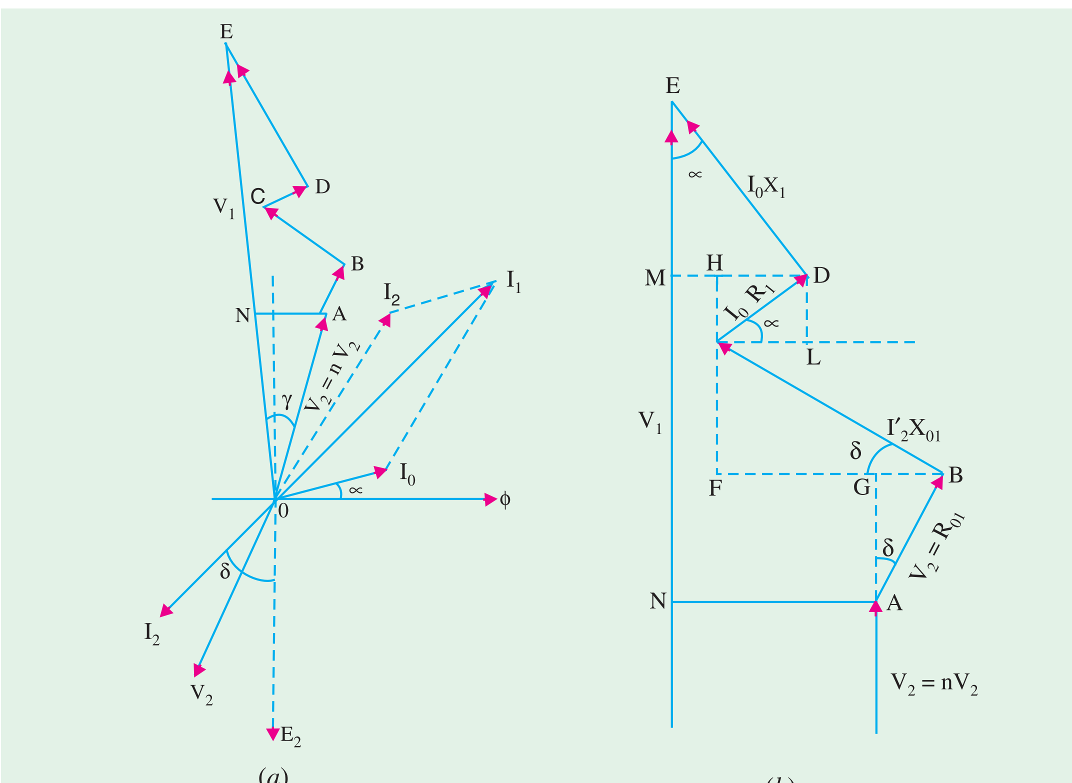


Fig. 10.90

(*b*) Phase Angle (γ)

To a very close approximation, value of γ is given by $\gamma = AN/OA$ —in radian

Now, $OA \cong OE$ provided ratio error is neglected. In that case,

$$\gamma = \frac{AN}{OE} = -\frac{GF + HM}{OE} \quad \text{...Fig. 10.90 }(b)$$

$$= -\frac{(BE - BG) + (DM - DH)}{OE}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_0 X_1 \sin\alpha - I_0 R_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_\omega X_1 - I_\mu R_1}{V_1}$$

The negative sign has been given because reversed secondary voltage *i.e.* V_2' lags behind V_1.

Example 10.44. *A current transformer with 5 primary turns and a nominal ratio of 1000/5 is operating with a total secondary impedance of 0.4 + j 0.3 Ω. At rated load, the iron loss and magnetising components of no-load primary current are 1.5 A and 6 A respectively. Calculate the ratio error and phase angle at rated primary current if the secondary has* **(a)** *1000 turns and* **(b)** *990 turns.*

Solution. Phasor diagram of Fig. 10.87 may please be referred to.

(*a*) $\tan\delta = 0.3/0.4$ or $\delta = \tan^{-1}(0.3/0.4) = 36°52'$

$\alpha = \tan^{-1}(1.5/6) = 14°2'$ $\quad \therefore (\alpha + \beta) = 50°54'$

$$I_0 = \sqrt{I_\omega^2 + I_\mu^2} = \sqrt{1.5^2 + 6^2} = 6.186 \text{ A}$$

$BC = I_0 \sin(\alpha + \beta) = 6.186 \times \sin 50°54' = 4.8$ A

Since β is amall, $OC \cong OA = 1000$ A $\quad \therefore I_2' = OC - BC = 1000 - 4.8 = 995.2$ A

$\therefore \quad \sigma = \dfrac{k_n I_2 - I_1}{I_1} = \dfrac{nI_2 - I_1}{I_1}$ because in this case $n (= N_2/N_1 = 1000/5)$ is equal to nominal ratio k_n (= 1000/5 A).

or $\quad \sigma = \dfrac{I_2 \quad I_1}{I_1} \quad \dfrac{995.2 \quad 1000}{1000} \quad 0.0048$ or **48%**

$\beta = \tan^{-1}(AC/OC)$

Now $\quad AC = I_0 \cos(\alpha + \beta) = 6.186 \cos 50°54' = 3.9$ A

centre $\quad \beta = \tan^{-1}(AC/OC) = \tan^{-1}(3.9/1000) =$ **0°13′**

(*b*) In this case, $\quad I_2 = 995.2 \times \dfrac{5}{990}; \; k_n . I_2 = \dfrac{1000}{5} \times 995.2 \times \dfrac{5}{990} = 1005.3$ A

$\therefore \quad \sigma = \dfrac{k_n I_2 \quad I_1}{I_1} \quad \dfrac{1005.3 \quad 1000}{1000} \quad 0.0053$ or **0.53%**

The value of phase angle β would not be significantly different from the value obtained in (*a*) above.

Example 10.45. *A relay current-transformer has a bar primary and 200 secondary turns. The secondary burden is an ammeter of resistance 1.2 Ω and resistance of 0.5 Ω and the secondary winding has a resistance of 0.2 Ω and reactance of 0.3 Ω. The core requires the equivalent of 100 AT for magnetisation and 50 AT for core losses.*

(*i*) *Find the primary current and the ratio error when the secondary ammeter indicates 5.0 A.*

(*ii*) *By how many turns should the secondary winding be reduced to eliminate the ratio error for this condition ?* **(Electrical Measurements, Bombay Univ.)**

Solution. Total secondary impedance is

$$\mathbf{Z_2} = 1.4 + j\,0.8 = 1.612 \angle 29°45' \qquad \therefore\ \delta = 29°45'$$
$$\mathbf{I0} = 100 + j50 = 111.8 \angle 26°34' \qquad \therefore\ \alpha = 26°34'$$

Turn ratio, $n = 200/I = 200$; Transformation ratio $R = n + \dfrac{I_0 \sin(\alpha + \delta)}{I_2}$

$$\therefore \quad R = 200 + \frac{111.8 \sin 56°19'}{5} = 218.6$$

(*i*) Primary current $= 5 \times 218.6 =$ **1093 A**

Ratio error $\sigma = \dfrac{I_0 \sin(\quad)}{nI_2} \quad \dfrac{111.8 \quad 0.8321}{200 \quad 5}$ **0.093 or 9.3%**

(*ii*) No. of secondary turns to be reduced $= I_0 \sin(\alpha + \delta)/I_2 = 93/5 =$ **19 (approx).**

Example 10.46. *A current transformer has 3 primary turns and 300 secondary turns. The total impedance of the secondary is (0.583 + j 0.25) ohm. The secondary current is 5 A. The ampere-turns required to supply excitation and iron losses are respectively 10 and 5 per volt induced in the secondary.*

Determine the primary current and phase angle of the transformer.

(Elect Meas; M.S. Univ. Baroda)

Solution. Z2 $= 0.583 = j\,0.25 = 0.6343 \angle 23°\,10'$ $\therefore E_2 = I_2Z_2 = 5 \times 0.6343 =$ **3.17 V**

Now, there are 10 magnetising *AT* per *secondary* volt induced in secondary.

$\therefore$ total magnetising $AT = 3.17 \times 10 = 31.7$; Similarly, iron-loss $AT = 3.17 \times 5 = 15.85$

Remembering that there are 3 primary turns, the magnetising and iron-loss components of primary current are as under :

Magnetising current, $I_\mu = 31.7/3 = 10.6$ A; iron-loss current $I_\omega = 15.85/3 = 5.28$ A

$$I_0 = \sqrt{10.6^2 + 5.28^2} = 11.84 \text{ A}$$

Now, $R = n \quad \dfrac{I_0 \sin(\quad)}{I_2}$

Here, $n = 300/3 = 100$; $\alpha = \tan^{-1}(I_\omega/I_\mu) = \tan^{-1}(5.28/10.6)$

$$= \tan^{-1}(0.498) = 26°30'$$

δ = secondary load angle $= 23°10'$ —found earlier

$$\therefore \quad R = 100 + \frac{11.86}{5}(\sin 49°40') = 100 + 1.81 = 101.81$$

$$I_1 = R \times I_2 = 101.81 \times 5 = \textbf{509.05 A}$$

$$\beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2}$$

$$= \frac{180}{\quad} \quad \frac{10.6 \cos 23 \;\; 10 \qquad 5.28 \sin 23 \;\; 10}{100 \quad 5} \quad \textbf{0.88°}$$

Example 10.47. *A current transformer with a bar primary has 300 turns in its secondary winding. The resistance and reactance of the secondary circuit are 1.5 Ω and 1.0 Ω respectively including the transformer winding. With 5 A flowing in the secondary circuit the magnetising ampere-turns required are 100 and iron loss is 1.2 W. Determine the ratio error at this condition.*

(Elect. Measure, A.M.I.E. Sec. B, 1992)

Solution. Turn ratio $n = 300/1 = 300$

Secondary impedance is $Z_2 = 1.5 + j\,1.0 = 1.8 \angle 33°42'$

Secondary induced e.m.f. $E_2 = I_2 Z_2 = 5 \times 1.8 = 9$ V

$$E_1 = E_2/n = 9/300 = 0.03 \text{ V}$$

Let us now find the magnetising and working components of primary no-load current I_0

Magnetising AT = 100. Since threre is one primary turn, $\therefore I_\mu = 100/1 = 100$ A

Now, $E_1 I_\omega = 1.2 \quad \therefore \quad I_\omega = 1.2/0.03 = 40$ A

$$I_0 = 100 + j\,40 = 107.7 \angle 21°48' \; ; \; \sigma = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

Now $\alpha = 21°48'$ and $\delta = 33°\,42'$

$$\therefore \quad \sigma = \frac{107.7 \sin 55 \;\; 30}{300 \;\;\; 5} \quad \mathbf{0.0592 \;\; or \;\; 5.92\%}$$

Phase angle
$$\beta = \frac{I_0 \cos(\alpha + \delta)}{nI_2} = \frac{107.7 \times \cos 55°30'}{1500}$$

$$= \frac{180}{\ } \; \frac{107.7 \quad 0.5664}{1500} \quad \mathbf{2°20}$$

Tutorial Problems No. 10.4

1. A current transformer with 5 primary turns has a secondary burden consisting of a resistance of 0.16 Ω and an inductive reactance of 0.12Ω. When primary current is 200 A, the magnetising current is 1.5 A and the iron-loss component is 0.4 A. Determine the number of secondary turns needed to make the current ratio 100/1 and also the phase angle under these conditions. **[407 : 0.275°]**

2. A current transformer having a 1-turn primary is rated at 500/5 A, 50 Hz, with an output of 1.5 VA. At rated load with the non-inductive burden, the in-phase and quadrature components (referred to the flux) of the exciting ampere-turns are 8 and 10 respectively. The number of turns in the secondary is 98 and and the resistance and leakage reactance of the secondary winding are 0.35 Ω and 0.3 Ω respectively. Calculate the current ratio and the phase angle error. **[501.95/5; 0.533°]**
(Elect. Inst. and Meas, M.S. Univ. Baroda)

3. A ring-core current transformer with a nominal ratio of 500/5 and a bar primary has a secondary resistance of 0.5 Ω and negligible secondary reactance. The resultant of the magnetising and iron-losss components of the primary current associated with a full-load secondary current of 5 A in a burden of 1.0 Ω (non-inductive) is 3 A at a power factor of 0.4. Calculate the true ratio and the phase-angle error of the transformer on full-load. Calculate also the total flux in the core, assuming that frequency is 50 Hz. **[501.2/5; 0.314°; 337 μWb]**

4. A current transformer has a single-turn primary and a 200-turn secondary winding. The secondary supplies a current of 5 A to a non-inductive burden of 1 Ω resistance, the requisite flux is set up in the core by 80 AT. The frequency is 50 Hz and the net cross-section of the core is 10 cm^2. Calculate the ratio and phase angle and the flux density in the core.
[200.64; 4°35′ 0.079 Wb/m2] ***(Electrical Measurements, Osmania Univ.)***

5. A potential transformer, ratio 1000/100-V, has the following constants :

primary resistance = 94.5 Ω ; secondary resistance = 0.86 Ω
primary reactance = 66.2 Ω ; equivalent reactance = 66.2 Ω
magnetising current = 0.02 A at 0.4 p.f.

Calculate (*i*) the phase angle at no-load between primary and secondary voltages (*ii*) the load in VA at u.p.f. at which the phase angle would be zero. **[(i) 0°4′ (ii) 18.1 VA]**

6. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to
(*i*) 0.5 A range Ammeter (*ii*) 0.100 V range Voltmeter? ***(Nagpur University, Summer 2002)***

7. What are the essential torques of an indicating instruments? Justify their necessity.
(Nagpur University, Winter 2002)

8. Discuss the necessity of damping in Indicating instrument and explain eddy current damping.
(U.P. Technical University 2002) (Nagpur University, Summer 2003)

9. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to (*i*) 0-5 A range Ammeter (*ii*) 0-100 V range Voltmeter?
(Nagpur University, Summer 2003)

10. What are the essential requirements of indicating type instruments? Explain each of them. *(U.P. TechnicalUniversity 2002) (Nagpur University, Winter 2003)*

11. What are the different operating systems required in an instrument? Explain damping system in detail. *(U.P. TechnicalUniversity 2002) (Nagpur University, Summer 2004)*

12. If a shunt for a moving coil instrument is a have a multiplicationfactor m, shown that its shunt resistance is given by

$$R_{sh} = \frac{R_m}{m - 1}$$

where R_m →resistance of the meter. *(Nagpur University, Summer 2004)*

13. Find the value of a series resistance to be connected to a basic d' Arsonval movement with internal resistance R_m = 100 ohm and full scale deflection current is 1 mA, for conversion into 0–500 volt. *(Nagpur University, Summer 2004)*

14. What is the basic principle of Induction type instruments? *(Anna University, April 2002)*

15. What is damping torque in instruments? *(Anna University, April 2002)*

16. Why does MI instruments have non-linear scale? *(Anna University, April 2002)*

17. Explain how can you measure power by the Ammeters. *(Anna University, April 2002)*

18. What is creeping in energy meter? *(Anna University, April 2002)*

19. What is the advantage of Induction type meters? *(Anna University, April 2002)*

20. What is a trivector meter? *(Anna University, April 2002)*

21. What is the difference between PMMC instrument and ballistic galvanometer? *(Anna University, April 2002)*

22. What is the basic principle of operation of reed type frequency meters? *(Anna University, April 2002)*

23. Why can't you measure low resistance with meggar? *(Anna University, April 2002)*

24. What are the advantages of electronic meters? *(Anna University, April 2002)*

25. Explain the principle of operation, construction and the expression for deflection of a moving iron instrument. *(Anna University, April 2002)*

26. Draw and explain the working of 2 element wattmeter. Explain how does the 2 element wattmeter reads total power in a 3 ϕ circuit. *(Anna University, April 2002)*

27. Write the equations for wattmeter readings W_1 and W_2 in 3 phase power measurement and therefrom for power factor. *(Anna University, Wimter 2002)*

28. Explain how power can be measured in a three phase circuit with the help of two wattmeters, for a balanced star connected load. Draw the phasor diagram. *(Anna University Winter 2002)*

29. State the working principle of a dynamometer type wattmeter and show its connections. *(V.T.U., Belgaum Karnataka University, February 2002)*

30. A.D.C. milliammeter having a resistance of 2Ω gives full scale deflection when the current is 50mA. How can it be used to measure (*i*) a current of 5A (*ii*) a voltage of 500 V. *(V.T.U., Belgaum Karnataka University, February 2002)*

31. With the help of a neat diagram, explain the construction and principle of operation of single phase energy meter. *(V.T.U., Belgaum Karnataka University, Wimter 2003)*

32. With a neat sketch explain the construction and working of a single phase induction type energymeter. *(V.T.U., Belgaum Karnataka University, Summer 2003)*

OBJECTIVE TESTS—10

1. The kWh meter can be classified as a/an- instrument :
(*a*) deflecting (*b*) digital
(*c*) recording (*d*) indicating

2. The moving system of an indicating type of electrical instrument is subjected to :
(*a*) a deflecting torque
(*b*) a controlling torque
(*c*) a damping torque
(*d*) all of the above

3. The damping force acts on the moving system of an indicating instrument only when it is :
(*a*) moving (*b*) stationary
(*c*) near its full deflection
(*d*) just starting to move.

4. The most efficient form of damping employed in electrical instruments is :
(*a*) air friction
(*b*) fluid friction
(*c*) eddy currents
(*d*) none of the above.

5. Moving iron instruments can be used for measuring :
(*a*) direct currents and voltages
(*b*) alternating current and voltages
(*c*) radio frequency currents
(*d*) both (*a*) and (*b*).

6. Permanent-magnet moving-coil ammetere have uniform scales because :
(*a*) of eddy current damping
(*b*) they are spring-controlled
(*c*) their deflecting torque varies directly as current
(*d*) both (*b*) and (*c*).

7. The meter that is suitable for *only* direct current measurements is :
(*a*) moving-iron type
(*b*) permanent-magnet type
(*c*) electrodynamic type
(*d*) hot-wire type.

8. A moving coil voltmeters measures—
(*a*) only a.c. voltages
(*b*) only d.c. voltages
(*c*) both a.c. and d.c. voltages
(Principles of Elect. Engg. Delhi Univ.)

9. The reading of the voltmeter in Fig. 10.91 would be nearest to–volt :
(*a*) 80
(*b*) 120
(*c*) 200
(*d*) 0

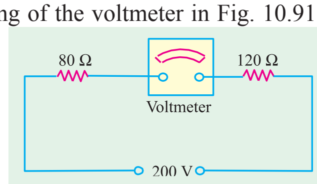


Fig. 10.91

10. The hot-wire ammeter :
(*a*) is used only for d.c. circuits
(*b*) is a high precision instrument
(*c*) is used only for a.c. circuits
(*d*) reads equally well on d.c. and/or a.c. circuits.

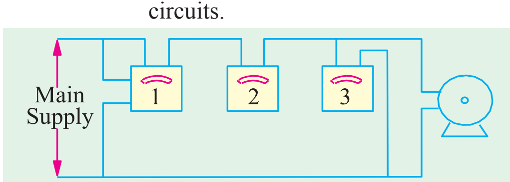


Fig. 10.92

11. If an energy meter disc makes 10 revolutions in 100 seconds when a load of 450 w is connected to it, the meter constant (in rev/k Wh) is
(*a*) 1000 (*b*) 500
(*c*) 1600 (*d*) 800
(GATE 2001)

12. The inductance of a certain moving-iron ammeter is expressed as $L = 10 + 3\theta - \frac{\theta^2}{4}$ μH, where θ is the deflection in radians from the zero position. The control spring torque in 25×10^{-6} Nm/radian. The deflection of the pointer in radian when the meter carries a current of 5A, is
(*a*) 2.4 (*b*) 2.0
(*c*) 1.2 (*d*) 1.0
(GATE 2003)

13. A dc potentiometer is designed to measure up to about 2 V with a slide wire of 800 mm. A standard cell of emf 1.18 V obtains balance at 600 mm. A test cell is seen to obtain balance at 680 mm. The emf of the test cell is
(*a*) 1.00 V (*b*) 1.34 V
(*c*) 1.50 V (*d*) 1.70 V
(GATE 2004)

14. A galvanometer with a full scale current of 10 mA has a resistance of 1000 Ω. The multiplying power (the ratio of measured current to galvanometer current) of a 100 Ω shunt with this galvanometer is
(*a*) 110 (*b*) 100
(*c*) 11 (*d*) 10
(GATE 2004)

15. A moving coil of a meter has 100 turns, and a length and depth of 10 mm and 20 mm respectively. It is positioned in a uniform radial flux density of 200mT. The coil carries a current of 50 mA. The torque on the coil is
(*a*) 200 μNm (*b*) 100 μNm
(*c*) 2 μNm (*d*) 1 μNm
(GATE 2004)

16. A dc A-h meter is rated for 15 A, 250 V. The meter constant is 14.4 A-sec/rev. The meter constant at rated voltage may be expressed as
(*a*) 3750 rev/kWh (*b*) 3600 rev/kWh
(*c*) 1000 rev/kWh (*d*) 960 rev/kWh
(GATE 2004)

ANSWERS

1. *c* **2.** *d* **3.** *a* **4.** *c* **5.** *d* **6.** *d* **7.** *b* **8.** *b* **9.** *c* **10.** *d*

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