C H A P T E R **4**

ELECTRO-STATICS

➤ Lightning and thunder are created when the static electricity concentrated on the clouds suddenly discharges

4.1. Static Electricity

In the preceding chapters, we concerned ourselves exclusively with electric current *i.e.* electricity in motion. Now, we will discuss the behaviour of static electricity and the laws governing it. In fact, electrostatics is that branch of science which deals with the phenomena associated with electricity at rest.

It has been already discussed that generally an atom is electrically neutral *i.e.* in a normal atom the aggregate of positive charge of protons is exactly equal to the aggregate of negative charge of the electrons.

If, somehow, some electrons are removed from the atoms of a body, then it is left with a preponderance of positive charge. It is then said to be positively-charged. If, on the other hand, some electrons are added to it, negative charge out-balances the positive charge and the body is said to be negatively charged.

In brief, we can say that positive electrification of a body results from a deficiency of the electrons whereas negative electrification results from an excess of electrons.

The total deficiency or excess of electrons in a body is known as its charge.

4.2. Absolute and Relative Permittivity of a Medium

While discussing electrostatic phenomenon, a certain property of the medium called its *permittivity* plays an important role. Every medium is supposed to possess two permittivities :

(*i*)   absolute permittivity (ε) and **(*ii*)** relative permittivity (ε_r).

For measuring relative permittivity, vacuum or free space is chosen as the reference medium. It has an absolute permittivity of 8.854×10^{-12} F/m

Absolute permittivity $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Relative permittivity, $\varepsilon_r = 1$

Charles Augustin de Coulomb[*]

Being a ratio of two similar quantities, ε_r has no units.

Now, take any other medium. If its relative permittivity, as compared to vacuum is ε_r, then its absolute permittivity is $\varepsilon = \varepsilon_0 \varepsilon_r$ F/m

If, for example, relative permittivity of mica is 5, then, its absolute permittivity is

$$\varepsilon = \varepsilon_0 \varepsilon_r = 8.854 \times 10^{-12} \times 5 = 44.27 \times 10^{-12} \text{ F/m}$$

4.3. Laws of Electrostatics

First Law. Like charges of electricity repel each other, whereas unlike charges attract each other.

Second Law.   According to this law, the force exerted between two *point* charges **(*i*)** is directly proportional to the product of their strengths **(*ii*)** is inversely proportional to the square of the distance between them.

This law is known as Coulomb's Law and can be expressed mathematically as :

$$F \propto \frac{Q_1 Q_2}{d^2} \quad \text{or} \quad F = k \frac{Q_1 Q_2}{d^2}$$

In vector form, the Coulomb's law can be written as $\left.\begin{aligned} \vec{F} &= \frac{Q_1 Q_2}{d^2} \hat{d} \\ &= \frac{Q_1 Q_2}{d^2} \vec{d} \end{aligned}\right\}$

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[*]   Coulomb is better known for his law which states that the force between two point charges is proportional to each charge and inversely proportional to the square of the distance between them.

where $\hat{d}$ is the unit vector *i.e.* a vector of unit length in the

direction of distance *d, i.e. $d = \dfrac{d}{\hat{d}}$* (where $\vec{d}$ is the vector notation

for *d*, which is a scalar notation).

Therefore, explicit forms of this law are :

$$\vec{F}_{21} = k\,\frac{Q_1 Q_2}{d_{12}^2}\,\hat{d}_{12} = k\,\frac{Q_1 Q_2}{d_{12}^3}\,\vec{d}_{12}$$

Fig. 4.1

where $\vec{F}_{21}$ is the force on Q_2 due to Q_1 and $\hat{d}_{12}$ is the unit vector
in direction from Q_1 to Q_2

and $\vec{F}_{12} = k\,\dfrac{Q_1 Q_2}{d_{21}^2}\,\hat{d}_{21} = k\,\dfrac{Q_1 Q_2}{d_{21}^2}\,\vec{d}_{21}$ where F_{12} is the force on Q_1 due to Q_2 and $\vec{d}_{21}$ is the unit
vector in the direction from Q_2 to Q_1.

where *k* is the constant of proportionality, whose value depends on the system of units employed. In S.I. system, as well as M.K.S.A. system $k = 1/4\pi\varepsilon$. Hence, the above equation becomes.

$$F = \frac{Q_1 Q_2}{4\,\pi\varepsilon d^2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

If Q_1 and Q_2 are in colomb, *d* in metre and ε in fard/metre, then *F* is in newtons

Now $\dfrac{1}{4\pi\varepsilon_0} = \dfrac{1}{4\pi\times 8.854\times 10^{-12}} = 8.9878\times 10^9 \approx 9\times 10^9$ (approx.)

Hence, Coulomb's Law can be written as

$$F = 9\times 10^9\,\frac{Q_1 Q_2}{\varepsilon_r d^2} \qquad\qquad \text{—in a medium}$$

$$= 9\times 10^9\,\frac{Q_1 Q_2}{d^2} \qquad\qquad \text{—in air or vacuum ...(i)}$$

If in Eq. (*i*) above

$$Q_1 = Q_2 = Q \text{ (say)}, d = 1 \text{ metre}; F = 9\times 10^9 \text{ N}$$

then $\qquad\qquad\qquad Q^2 = 1 \quad \text{or} \quad Q = \pm 1$ coulomb

Hence, one coulomb of charge may be defined as
that charge (or quantity of electricity) which when
placed in air (strictly vacuum) from an equal and
similar charge repels it with a force of 9×10^9 N.

Although coulomb is found to be a unit of conve-
nient size in dealing with electric current, yet, from
the standpoint of electrostatics, it is an enormous unit.
Hence, its submultiples like micro-coulomb (μ C) and
micro-microcoulomb ($\mu\,\mu$ C) are generally used.

$$1\,\mu\,C = 10^{-6}\,C;\ 1\,\mu\,\mu\,C = 10^{-12}\,C$$

Unlike charges attract and like charges repel
each other

It may be noted here that relative permittivity of air is one, of water 81, of paper between 2 and 3, of glass between 5 and 10 and of mica between 2.5 and 6.

Example 4.1. *Calculate the electrostatic force of repulsion between two α-particles when at a distance of 10^{-13} m from each other. Charge of an α-particles is 3.2×10^{-12} C. If mass of each particle is 6.68×10^{-27} N-m^2/kg^2.*

Solution. Here $\quad Q_1 = Q_2 = 3.2\times 10^{-19}\,C,\ d = 10^{-13}\,m$

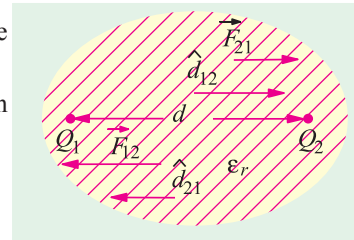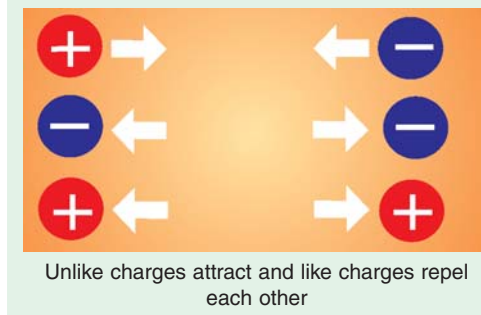$$F = 9\times 10^9\times\frac{3.2\times 10^{-19}\times 3.2\times 10^{-19}}{(10^{-13})^2} = \mathbf{9.2\times 10^{-2}\,N}$$

The force of gravitational attraction between the two particles is given by

$$F = G \frac{m_1 m_2}{d^2} = \frac{6.67 \times 10^{-11} \times (6.68 \times 10^{-27})^2}{(10^{-13})^2} = \mathbf{2.97 \times 10^{-37}} \ \mathbf{N}$$

Obviously, this force is negligible as compared to the electrostatic force between the two particles.

Example 4.2. *Calculate the distance of separation between two electrons (in vacuum) for which the electric force between them is equal to the gravitation force on one of them at the earth surface. mass of electron = 9.1×11^{-31} kg, charge of electron = 1.6×10^{-19} C.*

Solution. Gravitational force on one electron.

$$= mg \ \text{newton} = 9.1 \times 10^{-31} \times 9.81 \ \text{N}$$

Electrostatic force between the electrons

$$= 9 \times 10^9 \ \frac{Q^2}{d^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{d^2} N$$

Equating the two forces, we have

$$\frac{9 \times 10^9 \times 2.56 \times 10^{-38}}{d^2} = 9.1 \times 10^{-31} \times 9.81 \quad \therefore \ d = \mathbf{5.08 \ m}$$

Example 4.3. (*a*) *Three identical point charges, each $Q\Omega$ coulombs, are placed at the vertices of an equilateral triangle 10 cm apart. Calculate the force on each charge.*

(*b*) *Two charges Q coulomb each are placed at two opposite corners of a square. What additional charge "q" placed at each of the other two corners will reduce the resultant electric force on each of the charges Q to zero ?*

Solution. (*a*) The equilateral triangle with its three charges is shown in Fig. 4.2 (*a*). Consider the charge Q respectively. These forces are equal to each other and each is

$$F = 9 \times 10^9 \ \frac{Q^2}{0.1^2} = 9 \times 10^{11} \ Q^2 \ \text{newton}$$
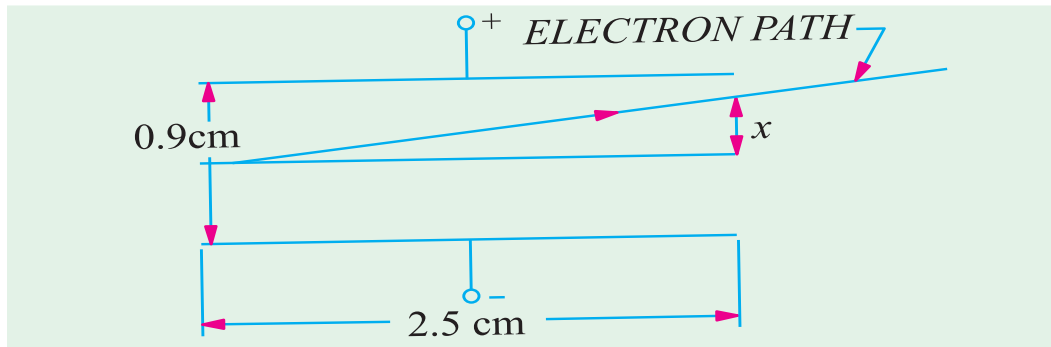

Fig. 4.2

Since the angle between these two equal forces is 60°, their resultant is

$$= 2 \times F \times \cos 60°/2 = \sqrt{3} \ F = \mathbf{9 \times 10^{11}} \times \sqrt{3} \ \mathbf{Q^2} \ \mathbf{Newton}$$

The force experienced by other charges is also the same.

(*b*) The various charges are shown in Fig. 4.2 (*b*). The force experienced by the charge Q at point C due to the charge Q at point A acts along ACM and is

$$= 9 \times 10^9 \ \frac{Q^2}{(\sqrt{2}d)^2} = 4.5 \times 10^9 \ Q^2/d^2 \ \text{newton} \qquad \qquad ...(i)$$

where d is the side of the square in metres.

If the charges q are negative, they will exert attractive forces on the charge Q at point C along CB and CD respectively. Each force is

$$= -9 \times 10^9 \frac{Qq}{d^2} \text{ newton}$$

Since these two forces are at right angles to each other, their resultant is

$$= -\sqrt{2} \times 9 \times 10^9 \frac{qQ}{d^2}$$

If net force on charge Q at point C is to be zero, then (*i*) must equal (*ii*),

$$\therefore \qquad 4.5 \times 10^9 \frac{Q^2}{d^2} = -9 \times 10^9 \sqrt{2} \frac{qQ}{d^2} \quad \therefore \quad q = -Q/2\sqrt{2} \text{ coulomb}$$

Example 4.4. *The small identical conducting spheres have charges of 2.0×10^9 C and -0.5×10^9 respectively. When they are placed 4 cm apart, what is the force between them ? If they are brought into contact and then separated by 4 cm, what is the force between them ?*

(Electromagnetic Theory, A.M.I.E. Sec B,)

Solution. $F = 9 \times 10^9 Q_1 Q_2/d^2 = 9 \times 10^9 \times (-0.5 \times 10^9)/0.04^2 = -56.25 \times 10^7$ N. When two identical spheres are brought into contact with each other and then separated, each gets half of the total charge. Hence,

$$Q_1 = Q_2 = [2 \times 10^9 + (-0.5 \times 10^9)/2] = 0.75 \times 10^9 \text{ C}$$

When they are separated by 4 cm,

$$F = 9 \times 10^9 \times (0.75 \times 10^9)^2/0.04^2 = \mathbf{0.316 \times 10^5 \text{ N}}$$

Example 4.5. *Determine resultant force on 3 µC charge due to -4µC and 10 nC charges. All these three point charges are placed on the vertices of equilateral triangle ABC of side 50 cm.*
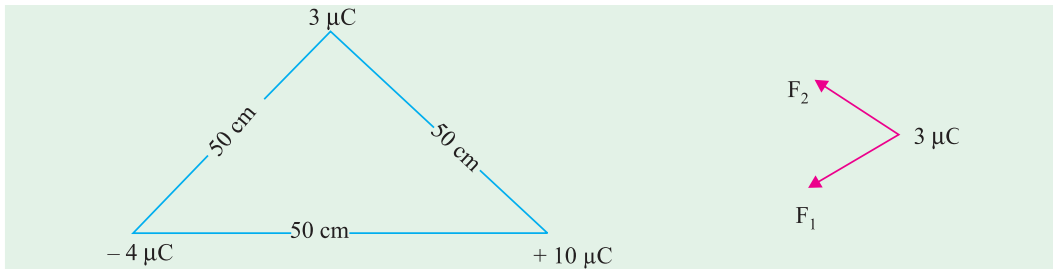
[Bombay University, 2001]

Fig. 4.3 (*a*)   Fig. 4.3 (*b*)

Solution. $\qquad F_2 = \dfrac{Q_1 Q_2}{4 \pi \varepsilon_0 d^2} \qquad \dfrac{3 \quad 10^6 \quad 10 \quad 10^9}{4 \quad 8.854 \quad 10^{12} \quad 0.50 \quad 0.50}$

$$= 1.08 \times 10^{-3} \text{ Newton, in the direction shown}$$

Similarly, $\qquad F_1 = 0.432$ Newton, in the direction shown.

Resultant of F_1 and F_2 has to be found out.

Example 4.6. *A capacitor is composed of 2 plates separated by a sheet of insulating material 3 mm thick and of relative permitivity 4. The distance between the plates is increased to allow the insertion of a second sheet of 5 mm thick and of relative permitivity ε_r. If the equivalent capacitance is one third of the original capacitance. Find the value of ε_r.* **[Bombay University, 2001]**

Solution. $\qquad C_1 \dfrac{\varepsilon_0 \varepsilon_r A}{d} = k(4/3), \quad \text{where} \quad k = \varepsilon_0 A \times 10^{+3}$

The composite capacitor [with one dielectric of $\varepsilon_{r1} = 4$ and other dielectric of ε_{r2} as relative

permitivity has a capacitance of $C/3$. Two capacitors are effectively in series. Let the second dielectric contribute a capacitor of C_2.

$$K.(4/9) = \frac{C_1 C_2}{C_1 + C_2} = \frac{K.(4/3).C_2}{K.(4/3) + C_2}$$

This gives

$$C_2 = 2/3\ K$$

$$(2/3)\ K = \frac{\varepsilon_0 \varepsilon_{r2} A}{5 \times 10^{-3}}$$

$$E_{r2} = 10/3 . K\ 1/e_0\ A \times 10^{-3}$$
$$= 10/3\ e_0\ A \times 103\ 1/e_0\ A \times 10^{-3}$$
$$= 10/3 = \textbf{3.33}$$

4.4. Electric Field

It is found that in the medium around a charge a force acts on a positive or negative charge when placed in that medium. If the charge is sufficiently large, then it may create such a huge stress as to cause the electrical rupture of the medium, followed by the passage of an arc discharge.
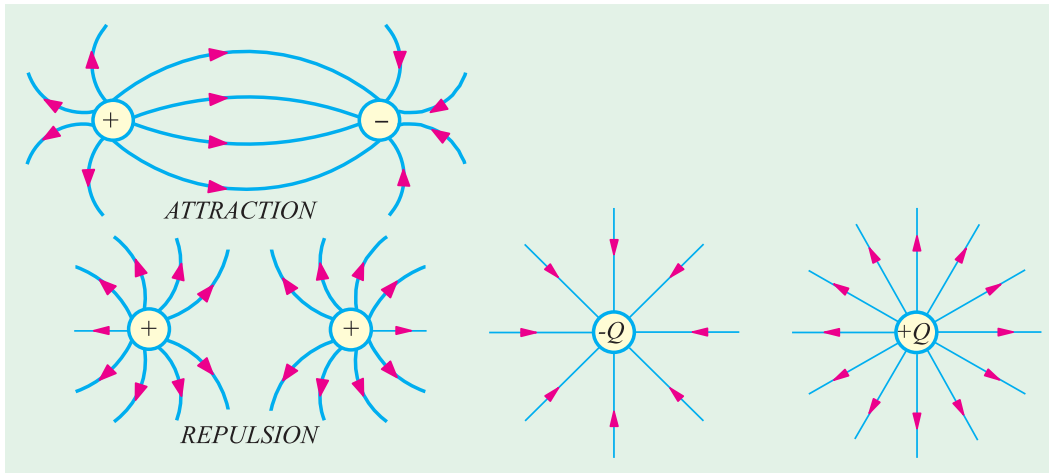

ATTRACTION

REPULSION

Fig. 4.4 (a)   **Fig. 4.4 (b)**

The region in which the stress exists or in which electric forces act, is called an electric field or electrostatic field.

The stress is represented by imaginary lines of forces. The direction of the lines of force at any point is the direction along which a unit positive charge placed at that point would move if free to do so. It was suggested by Faraday that the electric field should be imagined to be divided into *tubes of force* containing a fixed number of *lines* of force. He assumed these tubes to the elastic and having the property of contracting longitudinally the repelling laterally. With the help of these properties, it becomes easy to explain *(i)* why unlike charges attract each other and try to come nearer to each other and *(ii)* why like charges repel each other [Fig. 4.4 (a)].

However, it is more common to use the term lines of force. These lines are supposed to emanate from a positive charge and end on a negative charge [Fig. 4.4 (b)]. These lines always leave or enter a conducting surface normally.

4.5. Electrostatic Induction

It is found that when an uncharged body is brought near a charged body, it acquires some charge. This phenomenon of an uncharged body getting charged merely by the nearness of a charged body is known as *induction*. In Fig. 4.5, a positively-charged body A is brought close to a perfectly-insulated

uncharged body B. It is found that the end of B nearer to A gets negatively charged whereas further end becomes positively charged. The negative and positive charges of B are known as ***induced charges***. The negative charge of B is called 'bound' charge because it must remain on B so long as positive charge of A remains there. However, the positive charge on the farther end of B is called free charge. In Fig. 4.6, the body B has been earthed by a wire. The positive charge flows to earth leaving negative charge behind. If next A is removed, then this negative charge will also go to earth, leaving B uncharged. It is found that :

(*i*) a positive charge induces a negative charge and ***vice-versa.***

(*ii*) each of the induced charges is equal to the inducing charge.
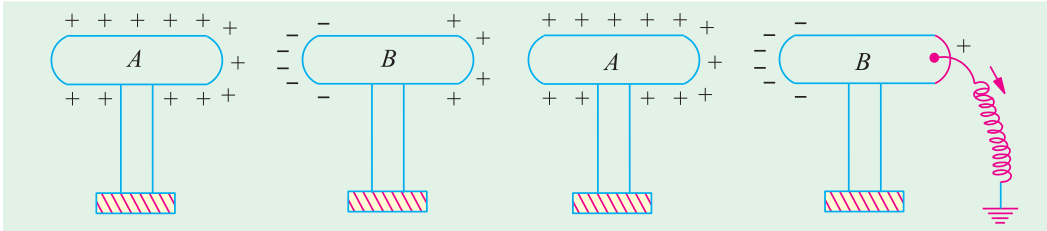

Fig. 4.5   Fig. 4.6

4.6. Electric Flux and Faraday Tubes

Consider a small closed curve in an electric field (Fig. 4.7). If we draw lines of force through each point of this closed curve, then we get a tube as sho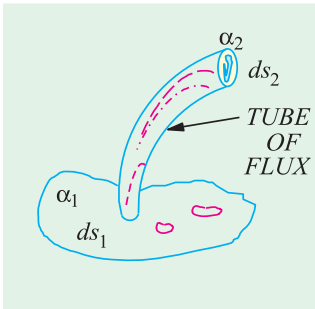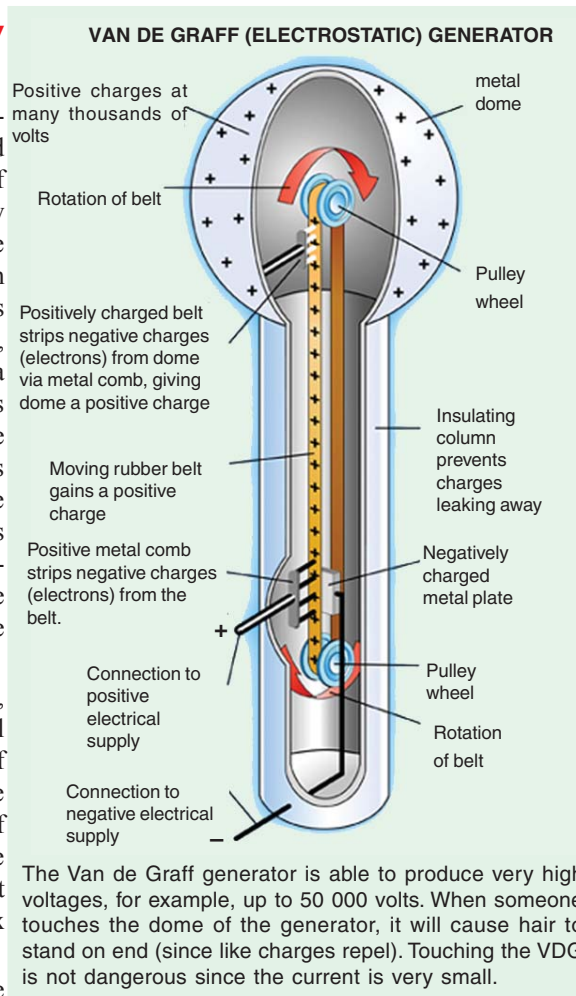wn in the figure. It is called the ***tube of the electric flux***. It may be defined as the region of space enclosed within the tubular surface formed by drawing lines of force through every point of a small closed curve in the electric field.

Fig. 4.7

Since lines of force end on conductors, the two ends of a flux tube will consist of small area ds_1 and ds_2 on the conductor surfaces. If surface charge densities over these areas are σ_1 and $-\sigma_2$, then charges at the two ends of the flux tube will be $\sigma_1\, ds_1$ and $-\sigma_2\, ds_2$. These charges are assumed to be always equal but opposite to each other. The strength of a flux tube is represented by the charge at its ends.

A ***unit tube of flux*** is one in which the

VAN DE GRAFF (ELECTROSTATIC) GENERATOR

Positive charges at many thousands of volts

Rotation of belt

Positively charged belt strips negative charges (electrons) from dome via metal comb, giving dome a positive charge

Moving rubber belt gains a positive charge

Positive metal comb strips negative charges (electrons) from the belt.

Connection to positive electrical supply

Connection to negative electrical supply

metal dome

Pulley wheel

Insulating column prevents charges leaking away

Negatively charged metal plate

Pulley wheel

Rotation of belt

The Van de Graff generator is able to produce very high voltages, for example, up to 50 000 volts. When someone touches the dome of the generator, it will cause hair to stand on end (since like charges repel). Touching the VDG is not dangerous since the current is very small.

end charge is one unit of charge.

In the S.I. system of units, one such tube of flux is supposed to start from a positive charge of one coulomb and terminate on a negative charge of the same amount.

A *unit tube of flux* is known as Faraday tube. If the charge on a conductor is ± Q coulombs, then the *number of Faraday tubes starting* or *terminating on it also Q.*

The number of Faraday tubes of flux passing through a surface in an electric field is called the *electric flux* (or dielectric flux) through that surface. Electric flux is represented by the symbol ψ. Since electric flux is numerically equal to the charge, it is measured in coulombs.

Hence,   ψ = Q coulombs

Note. It may also be noted that 'tubes of flux' passing per unit area through a medium are also supposed to measure the 'electric displacement' of that dielectric medium. In that case, they are referred to as *lines of displacement* and are equal to ε times the lines of force (Art. 4.8). It is important to differentiate between the 'tubes of flux' and 'lines of force' and to remember that if Q is the charge, then

tubes of flux = Q and lines of force = Q/ε

4.7. Field Strength or Field Intensity or Electric Intensity (*E*)

Electric intensity at any point within an electric field may be defined in either of the following three ways :

(*a*) It is given by the force experienced by a unit positive charge placed at that point. Its direction is the direction along which the force acts.

Obviously, the unit of *E* is newton/coulomb (*N/C*).

For example, if a charge of *Q* coulombs placed at a particular point *P* within an electric field instances a force of *F* newton, then electric field at that point is given by

$$E = F/Q \ N/C$$

The value of *E* within the field due to a point charge can be found with help of Coulomb's laws. Suppose it is required to find the electric field at a point *A* situated at a distance of *d* metres from a charge of *Q* coulombs. Imagine a positive charge of one coulomb placed at that point (Fig. 4.8). The force experienced by this charge is

$$F = \frac{Q \times 1}{4 \pi \varepsilon_0 \varepsilon_r d^2} N \quad \text{or} \quad \vec{F} = \frac{Q \times 1}{4 \pi \varepsilon_0 \varepsilon_r d_{PA}^2} \hat{d}_{PA}$$

$$\therefore \qquad E = \left. \begin{array}{l} \dfrac{Q \times 1}{4 \pi \varepsilon_0 \varepsilon_r d_{PA}^2} \ N/C \\[4mm] = 9 \times 10^9 \dfrac{Q}{\varepsilon_r d_{PA}^2} \ N/C \end{array} \right\} \text{ in a medium}$$
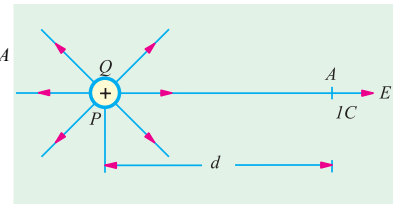

Fig. 4.8

or in vector notation,

$$\vec{E}(d) = 9 \times 10^9 \frac{Q}{\varepsilon_r d^2} \hat{d} \quad \text{where } \vec{E}(d) \text{ denotes } \vec{E} \text{ as a function of } d$$

$$\left. \begin{array}{l} = \dfrac{Q}{4\pi \varepsilon_0 d^2} \ N/C \\[4mm] = 9 \times 10^9 \dfrac{Q}{d^2} \ N/C \end{array} \right\} \text{ in air}$$

(*b*) Electric intensity at a point may be defined as equal to the *lines of force* passing normally through a unit cross-section at that point. Suppose, there is a charge of *Q* coulombs. The number of

lines of force produced by it is Q/ε. If these lines fall normally on an area of A m^2 surrounding the point, then electric intensity at that point is

$$E = \frac{Q/\varepsilon}{A} = \frac{Q}{\varepsilon A}$$

Now $\qquad\qquad Q/A = D$ —the flux density over the area

$\therefore\qquad\qquad E = \dfrac{D}{\varepsilon} = \dfrac{D}{\varepsilon_0 \varepsilon_r}$ —in a medium

$\qquad\qquad\quad = \dfrac{D}{\varepsilon_0}$ —in air

The unit of E is volt/metre.

(c) Electric intensity at any point in an electric field is *equal to the potential gradient at that point.*

In other words, E is equal to the rate of fall of potential in the direction of the lines of force.

$\therefore\qquad\qquad E = \dfrac{-dV}{dx}$

Obviously, the unit of E is volt/metre.

It may be noted that E and D are vector quantities having magnitude and direction.

$\therefore$ In vector notation, $\qquad \vec{D} = \varepsilon_0 \vec{E}$

Example 4.7. *Point charges in air are located as follows :*

$+5 \times 10^{-8}$ C at (0, 0) metres, $+4 \times 10^{-8}$ C at (3, 0) metres and -6×10^{-8} C at (0, 4) metres. Find electric field intensity at (3, 4) metres.

Solution. Electric intensity at point D (3, 4) due to positive charge at point A is

$E_1 = 9 \times 10^9 \, Q/d^2 = 9 \times 10^9 \times 5 \times 10^{-8}/5^2 = 18$ V/m

As shown in Fig. 4.9, it acts along AD.

Similarly, electric intensity at point D due to positive charge at point B is $E_2 = 9 \times 10^9 \times 4 \times 10^{-8}/4^2 = 22.5$ V/m. It acts along BD.

$E_1 = 9 \times 10^9 \times 6 \times 10^{-8}/3^2 = 60$ V/m. It acts along DC.

The resultant intensity may be found by resolving E_1, E_2 and E_3 into their X-and Y-components. Now, tan $\theta = 4/3$; $\theta = 53°8'$.

X-component $= E_1 \cos \theta - E_2 = 18 \cos 53°8' - 60$

$\qquad\qquad\quad = -49.2$

Y-component $= E_1 \sin \theta + E_2 = 18 \sin 53°8' + 22.5 = 36.9$

$\therefore\qquad E = \sqrt{(-49.2)^2 + 36.9^2} = 61.5$ V/m.

It acts along DE such that tan $\phi = 36.9/49.2 = 0.75$.
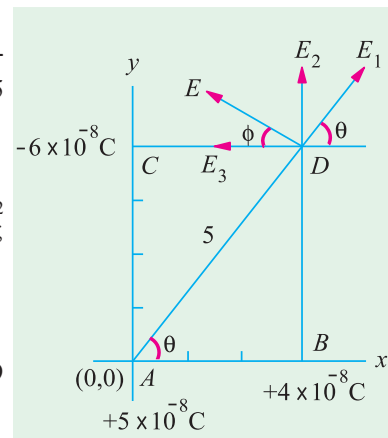
Hence $\phi = $ **36.9°.**

Fig. 4.9

Example 4.8. *An electron has a velocity of 1.5×10^7 m/s at right angles to the uniform electric field between two parallel deflecting plates of a cathode-ray tube. If the plates are 2.5 cm long and spaced 0.9 cm apart and p.d. between the plates is 75 V, calculate how far the electron is deflected sideways during its movement through the electric field. Assume electronic charge to be 1.6×10^{-19} coulomb and electronic mass to be 9.1×10^{-31} kg.*

Solution. The movement of the electron through the electric field is shown in Fig. 4.10. Electric intensity between the plates is $E = dV/dx = 75/0.009 = 8,333$ V/m.

Force on the electron is $F = QE$
$= 8,333 \times 1.6 \times 10^{-19} = 1.33 \times 10^{-15}$ N.

Since the deflection x is small as compared to the length of the plates, time taken by the electron to travel through the electric field is
$= 0.025/1.5 \times 10^7 = 1.667 \times 10^{-9}$ s

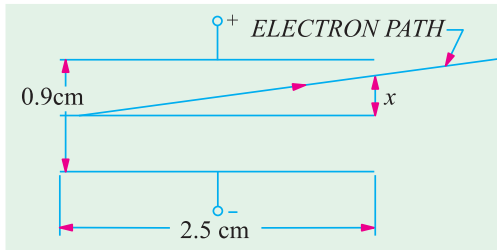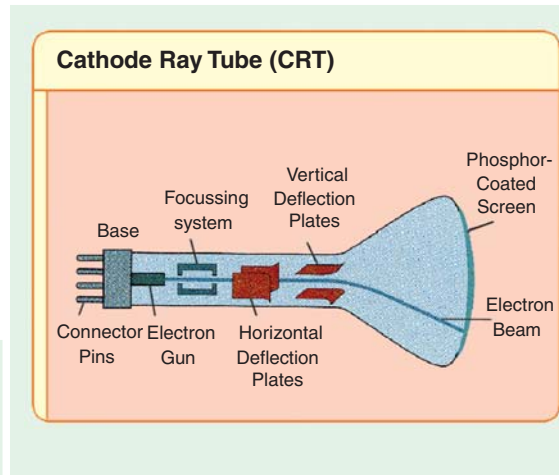**Cathode Ray Tube (CRT)**

Phosphor-Coated Screen

Vertical Deflection Plates

Focussing system

Base

Connector Pins

Electron Gun

Horizontal Deflection Plates

Electron Beam

Fig. 4.10

ELECTRON PATH

0.9cm

x

2.5 cm

Now, force = mass × acceleration

∴ Transverse acceleration is

$= \dfrac{1.33 \times 10^{-15}}{9.1 \times 10^{-31}} = 1.44 \times 10^{15}$ m/s^2

Final transverse velocity of the electron = acceleration × time
$= 1.44 \times 10^{-15} \times 1.667 \times 10^{-9} = 2.4 \times 10^6$ m/s

∴ sideways or transverse movement of the electron is

$$x = \text{(average velocity)} \times \text{time}$$
$$= \frac{1}{2} \times 2.4 \times 10^6 \times 1.667 \times 10^{-9} = \textbf{2 mm (approx.)}*$$

4.8. Electric Flux Density or Electric Displacement

It is given by the normal flux per unit area.

If a flux of Ψ coulombs passes normally through an area of A m^2, then flux density is

$$D = \frac{\Psi}{A} \text{ C/m}^2$$

It is related to electric field intensity by the relation

$$D = \varepsilon_0 \varepsilon_r E \qquad \qquad \text{...in a medium}$$
$$= \varepsilon_0 E \qquad \qquad \text{...in free space}$$

In other words, the product of electric intensity E at any point within a dielectric medium and the absolute permittivity ε ($= \varepsilon_0, \varepsilon_r$) at the same point is called the ***displacement*** at that point.

Like electric intensity E, electric displacement $D**$ is also a vector quantity (see 4.7) whose direction at every point is the same as that of E but whose magnitude is $\varepsilon_0 \varepsilon_r$ times E. As E is represented by lines of force, similarly D may also be represented by lines called lines of electric

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*   The above result could be found by using the general formula

$$x = \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{V}{d}\right)\left(\frac{l}{v}\right)^2 \text{ metres}$$

where e/m = ratio of the charge and mass of the electron
   V = p.d. between plates in volts; d = separation of the plates in metres
   l = length of the plates in metres; v = axial velocity of the electron in m/s.

**   A more general definition of displacement D is that $D = e_0 \, e_r \, E + P$ where P is the polarisation of the dielectric and is equal to the dipole moment per unit volume.

displacement. The tangent to these lines at any point gives the direction of D at that point and the number of lines per unit area perpendicular to their direction is numerically equal to the electric displacement at that point. Hence, the number of lines of electric displacement per unit area (D) is ε_0, ε_r times the number of lines of force per unit area at that point.

It should be noted that whereas the value of E depends on the permittivity of the surrounding medium, that of D is independent of it.

One useful property of D is that its surface integral over any closed surface equals the enclosed charge (Art. 4.9).

Let us find the value of D at a point distant r metres from a point charge of Q coulombs. Imagine a sphere of radius r metres surrounding the charge. Total flux = Q coulombs and it falls normally on a surface area of $4 \pi r^2$ metres. Hence, electric flux density.

$$D = \frac{\Psi}{4 \pi r^2} = \frac{Q}{4 \pi r^2} \text{ coulomb/metre}^2 \text{ or } D = \frac{Q}{4 \pi r^2} r = r \text{ (in vector notation)}$$

4.9. Gauss* Law

Consider a point charge Q lying at the centre of a sphere of radius r which surrounds it com-pletely [Fig. 4.11 (a)]. The total number of *tubes of flux* originating from the charge is Q (but number of lines of force is Q/ε_0) and are normal to the surface of the sphere. The electric field E which equals $Q/4 \pi \varepsilon_0 r^2$ is also normal to the surface. As said earlier, total number of lines of force passing perpendicularly through the whole surface of the sphere is

$$= E \times \text{Area} = \frac{Q}{4\pi\varepsilon_0 r^2} \times 4 \pi r^2 = \frac{Q}{\varepsilon_0}$$
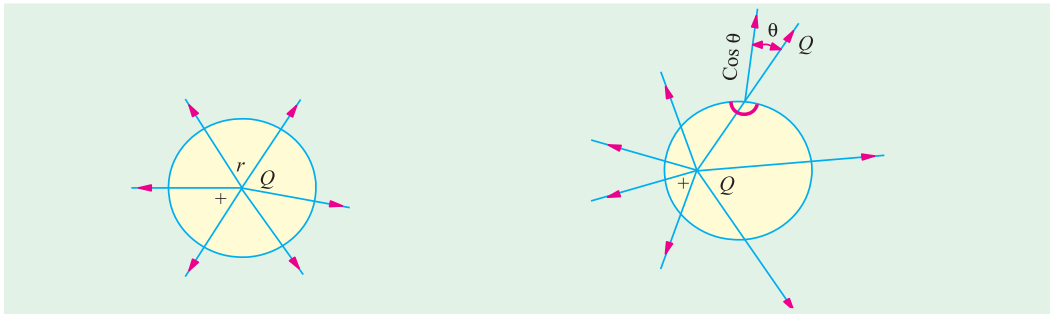

Fig. 4.11

Now, suppose we draw another sphere surrounding the charge [Fig. 4.11 (b)] but whose centre does not lie at the charge but elsewhere. In this case also, the number of tubes of flux emanating from the charge is Q and lines of force is $Q.\varepsilon_0$ though they are not normal to the surface. These can, however, be split up into $\cos \theta$ components and $\sin \theta$ components. If we add up $\sin \theta$ components all over the surface, they will be equal to zero . But if add up $\cos \theta$ components over the whole surface of the sphere, the normal flux will again come out to be Q (or lines of force will come out to be Q/ε_0). Hence, it shows that irrespective of where the charge Q is placed within a closed surface completely surrounding it, the total normal flux is Q and the total number of lines of force passing out normally is Q/ε_0.

In fact, as shown in Fig. 4.12, if there are placed charges of value Q_1, Q_2, $-Q_3$ inside a closed surface, the total *i.e.* net charge enclosed by the surface is $(Q_1 + Q_2 - Q_3)/\varepsilon_0$ through the closed surface.

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* After the German mathematician and astronomer Karel Freidrich Gauss (1777-1855).

This is the meaning of Gauss's law which may be stated thus : the surface integral of the normal component of the electric intensity E over a closed surface is equal to $1/\varepsilon_0$ times the total charge inside it.

Mathematically, $\int E_n ds = Q/\varepsilon_0$ (where the circle on the integral sign indicates that the surface of integration is a closed surface).

or	$\int \varepsilon_n \varepsilon_0 \, ds = Q$, i.e.	$\int D_n \, ds = Q$	$[\therefore D_n = \varepsilon_0 E_n]$
or	$\int \varepsilon_0 \, E \cos \theta \, ds = Q$, i.e.	$\int D \cos \theta \, ds = Q$	
or	$\int \varepsilon_0 \, E_{ds} \cos \theta = Q$, i.e.,	$\int D \, ds \cos \theta = Q$	

when E and D are not normal to the surface but make an angle θ with the normal (perpendicular) to the surface as shown in Fig. 4.13.

Proof. In Fig. 4.13, let a surface S completely surround a quantity of electricity or charge Q. Consider a small surface area ds subtending a small solid angle $d\omega$ at point charge Q. The field intensity at ds is $E = \dfrac{Q}{4 \, \pi \, \varepsilon_0 \, d^2}$ where d is the distance between Q and ds.

In vector notation, $\int \varepsilon_0 \, \vec{E} . \vec{ds} = Q$ i.e. $\int \vec{D} \, \vec{ds} = Q = \int_v \rho \, dv$ (where ρ is the volume density of charge in the volume enclosed by closed surface S).

Thus $\int_s \vec{D} . \vec{ds} = \int_v \rho \, dv$ is the vector statement of Gauss Law* and its alternative statement is

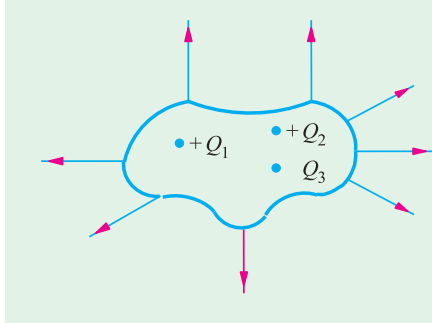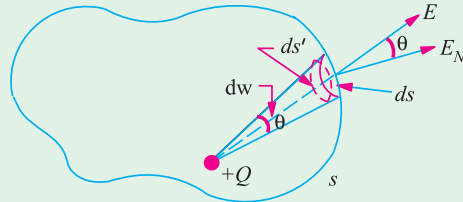$$\nabla D = \rho$$

Fig. 4.12   Fig. 4.13

The normal component of the intensity $E_n = E \cos \theta$

$\therefore$ No. of lines of force passing normally through the area ds is

$$= E_n . d_s = E \, ds \cos \theta = E. ds \text{ in vector notation}$$

Now $\quad\quad ds \cos \theta = ds' \quad \therefore E.ds' = \dfrac{Q}{4 \pi \varepsilon_0 \, d^2} \, .ds'$

Now $\quad\quad ds' / d^2 = d\omega$

Hence, the number of lines of force passing normally is $= \dfrac{Q}{4 \, \pi \varepsilon_0} \, d\omega$

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*   This results from the application of the Divergence theorem, also called the Gauss' Theorem, *viz.*,

$\int_s \nabla D \, dv = \int D. ds$ where vector operator called 'del' is defined as

$$\nabla = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z$$

Total number of lines of force over the whole surface

$$= \frac{Q}{4\pi\varepsilon_0} \int_s d\omega = \frac{Q}{4\pi\varepsilon_0} \times 4\pi = \frac{Q}{\varepsilon_0}$$

where sign $\oint$ denotes integration around the whole of the closed surface *i.e.* surface integral.

If the surface passes through a material medium, then the above law can be generalized to include the following :

the surface integral of the normal component of D over a closed surface equals the free charge enclosed by the surface.

As before $D = \dfrac{Q}{4\pi\varepsilon d^2}$. The normal component $D_n = D\cos\theta = \dfrac{Q}{4\pi d^2} \times \cos\theta$

Hence, the normal electric flux from area *ds* is

$$d\psi = D_n \times ds = \frac{Q}{4\pi d^2} . \cos\theta . ds = \frac{Q}{4\pi d^2} . ds'$$

$\therefore$ $\qquad d\psi = \dfrac{Q}{4\pi}\left(\dfrac{ds'}{d^2}\right) = \dfrac{Q}{4\pi}d\omega$

or $\qquad \psi = \int \dfrac{Q}{4\pi}.d\omega = \dfrac{Q}{4\pi}\int d\omega = \dfrac{Q}{4\pi} \times 4\pi = Q \qquad \therefore \; \Psi = Q$

which proves the statement made above.

Hence, we may state Gauss's law in two slightly different ways.

$$\int_3 E_n . ds = \int_3 E . \cos\theta . ds = Q/\varepsilon_0 \;\; \text{or} \;\; \varepsilon_0 \int_3 E_n . ds = Q$$

and $\qquad \int_3 D_n . ds = \int_3 D_n . ds = Q$

(vector statement is given above)

4.10. The Equations of Poisson and Laplace

These equations are useful in the solution of many problems concerning electrostatics especially the problem of space charge* present in an electronic valve. The two equations can be derived by applying Gauss's theorem. Consider the electric field set up between two charged plates *P* and *Q* [Fig. 4.14 (*a*)]. Suppose there is some electric charge present in the space between the two plates. It is, generally, known as the space charge. Let the space charge density be ρ coulomb/metre³. It will be assumed that the space charge density varies from one point of space to the another but is uniform throughout any thin layer taken parallel to the plates *P* and *Q*. If *X*-axis is taken

Fig. 4.14

perpendicular to the plates, then ρ is assumed to depend on the value of *x*. It will be seen from Fig. 4.14 (*a*) that the value of electric intensity *E* increases with *x* because of the space charge.

Now, consider a thin volume element of cross-section *A* and thickness Δx as shown in Fig. 4.14 (*b*). The values of electric intensity at the two opposite faces of this element are *E* and $(E + \Delta E)$. If *dE/dx* represents the rate of increases of electric intensity with distance, then

$$\Delta E = \frac{\partial E}{\partial x} \times \Delta x \;\; \therefore \;\; E + \Delta E = E + \frac{\partial E}{\partial x} \times \Delta x$$

The surface integral of electric intensity over the right-hand face of this element is

* Such a space charge exists in the space between the cathode and anode of a vacuum tube.
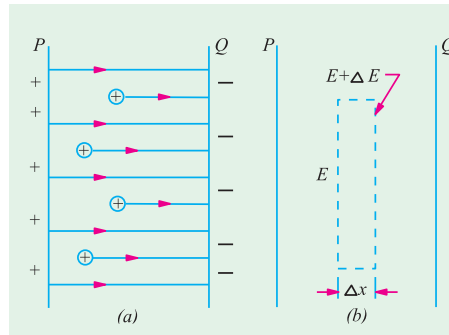
$$= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A$$

The surface integral over the left-hand face of the element is $= -E \times A$

The negative sign represents the fact that E is directed inwards over this face.

The surface integral over the entire surface, *i.e.,* the closed surface of the element is

$$= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A - E \times A = A . \Delta x . \frac{\partial E}{\partial x}.$$ From symmetry it is evident that along with y and z there is no field.

Now, according to Gauss's theorem (Art. 4.9), the surface integral of electric intensity over a closed surface is equal to $1/\varepsilon_0$ time the charge within that surface.

Volume of the element, $dV = A \times \Delta x$; charge $= \rho A . \Delta x$

$$\therefore \quad A . \Delta x . \frac{\partial E}{\partial x} = \rho A . \Delta x . \frac{1}{\varepsilon_0} \quad \text{or} \quad \frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$$

Now $$E = -\frac{\partial V}{\partial x} \therefore \frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{dV}{dx} \right) = -\frac{\partial^2 V}{\partial x^2} \quad \therefore \quad \frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon_0}$$

It is known as Poisson's equation in one dimension where potential varies with x.

When V varies with x, y and z, then $\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2} = -\dfrac{\rho}{\varepsilon_0} = \nabla^2 V$ in vector notation.

If, as a special case, where space charge density is zero, then obviously,

$$\partial^2 V / \partial x^2 = 0$$

In general, we have $\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2} = 0$ or $\nabla^2 V = 0$ in vector notation where ∇^2 is defined (in cartesian co-ordinates) as the operation

$$\nabla^2 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

It is known as Laplace's equation.

4.11. Electric Potential and Energy

We know that a body raised above the ground level has a certain amount of mechanical potential energy which, by definition, is given by the amount of work done in raising it to that height. If, for example, a body of 5 kg is raised against gravity through 10 m, then the potential energy of the body is $5 \times 10 = 50$ m-kg. wt. $= 50 \times 9.8 = 490$ joules. The body falls because there is attraction due to gravity and always proceeds from a place of higher potential energy to one of lower potential energy. So, we speak of gravitational potential energy or briefly 'potential' at different points in the earth's gravitational field.

Now, consider an electric field. Imagine an isolated positive charge Q placed in air (Fig. 4.15). Like earth's gravitational field, it has its own electrostatic field which theoretically extends upto infinity. If the charge X is very far away from Q, say, at infinity, then force on it is practically zero. As X is brought nearer to Q, a force of repulsion acts on it (as similar charges repel each other), hence work or energy is required to bring it to a point like A in the electric field. Hence, when at point A, charge X has
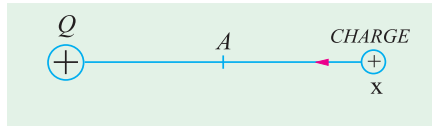
Using Van De Graff Generator, artificial lightning can be created in the laboratory, in a miature scale.

Fig. 4.15

some amount of electric potential energy. Similar other points in the field will also have some potential energy. In the gravitational field, usually 'sea level' is chosen as the place of 'zero' potential. In electric field infinity is chosen as the theoretical place of 'zero' potential although, in practice, earth is chosen as 'zero' potential, because earth is such a large conductor that its potential remains practically constant although it keeps on losing and gaining electric charge every day.

4.12. Potential and Potential Difference

As explained above, the force acting on a charge at infinity is zero, hence 'infinity' is chosen as the theoretical place of zero electric potential. Therefore, potential at any point in an electric field may be defined as

numerically equal to the work done in bringing a positive charge of one coulomb from infinity to that point against the electric field.

The unit of this potential will depend on the unit of charge taken and the work done.

If, in shifting one coulomb from infinity to a certain point in the electric field, the work done is one joule, then potential of that ponit is one volt.

Obviously, potential is work per unit charge,

$$\therefore \quad 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Similarly, potential difference (p.d.) of one volt exists between two points if one joule of work is done in shifting a charge of one coulomb from one point to the other.

4.13. Potential at a Point

Consider a positive point charge of Q coulombs placed in air. At a point x metres from it, the force on one coulomb positive charge is $Q/4\pi\varepsilon_0 x^2$ (Fig. 4.16). Suppose, this one coulomb charge is moved towards Q through a small distance dx. Then, work done is

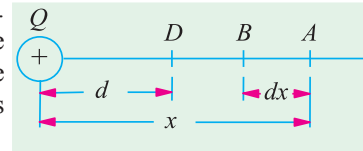**Fig. 4.16**

$$dW = \frac{Q}{4\pi\varepsilon_0 r^2} \times (-dx)$$

The negative sign is taken because dx is considered along the negative direction of x.

The total work done in bringing this coulomb of positive charge from infinity to any point D which is d metres from Q is given by

$$W = -\int_{x=\infty}^{x=d} Q \cdot \frac{dx}{4\pi\varepsilon_0 x^2} = -\frac{Q}{4\pi\varepsilon_0} \int_\infty^d \frac{dx}{x^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0} \left|-\frac{1}{x}\right|_\infty^d = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{d} - \left(-\frac{1}{\infty}\right)\right] = \frac{Q}{4\pi\varepsilon_0 d} \text{ joules}$$

By definition, this work in joules in numerically equal to the potential of that point in volts.

$$\therefore \quad V = \frac{Q}{4\pi\varepsilon_0 d} = 9 \times 10^9 \frac{Q}{d} \text{ volt} \qquad \text{—in air}$$

and

$$V = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d} = 9 \times 10^9 \frac{Q}{\varepsilon_r d} \text{ volt} \qquad \text{—in medium}$$

We find that as d increases, V decreases till it becomes zero at infinity.

4.14. Potential of a Charged Conducting Sphere

The above formula $V = Q/4\pi\varepsilon_0 \varepsilon_r d$ applies only to a charge concentrated at a point. The problem

of finding potential at a point outside a charged sphere sounds difficult, because the charge on the sphere is distributed over its entire surface and so, is not concentrated at a point. But the problem is easily solved by nothing that the lines of force of a charged sphere, like A in by noting that the lines of force of a charged sphere, like A in Fig. 4.17 spread out normally from its surface. If produced backwards, they meet at the centre of A. Hence for finding the potentials at points outside the sphere, we can imagine the charge on the sphere as concentrated at its centre O. If r is the radius of sphere in metres and Q its charge in coulomb then, potential of its surface is $Q/4\pi \ \varepsilon_0 \ r$ volt and electric intensity is $Q/4\pi\varepsilon_0 \ r^2$. At any other point '$d$' metres from the centre of the sphere, the corresponding values are $Q/4\pi \ \varepsilon_0 \ d$ and $Q/4\pi\varepsilon_0 \ d^2$ respectively with $d > r$ as shown in Fig. 4.18 though its starting point is coincident with that of r. The variations of the potential and electric intensity with distance for a charged sphere are shown in Fig. 4.18.
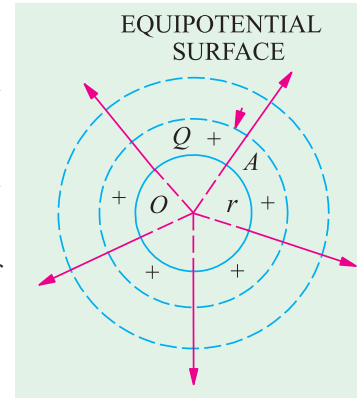

Fig. 4.17

4.15. Equipotential Surfaces

An equipotential surface is a surface in an electric field such that all points on it are at the same potential. For example, different spherical surfaces around a charged sphere are equipotential surfaces. One important property of an equipotential surface is that the direction of the electric field strength and flux density is always at right angles to the surface. Also, electric flux emerges out normal to such a surface. If, it is not so, then there would be some component of E along the surface resulting in potential difference between various points lying on it which is contrary to the definition of an equipotential surface.
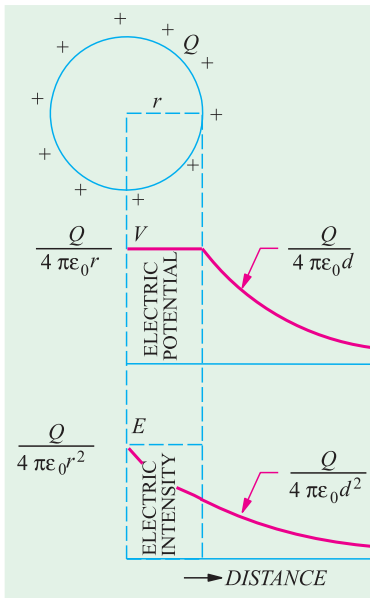
Fig. 4.18
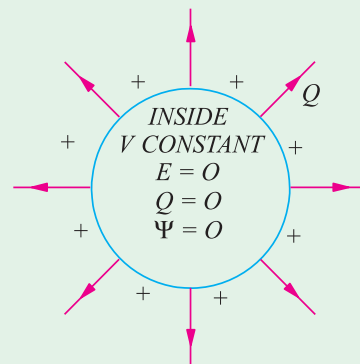
Fig. 4.19

4.16. Potential and Electric Intensity Inside a Conducting Sphere

It has been experimentally found that when charge is given to a conducting body say, a sphere then it resides entirely on its outer surface *i.e.*, within a conducting body whether hollow or solid, the charge is zero. Hence, **(*i*)** flux is zero **(*ii*)** field intensity is zero **(*iii*)** all points within the conductor are at the same potential as at its surface (Fig. 4.19).

Example 4.9. *Three concentric spheres of radii 4, 6 and 8 cm have charges of + 8, –6 and + 4 μμC respectively. What are the potentials and field strengths at points, 2, 5, 7 and 10 cm from the centre.*

Solution. As shown in Fig. 4.20, let the three spheres be marked A, B and C. It should be remembered that (*i*) the field intensity outside a sphere is the same as that obtained by considering the charge at its centre (*ii*) inside the sphere, the field strength is zero (*iii*) potential anywhere inside a sphere is the same as at its surface.

(*i*) Consider point 'a' at a distance of 2 cm from the centre O. Since it is inside all the spheres, field strength at this point is zero.

Fig. 4.20

Potential at 'a'

$$= \sum \frac{Q}{4\pi\varepsilon_0 d} = 9 \times 10^9 \sum \frac{Q}{d}$$

$$= 9 \times 10^9 \left(\frac{8 \times 10^{-12}}{0.04} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{1.35\ V}$$

(*ii*) Since point 'b' is outside sphere A but inside B and C.

$$\therefore \quad \text{Electrical field} = \frac{Q}{4\pi\varepsilon_0 d^2} = 9 \times 10^9 \frac{Q}{d}\ \text{N/C}$$

$$= 9 \times 10^9 \times \frac{8 \times 10^{-12}}{0.05^2} = \mathbf{28.8\ N/C}$$

Potential at   'b' $= 9 \times 10^9 \times \left(\frac{8 \times 10^{-12}}{0.05} - \frac{6 \times 10^{-12}}{0.06} + \frac{4 \times 10^{-12}}{0.08} \right) = \mathbf{0.99\ V}$

(*iii*) The field strength at point 'c' distant 7 cm from centre O

$$= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.07^2} - \frac{6 \times 10^{-12}}{0.07^2} \right] = \mathbf{3.67\ N/C}$$

Potential at 'c' $= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.07} - \frac{6 \times 10^{-12}}{0.07} + \frac{4 \times 10^{-12}}{0.08} \right] = \mathbf{0.71\ V}$

(*iv*) Field strength at 'd' distant 10 cm from point O is

$$= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.1^2} - \frac{6 \times 10^{-12}}{0.1^2} + \frac{4 \times 10^{-12}}{0.1^2} \right] = \mathbf{5.4\ N/C}$$

Potential at   'd' $= 9 \times 10^9 \times \left[\frac{8 \times 10^{-12}}{0.1} - \frac{6 \times 10^{-12}}{0.1} + \frac{4 \times 10^{-12}}{0.1} \right] = \mathbf{0.54\ V}$
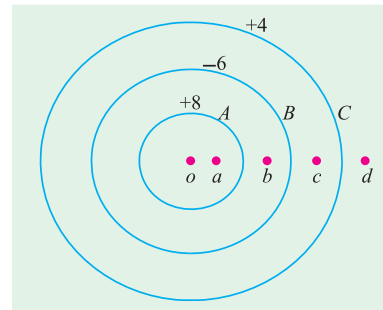
Example 4.10. *Two positive point charges of 12×10^{-10} C and $8 \cdot \times 10^{-10}$ C are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.*

Solution. Suppose the 12×10^{-10} C charge to be fixed. Now, the potential of a point 10 cm from this charge $= 9 \times 10^9 \dfrac{12 \times 10^{-10}}{0.1} = 108\ V$

The potential of a point distant 6 cm from it

$$= 9 \times 10^9 \times \frac{12 \times 10^{-10}}{0.06} = 180\ V$$

$\therefore$ potential difference $= 180 - 108 = 72\ V$

Work done $= \text{charge} \times \text{p.d.} = 8 \times 10^{-10} \times 72 = \mathbf{5.76 \times 10^{-8}\ joule}$

Example 4.11. *A point charge of 10^{-9} C is placed at a point A in free space. Calculate :*
(i) the intensity of electrostatic field on the surface of sphere of radius 5 cm and centre A.

    (*ii*) *the difference of potential between two points 20 cm and 10 cm away from the charge at A.*

                                                   (Elements of Elect.-I, Banglore Univ. 1987)

Solution. (*i*) $E = Q/4\pi\varepsilon_0 r^2 = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times (5 \times 10^{-2})^2 = \textbf{3,595 V/m}$

(*ii*) Potential of first point $= Q/4\pi\varepsilon_0 d = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.2 = 45$ V

Potential of second point $= 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.1 = 90$ V

∴   p.d. between two points $= 90 - 45 = \textbf{45 V}$

4.17. Potential Gradient

It is defined as the rate of change of potential with distance in the direction of electric force

i.e. $\dfrac{dV}{dx}$

Its unit is volt/metre although volt/cm is generally used in practice. Suppose in an electric field of strength E, there are two points dx metre apart. The p.d. between them is

$$dV = E \cdot (- dx) = - E \cdot dx \quad \therefore \quad \boxed{E = -\dfrac{dV}{dx}} \qquad \qquad \textit{...(i)}$$

The +ve sign indicates that the electric field is directed outward, while the potential increases inward.

Hence, it means that electric intensity at a point is equal to the negative potential gradient at that point.

4.18. Breakdown Voltage and Dielectric Strength

An insulator or dielectric is a substance within which there are no mobile electrons necessary for electric conduction. However, when the voltage applied to such an insulator exceeds a certain value, then it breaks down and allows a heavy electric current (much larger than the usual leakage current) to flow through it. If the insulator is a solid medium, it gets punctured or cracked.

The disruptive or breakdown voltage of an insulator is the minimum voltage required to break it down.[*]

Dielectric strength of an insulator or dielectric medium is given by the *maximum potential difference which a unit thickness of the medium can withstand without breaking down*.

In other words, the dielectric strength is given by the potential gradient necessary to cause breakdown of an insulator. Its unit is volt/metre (V/m) although it is usually expressed in kV/mm.

For example, when we say that the dielectric strength of air is 3 kV/mm, then it means that the maximum p.d. which one mm thickness of air can withstand across it without breaking down is 3 kV or 3000 volts. If the p.d. exceeds this value, then air insulation breaks down allowing large electric current to pass through.

Dielectric strength of various insulating materials is very important factor in the design of high-voltage generators, motors and transformers. Its value depends on the thickness of the insulator, temperature, moisture, content, shape and several other factors.

For example doubling the thickness of insulation does not double the safe working voltage in a machine.[**]

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[*]   Flashover is the disruptive discharge which taken places over the surface of an insulator and occurs when the air surrounding it breaks down. Disruptive conduction is luminous.

[**]   The relation between the breakdown voltage V and the thickness of the dielectric is given approximately by the relation $V = At^{2/3}$

      where A is a constant depending on the nature of the medium and also on the thickness t. The above statement is known as Baur's law.

Note. It is obvious that the electric intensity E, potential gradient and dielectric strength are dimensionally equal.

4.19. Safety Factor of a Dielectric

It is given by the ratio of the dielectric strength of the insulator and the electric field intensity established in it. If we represent the dielectric strength by E_{bd} and the actual field intensity by E, then safety factor
$$k = E_{bd}/E$$

For example, for air $E_{bd} = 3 \times 10^6$ V/m. If we establish a field intensity of 3×10^5 V/m in it, then, $k = 3 \times 10^6/3 \cdot 10^5 = 10$.

4.20. Boundary Conditions

There are discontinuities in electric fields at the boundaries between conductors and dielectrics of different permittivities. The relationships existing between the electric field strengths and flux densities at the boundary are called the boundary conditions.

With reference to Fig. 4.21, first boundary conditions is that the normal component of flux density is continuous across a surface.

As shown, the electric flux approaches the boundary BB at an angle θ_1 and leaves it at θ_2. D_{1n} and D_{2n} are the normal components of D_1 and D_2. According to first boundary condition,
$$D_{1n} = D_{2n} \qquad ...(i)$$

Fig. 4.21

The second boundary condition is that the tangential field strength is continuous across the boundary
$$\therefore \qquad E_{1t} = E_{2t} \qquad ...(ii)$$

In Fig. 4.21, we see that
$$D_{1n} = D_1 \cos \theta_1 \quad \text{and} \quad D_{2n} = D_2 \cos \theta_2$$

Also $\qquad E_1 = D_1/\varepsilon_1 \qquad$ and $\quad E_{1t} = D_1 \sin \theta_2/\varepsilon_1$

Similarly, $\quad E_2 = D_2/\varepsilon_2 \qquad$ and $\quad E_{2t} = D_2 \sin \theta_2/\varepsilon_2$

$$\therefore \qquad \frac{D_{1n}}{E_1 t} = \frac{\varepsilon_1}{\tan \theta_1} \qquad \text{and} \qquad \frac{D_{2n}}{E_{2t}} = \frac{\varepsilon_2}{\tan \theta_2}$$

Since $\quad D_{1n} = D_{2n} \qquad$ and $\qquad E_{1t} = E_{2t} \quad \therefore \quad \dfrac{\tan \theta_1}{\tan \theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}$

This gives the law of electric flux refraction at a boundary.

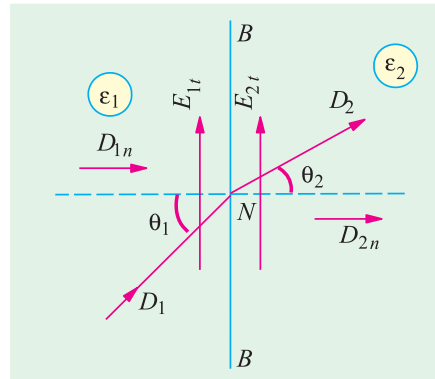It is seen that if $\varepsilon_1 > \varepsilon_2$, $\theta_1 > \theta_2$.

Table No. 4.1
Dielectric Constant and Strength
(*indicates average value)

Insulating material	Dielectric constant or relative permittivity (e_r)	Dielectric Strength in kV/mm
Air	1.0006	3.2
Asbestos*	2	2
Bakelite	5	15
Epoxy	3.3	20

Glass	5-12	12-100
Marble*	7	2
Mica	4-8	20-200
Micanite	4-5-6	25-35
Mineral Oil	2.2	10
Mylar	3	400
Nylon	4.1	16
Paper	1.8-2.6	18
Paraffin wax	1.7-2.3	30
Polyethylene	2.3	40
Polyurethane	3.6	35
Porcelain	5-6.7	15
PVC	3.7	50
Quartz	4.5-4.7	8
Rubber	2.5-4	12-20
Teflon	2	20
Vacuum	1	infinity
Wood	2.5-7	---

Example 4.12. *Find the radius of an isolated sphere capable of being charged to 1 million volt potential before sparking into the air, given that breakdown voltage of air is 30,000 V/cm.*

Solution. Let r metres be the radius of the spheres, then

$$V = \frac{Q}{4\pi\varepsilon_0 r} = 10^6 \text{ V} \qquad ...(i)$$

Breakdown voltage $= 30,000 \text{ V/cm} = 3 \times 10^6 \text{ V/m}$

Since electric intensity equals breakdown voltage

$$\therefore \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2} = 3 \times 10^6 \text{ V/m} \qquad ...(ii)$$

Dividing (i) by (ii), we get $r = 1/3 = $ **0.33 metre**

Example 4.13. *A parallel plate capacitor having waxes paper as the insulator has a capacitance of 3800 pF, operating voltage of 600 V and safety factor of 2.5. The waxed paper has a relative permittivity of 4.3 and breakdown voltage of 15 · 10^6 V/m. Find the spacing d between the two plates of the capacitor and the plate area.*

Solution. Breakdown voltage V_{bd} = operating voltage × safety factor = 600 · 2.5 = 1500 V

$V_{bd} = d \times E_{bd}$ or $d = 1500/15 \times 10^6 = 10^{-4}$ m = 0.1 mm

$C = \varepsilon_0 \varepsilon_r A/d$ or $A = Cd/\varepsilon_0 \varepsilon_r = 3800 \times 10^{-9} \times 10^{-4}/8.854 \times 10^{-12} \cdot 4.3 = $ **0.01 m^2**

Example 4.14. *Two brass plates are arranged horizontally, one 2 cm above the other and the lower plate is earthed. The plates are charged to a difference of potential of 6,000 volts. A drop of oil with an electric charge of 1.6 × 10^{-19} C is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop ?*

Solution. The electric intensity is equal to the potential gradient between the plates.

$$g = 6,000/2 = 3,000 \text{ volt/cm} = 3 \times 10^5 \text{ V/m}$$

$$\therefore \qquad E = 3 \times 10^5 \text{ V/m} \quad \text{or} \quad \text{N/C}$$

$$\therefore \qquad \text{force on drop} = E \times Q = 3 \times 10^5 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14} \text{ N}$$

$$\text{Wt. of drop} = mg \text{ newton}$$

$$\therefore \qquad m \times 9.81 = 4.8 \times 10^{-14} \quad \therefore \ m = \textbf{4.89} \times \textbf{10}^{\textbf{-15}} \textbf{ kg}$$

Example 4.15. *A parallel-plate capacitor has plates 0.15 mm apart and dielectric with relative permittivity of 3. Find the electric field intensity and the voltage between plates if the surface charge is 5×10^{-4} $\mu C/cm^2$.*

(Electrical Engineering, Calcutta Univ.)

Solution. The electric intensity between the plates is

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} \text{ volt/metre;}$$

Now, $\sigma = 5 \times 10^{-4} \mu C/cm^2 = 5 \times 10^{-6} C/m^2$

Since, charge density equals flux density

$$\therefore \quad E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3} = 188,000 \text{ V/m} = \textbf{188 kV/m}$$

Now potential difference $V = E \times dx = 188,000 \times (0.15 \times 10^{-3}) = \textbf{2.82 V}$

Example 4.16. *A parallel-plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon ($\varepsilon_r = 2.0$) 6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100 V is applied across the capacitor, find the electric field (E_0) in the air, electric field E_t in Teflon, flux density D_a in air, flux density D_t in Teflon and potential difference V_t across Teflon slab.* **(Circuit and Field Theory, A.M.I.E. Sec. B)**

Solution.
$$C = \frac{\varepsilon_0 A}{(d_1/\varepsilon_{e1} + d_2/\varepsilon_{r2})} = \frac{8.854 \times 10^{-12} \times (0.5)^2}{(6 \times 10^{-3}/2) + (4 \times 10^{-3}/1)} = 3.16 \times 10^{-10} \text{ F}$$

$$Q = CV = 3.16 \times 10^{-10} \times 100 = 31.6 \times 10^{-9} C$$

$$D = Q/A = 31.6 \times 10^{-9}/(0.5)^2 = 1.265 \times 10^{-7} C/m^2$$

The charge or flux density will be the same in both media *i.e.* $D_a = D_t = D$

In air, $\quad E_0 = D/\varepsilon_0 = 1.265 \times 10^{-7}/8.854 \times 10^{-12} = 14,280 \text{ V/m}$

In Teflon, $\quad E_t = D/\varepsilon_0 \varepsilon_r = 14,280/2 = 7,140 \text{ V/m}$

$$V_t = E_t \times d_t = 7,140 \times 6 \times 10^{-3} = \textbf{42.8 V}$$

Example 4.17. *Calculate the dielectric flux in micro-coulombs between two parallel plates each 35 cm square with an air gap of 1.5 mm between them, the p.d. being 3,000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7,500 V.*

(Elect. Engg.-I, Nagpur Univ.)

Solution. The capacitance of the two parallel plates is

$C = \varepsilon_0 \varepsilon_r A/d \quad$ Now, $\varepsilon_r = 1 \quad$ —for air

$A = 35 \times 35 \times 10^{-4} = 1,225 \times 10^{-4} m^2; d = 1.5 \times 10^{-2} m$

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 1,225 \times 10^{-4}}{1.5 \times 10^{-3}} \text{ F} = 7.22 \times 10^{-16} \text{ F}$$

Charge $Q = CV = 7.22 \times 10^{-10} \times 3,000$ coulomb

Capacitor

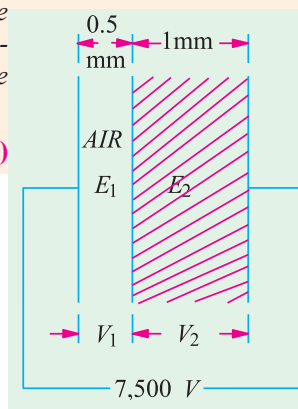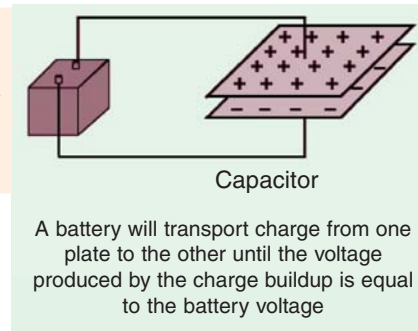A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage

0.5 mm |←1mm→|

AIR

E_1 | E_2

V_1 | V_2

7,500 V

Fig. 4.22

Dielectric flux $= 7.22 \times 3,000 \times 10^{-10}$ C

$\qquad = 2.166 \times 10^{-6}$ C $= \mathbf{2.166 \ \mu C}$

With reference to Fig. 4.23, we have

$$V_1 = E_1 x_1 = 0.5 \times 10^{-3} E_1 \ ; \ V_2 = 10^{-3} E_2$$

Now $\qquad V = V_1 + V_2$

$\therefore \qquad 7,500 = 0.5 \times 10^{-3} E_1 + 10^{-3} E_2$

or $E_1 + 2 E_2 = 15 \times 10^6$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ...(*i*)

Also $\qquad D = \varepsilon_0 \varepsilon_{r1} E_1 = \varepsilon_0 \varepsilon_{r2} E_2 \qquad \therefore \ E_1 = 6 E_2$ $\qquad\qquad\qquad\qquad$ ...(*ii*)

From (*i*) and (*ii*), we obtain $E_1 = \mathbf{11.25 \times 10^6}$ *V/m*$; E_2 = \mathbf{1.875 \times 10^6}$ *V/m*

Example 4.18. *An electric field in a medium with relative permittivity of 7 passes into a medium of relative permittivity 2. If E makes an angle of 60° with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric ?*

(Elect. Engg. Nagpur Univ.)

Solution. As seen from Art. 4.19.

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \quad \text{or} \quad \frac{\tan 60°}{\tan \theta_2} = \frac{7}{2} \quad \therefore \ \tan \theta_2 = \sqrt{3} \times 2/7 = 4.95 \text{ or } \theta_2 = \mathbf{26°20'}$$

Example 4.19. *Two parallel sheets of glass having a uniform air gap between their inner surfaces are sealed around their edges (Fig. 4.23). They are immersed in oil having a relative permittivity of 6 and are mounted vertically. The glass has a relative permittivity of 3. Calculate the values of electric field strength in the glass and the air when that in the oil is 1.2 kV/m. The field enters the glass at 60° to the horizontal.*

Solution. Using the law of electric flux refraction, we get (Fig. 4.23).

$$\tan \theta_2 / \tan \theta_1 = \varepsilon_2 / \varepsilon_1 = \varepsilon_0 \varepsilon_{r2} / \varepsilon_0 \varepsilon_{r1} = (\varepsilon_{r2} / \varepsilon_{r1})$$

$\therefore \qquad \tan \theta_2 = (6/3) \tan 60°$

$\qquad\qquad = 2 \times 1.732 = 3.464;$

$\qquad \theta_2 = 73.9°$

Similarly

$\tan \theta_3 = (\varepsilon_{r3} / \varepsilon_{r2}) \tan \theta_2 = (1/6) \tan 73.9°$

$\qquad = 0.577; \ \therefore \ \theta_3 = 30°$

As shown in Art. 4.20.

$\qquad D_{1n} = D_{2n}$ or $D_1 \cos \theta_1 = D_2 \cos \theta_2$

$\therefore \quad D_2 = D_1 \times \cos \theta_1 / \cos \theta_2$ or $\varepsilon_0 \varepsilon_{r2} E_2$

$\qquad\qquad = \varepsilon_0 \varepsilon_{r1} E_1 \times \cos \theta_1 / \cos \theta_2$

$\therefore \ 6 E_2 = 3 \times 1.2 \times 10^3 \times \cos 60° / \cos 73.9°$

$\therefore \quad E_2 = 1082$ V/m

Now, $\varepsilon_0 \varepsilon_{r3} E_3 \cos \theta_3 = \varepsilon_0 \varepsilon_{r2} E_2 \cos \theta_2$

$\therefore \quad E_3 = E_2 (\varepsilon_{r2} / \varepsilon_{r3}) \times (\cos \theta_2 / \cos \theta_3)$

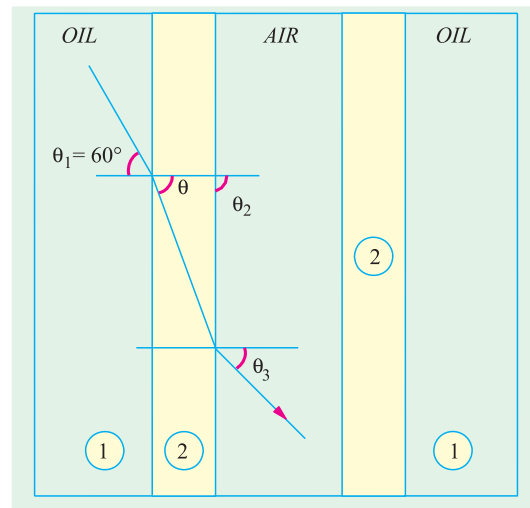$= 1082 \ (6/1) \ (\cos 73.9° / \cos 30°) = \mathbf{2079V/m}$

Fig. 4.23

Tutorial Problems No. 4.1

1. Two parallel metal plates of large area are spaced at a distance of 1 cm from each other in air and a p.d. of 5,000 V is maintained between them. If a sheet of glass 0.5 cm thick and having a relative permittivity of 6 is introduced between the plates, what will be the maximum electric stress and where will it occur ?   **[8.57 kV/cm; in air]**

2. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 1.5 cm and relative permittivity 7 is introduced between the plates.   **[1.4×10^8 V/m)**

3. A capacitor is made up of two parallel circular metal discs separated by three layers of dielectric of equal thickness but having relative permittivities of 3, 4 and 5 respectively. The diameter of each disc is 25.4 cm and the distance between them is 6 cm. Calculate the potential gradient in each dielectric when a p.d. of 1,500 V is applied between the discs.   **[319.2; 239.4; 191.5 kV/m]**

4. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 0.5 cm and relative permittivity 5 is introduced between the plates.   **[0.625×10^4 V/m]**

5. The capacitance of a capacitor formed by two parallel metal plates, each having an effective surface area of 50 cm^2 and separated by a dielectric 1 mm thick, is 0.0001 μF. The plates are charged to a p.d. of 200 V. Calculate (*a*) the charge stored (*b*) the electric flux density (*c*) the relative permittivity of the dielectric.   **[(*a*) 0.02 μC (*b*) 4 μC/m^2 (*c*) 2.26]**

6. A capacitor is constructed from two parallel metallic circular plates separated by three layers of dielectric each 0.5 cm thick and having relative permittivity of 4, 6 and 8 respectively. If the metal discs are 15.25 cm in diameter, calculate the potential gradient in each dielectric when the applied voltage is 1,000 volts.   **(*Elect. Engg.-I Delhi Univ.*)**

7. A point electric charge of 8 μ*C* is kept at a distance of 1 metre from another point charge of −4 μC in free space. Determine the location of a point along the line joining two charges where in the electric field intensity is zero.

   (*Elect. Engineering, Kerala Univ.*)

8. In a given R-L circuit, R = 35Ω and L = 0.1H. Find (*i*) current through the circuit (*ii*) power factor if a 50 Hz frequency, voltage V = 220∠30° is applied across the circuit.

   (*RGPV, Bhopal 2001*)

9. Three voltage represented by e_1 = 20 sin ω*t*, e_2 = 30 sin (ω*t* = 45°) and e_3 = sin (ω*t* + 30°) are connected in series and then connected to a load of impedance (2 + *j* 3) Ω Find the resultant current and power factor of the circuit. Draw the phasor diagram.

   (*B.P.T.U. Orissa 2003*) (*RGPV Bhopal 2001*)

OBJECTIVE TESTS – 4

1. The unit of absolute permittivity of a medium is
   (*a*) joule/coulomb
   (*b*) newton-metre
   (*c*) farad/metere
   (*d*) farad/coulomb

2. If relative permittivity of mica is 5, its absolute permittivity is
   (*a*) 5 ε_0 (*b*) 5/ε_0
   (*c*) ε_0/5   (*d*) 8.854× 10^{-12}

3. Two similar electric charges of 1 C each are placed 1 m apart in air. Force of repulsion

between them would be nearly...... newton
   (*a*) 1
   (*b*) 9×10^9
   (*c*) 4 π
   (*d*) 8.854 × 10^{-12}

4. Electric flux emanating from an electric charge of + *Q* coulomb is
   (*a*) Q/ε_0 (*b*) Q/ε_r
   (*c*) $Q/\varepsilon_0\varepsilon_r$ (*d*) *Q*

5. The unit of electric intensity is
   (*a*) joule/coulomb
   (*b*) newton/coulomb

(c) volt/metre

(d) both (b) and (c)

6. If D is the electric flux density, then value of electric intensity in air is

(a) D/ε_0 (b) $D/\varepsilon_0\varepsilon_r$

(c) dV/dt (d) $Q/\varepsilon A$

7. For any medium, electric flux density D is related to electric intensity E by the equation

(a) $D = \varepsilon_0 E$ (b) $D = \varepsilon_0\varepsilon_r E$

(c) $D = E/\varepsilon_0\varepsilon_r$ (d) $D = \varepsilon_0 E/\varepsilon_r$

8. Inside a conducting sphere,...remains constant

(a) electric flux

(b) electric intensity

(c) charge

(d) potential

9. The SI unit of electric intensity is

(a) N/m

(b) V/m

(c) N/C

(d) either (b) or (c)

10. According to Gauss's theorem, the surface integral of the normal component of electric flux density D over a closed surface containing charge Q is

(a) Q (b) Q/ε_0

(c) $\varepsilon_0 Q$ (d) Q^2/ε_0

11. Which of the following is zero inside a charged conducting sphere ?

(a) potential

(b) electric intensity

(c) both (a) and (b)

(d) both (b) and (c)

12. In practice, earth is chosen as a place of zero electric potential because it

(a) is non-conducting

(b) is easily available

(c) keeps lossing and gaining electric charge every day

(d) has almost constant potential.

ANSWERS

1. *c*	2. *a*	3. *b*	4. *d*	5. *d*	6. *a*
7. *b*	8. *d*	9. *d*	10. *a*	11. *c*	12. *d*

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