Contents

CHAPTER 3

WORK, POWER AND ENERGY

Learning Objectives

- Effect of Electric Current
- Joule's Law of Electric Heating
- Thermal Efficiency
- S-I. Units
- Calculation of Kilo-watt Power of a Hydroelectric Station

Today, life without electricity is highly unimaginable. Electric locomotives, heaters, and fans are some of the appliances and machines which convert electricity into work and energy

3.1. Effect of Electric Current

It is a matter of common experience that a conductor, when carrying current, becomes hot after some time. As explained earlier, an electric current is just a directed flow or drift of electrons through a substance. The moving electrons as they pass through molecules of atoms of that substance, collide with other electrons. This electronic collision results in the production of heat. This explains why passage of current is always accompanied by generation of heat.

3.2. Joule's Law of Electric Heating

The amount of work required to maintain a current of I amperes through a resistance of R ohm for t second is

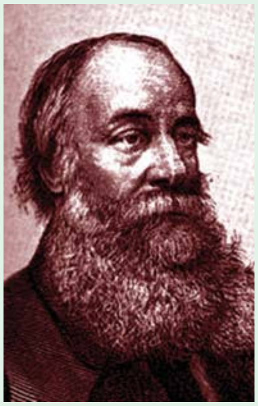
$$\begin{aligned} \text{W.D.} &= I^2 Rt \text{ joules} \\ &= VIt \text{ joules} && (\because\ R = V/I) \\ &= Wt \text{ joules} && (\because\ W = VI) \\ &= V^2t/R \text{ joules} && (\because\ I = V/R) \end{aligned}$$

This work is converted into heat and is dissipated away. The amount of heat produced is

$$H = \frac{\text{work done}}{\text{mechanical equivalent of heat}} = \frac{W.D.}{J}$$

where J = 4,186 joules/kcal = 4,200 joules / kcal (approx)

$$\therefore \quad H = I^2Rt/4{,}200 \text{ kcal} = VIt/4{,}200 \text{ kcal}$$
$$= Wt/4{,}200 \text{ kcal} = V^2t/4{,}200\,R \text{ kcal}$$



James Joule*

3.3. Thermal Efficiency

It is defined as the ratio of the heat actually utilized to the total heat produced electrically. Consider the case of the electric kettle used for boiling water. Out of the total heat produced **(*i*)** some goes to heat the apparatus itself *i.e.* kettle **(*ii*)** some is lost by radiation and convection etc. and **(*iii*)** the rest is utilized for heating the water. Out of these, the heat utilized for useful purpose is that in **(*iii*)**. Hence, thermal efficiency of this electric apparatus is the ratio of the heat utilized for heating the water to the total heat produced.

Hence, the relation between heat produced electrically and heat absorbed usefully becomes

$$\frac{VIt}{J} \times \eta = ms\,(\theta_2 - \theta_1)$$

In an electric kettle, electric energy is converted into heat energy.

Example 3.1. *The heater element of an electric kettle has a constant resistance of 100 Ω and the applied voltage is 250 V. Calculate the time taken to raise the temperature of one litre of water from 15ºC to 90ºC assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100 g, find how long will it take to raise a second litre of water through the same temperature range immediately after the first.*

(Electrical Engineering, Calcutta Univ.)

* James Joule was born in Salford, England, in 1818. He was a physicist who is credited with discovering the law of conservation of energy. Joule's name is used to describe the international unit of energy known as the joule.

Solution. Mass of water $= 1000 \text{ g} = 1 \text{ kg}$ $(\because 1 \text{ cm}^3 \text{ weight } 1 \text{ gram})$

Heat taken by water $= 1 \times (90 - 15) = 75$ kcal

Heat taken by the kettle $= 0.1 \times (90 - 15) = 7.5$ kcal

Total heat taken $= 75 + 7.5 = 82.5$ kcal

Heat produced electrically $H = I^2Rt/J$ kcal

Now, $I = 250/100 = 2/5$ A, $J = 4{,}200$ J/kcal; $H = 2.5^2 \times 100 \times t/4200$ kcal

Heat actually utilized for heating one litre of water and kettle

$$= 0.85 \times 2.5^2 \times 100 \times t/4{,}200 \text{ kcal}$$

$$\therefore \quad \frac{0.85 \times 6.25 \times 100 \times t}{4{,}200} = 82.5 \quad \therefore \quad t = \mathbf{10 \text{ min } 52 \text{ second}}$$

In the second case, heat would be required only for heating the water because kettle would be already hot.

$$\therefore \quad 75 = \frac{0.85 \times 6.25 \times 100 \times t}{4{,}200} \quad \therefore \quad t = \mathbf{9 \text{ min } 53 \text{ second}}$$

Example 3.2. *Two heater A and B are in parallel across supply voltage V. Heater A produces 500 kcal in 200 min. and B produces 1000 kcal in 10 min. The resistance of A is 10 ohm. What is the resistance of B ? If the same heaters are connected in series across the voltage V, how much heat will be prduced in kcal in 5 min ?* **(Elect. Science - II, Allahabad Univ. 1992)**

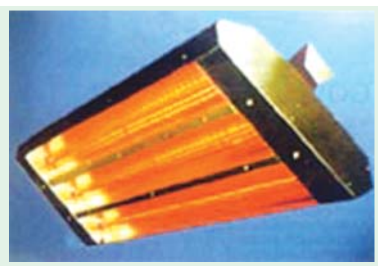
Solution. Heat produced $= \dfrac{V^2 t}{JR}$ kcal

For heater A, $$500 = \frac{V^2 \times (20 \times 60)}{10 \times J} \quad \ldots(i)$$

For heater B, $$1000 = \frac{V^2 \times (10 \times 60)}{R \times J} \quad \ldots(ii)$$

From Eq. ***(i)*** and ***(ii)***, we get, $R =$ **2.5 Ω**

(*a*)



(*b*)

(*c*)

(*d*)

In this *a*, *b*, and *c* are heaters which convert electric energy into heat; and *d* is the electric bulb which coverts electric energy into light and heat

When the two heaters are connected in series, let H be the amount of heat produced in kcal. Since combined resistance is (10 + 2.5) = 12.5 Ω, hence

$$H = \frac{V^2 \times (5 \times 60)}{12.5 \times J} \quad \text{...}(iii)$$

Dividing Eq. **(*iii*)** by Eq. **(*i*)**, we have H = **100 kcal.**

Example 3.3. *An electric kettle needs six minutes to boil 2 kg of water from the initial temperature of 20ºC. The cost of electrical energy required for this operation is 12 paise, the rate being 40 paise per kWh. Find the kW-rating and the overall efficiency of the kettle.*

(F.Y. Engg. Pune Univ.)

Solution. Input energy to the kettle = $\frac{12 \text{ paise}}{40 \text{ paise/kWh}}$ = 0.3 kWh

$$\text{Input power} = \frac{\text{energy in kWh}}{\text{Time in hours}} = \frac{0.3}{(6/60)} = 3 \text{ kW}$$

Hence, the power rating of the electric kettle is 3 kW

Energy utilised in heating the water

= mst = 2 × 1 × (100 –20) = 160 kcal = 160 /860 kWh = 0.186 kWh.

Efficiency = output/input = 0.186/0.3 = 0.62 = **62%.**

3.4. S.I. Units

1. Mass. It is quantity of matter contained in a body.

Unit of mass is kilogram (kg). Other multiples commonly used are :

1 quintal = 100 kg, 1 tonne = 10 quintals = 1000 kg

2. Force. Unit of force is newton (N). Its definition may be obtained from Newton's Second Law of Motion *i.e.* $F = ma$.

If m = 1 kg ; a = 1m/s^2, then F = 1 newton.

Hence, one newton is that force which can give an acceleration of 1 m/s^2 to a mass of 1 kg.

Gravitational unit of force is kilogram-weight (kg-wt). It may be defined as follows :

or

It is the force which can impart an acceleration of 9.8 m/s^2 to a mass of 1 kg.

It is the force which can impart an acceleration of 1 m/s^2 to a mass of 9.8 kg.

Obviously, 1 kg-wt. = 9.8 N

3. Weight. It is the force with which earth pulls a body downwards. Obviously, its units are the same as for force.

(*a*) Unit of weight is newton (N)

(*b*) Gravitational unit of weight is kg-wt.*

Note. If a body has a mass of m kg, then its weight, W = mg newtons = 9.8 newtons.

4. Work, If a force F moves a body through a distance S in its direction of application, then

Work done $W = F \times S$

(*a*) Unit of work is joule (J).

If, in the above equation, F = 1 N : S = 1 m ; then work done = 1 m.N or joule.

Hence, one joule is the work done when a force of 1 N moves a body through a distance of 1 m in the direction of its application.

(*b*) Gravitational unit of work is m-kg. wt or m-kg**.

* Often it is referred to as a force of 1 kg, the word 'wt' being omitted. To avoid confusion with mass of 1 kg, the *force* of 1 kg is written in engineering literature as kgf instead of kg. wt.

** Generally the work '*wt*' is omitted and the unit is simply written as m-kg.

If F = 1 kg-wt; S = 1 m; then W.D. = 1 m-kg. Wt = 1 m-kg.

Hence, one m-kg is the work done by a force of one kg-wt when applied over a distance of one metre.

Obviously, 1 m-kg = 9.8 m-N or J.

5. Power. It is the rate of doing work. Its units is watt (W) which represents 1 joule per second.

$$1 \text{ W} = 1 \text{ J/s}$$

If a force of F newton moves a body with a velocity of ν m./s then

$$\text{power} = F \times \nu \text{ watt}$$

If the velocity ν is in km/s, then

$$\text{power} = F \times \nu \text{ kilowatt}$$

6. Kilowatt-hour (kWh) and kilocalorie (kcal)

$$1 \text{ kWh} = 1000 \times 1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J}$$

$$1 \text{ kcal} = 4{,}186 \text{ J} \quad \therefore\ 1 \text{ kWh} = 36 \times 10^5/4{,}186 = 860 \text{ kcal}$$

7. Miscellaneous Units

(*i*) 1 watt hour (Wh) = $1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600 \text{ J}$

(*ii*) 1 horse power (metric) = 75 m-kg/s = 75 × 9.8 = 735.5 J/s or watt

(*iii*) 1 kilowatt (kW) = 1000 W and 1 megawatt (MW) = 10^6 W

3.5. Calculation of Kilo-watt Power of a Hydroelectric Station

Let Q = water discharge rate in cubic metres/second (m^3/s), H = net water head in metre (m). g = 9.81, η; overall efficiency of the hydroelectric station expressed as a fraction.

Since 1 m^3 of water weighs 1000 kg., discharge rate is 1000 Q kg/s.

When this amount of water falls through a height of H metre, then energy or work available per second or available power is

$$= 1000\, QgH \text{ J/s} \quad \text{or} \quad W = QgH \text{ kW}$$

Since the overall station efficiency is η power actually available is = 9.81 ηQH kW.

Example 3.4. *A de-icing equipment fitted to a radio aerial consists of a length of a resistance wire so arranged that when a current is passed through it, parts of the aerial become warm. The resistance wire dissipates 1250 W when 50 V is maintained across its ends. It is connected to a d.c. supply by 100 metres of this copper wire, each conductor of which has resistance of 0.006 Ω/m.*

Calculate

(*a*) *the current in the resistance wire*

(*b*) *the power lost in the copper connecting wire*

(*c*) *the supply voltage required to maintain 50 V across the heater itself.*

Solution. (*a*) Current = wattage/voltage = 1250/50 = **25 A**

(*b*) Resistance of one copper conductor = 0.006 × 100 = 0.6 Ω

Resistance of both copper conductors = 0.6 × 2 = 1.2 Ω

Power loss = I^2R watts = 252 × 1.2 = **750 W**

(*c*) Voltage drop over connecting copper wire = IR volt = 25 × 1.2 = 30 V

∴ Supply voltage required = 50 + 30 = **80 V**

Example 3.5. *A factory has a 240-V supply from which the following loads are taken :*

Lighting : *Three hundred 150-W, four hundred 100 W and five hundred 60-W lamps*

Heating : *100 kW*

Motors : *A total of 44.76 kW (60 b.h.p.) with an average efficiency of 75 percent*

Misc. : *Various load taking a current of 40 A.*

Assuming that the lighting load is on for a period of 4 hours/day, the heating for 10 hours per day and the remainder for 2 hours/day, calculate the weekly consumption of the factory in kWh when working on a 5-day week.

What current is taken when the lighting load only is switched on ?

A factory needs electric power for lighting and running motors

Solution. The power consumed by each load can be tabulated as given below :

		Power consumed
Lighting	300×150	$= 45{,}000 = 45$ kW
	400×100	$= 40{,}000 = 40$ kW
	500×60	$= 30{,}000 = 30$ kW
		Total $= 115$ kW
Heating		$= 100$ kW
Motors		$= 44.76/0.75 = 59.7$ kW
Misc.		$= 240 \times 40/1000 = 9.6$ kW

Similarly, the energy consumed/day can be tabulated as follows :

	Energy consumed / day	
Lighting	$= 115\text{ kW} \times 4\text{ hr}$	$= 460$ kWh
Heating	$= 100\text{ kW} \times 10\text{ hr}$	$= 1{,}000$ kWh
Motors	$= 59.7\text{ kW} \times 2\text{ hr}$	$= 119.4$ kWh
Misc.	$= 9.6\text{ kW} \times 2\text{ hr}$	$= 19.2$ kWh
Total daily consumption		$= 1{,}598.6$ kWh
Weekly consumption		$= 1{,}598.6 \times 5 =$ **7,993 kWh**
Current taken by the lighting load alone		$= 115 \times 1000/240 =$ **479 A**

Example 3.6. *A Diesel-electric generating set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35%* **(a)** *calculate the mass of oil required per hour* **(b)** *the electric energy generated per tonne of the fuel.*

Diesel electric generator set

Solution. Output = 25 kW, Overall $\eta = 0.35$,

Input $= 25/0.35 = 71.4$ kW

$\therefore$ input per hour $= 71.4\text{ kWh} = 71.4 \times 860 = 61{,}400$ kcal

Since 1 kg of fuel-oil produces 12,500 kcal

(*a*) $\therefore$ mass of oil required $= 61{,}400/12{,}500 =$ **4.91 kg**

(*b*) 1 tonne of fuel $= 1000$ kg

Heat content $= 1000 \times 12{,}500 = 12.5 \times 10^6$ kcal

$= 12.5 \times 10^6/860 = 14{,}530$ kWh

Overall $\eta = 0.35\%$ $\therefore$ energy output $= 14{,}530 \times 0.35 =$ **5,088 kWh**

Example 3.7. *The effective water head for a 100 MW station is 220 metres. The station supplies full load for 12 hours a day. If the overall efficiency of the station is 86.4%, find the volume of water used.*

Solution. Energy supplied in 12 hours $= 100 \times 12 = 1200$ MWh

$= 12 \times 10^5\text{ kWh} = 12 \times 10^5 \times 3^5 \times 10^5\text{ J} = 43.2 \times 10^{11}$ J

Overall $\eta = 86.4\% = 0.864$ $\therefore$ Energy input $= 43.2 \times 10^{11}/0.864 = 5 \times 10^{12}$ J

Suppose m kg is the mass of water used in 12 hours, then $m \times 9.81 \times 220 = 5 \times 10^{12}$

$\therefore$ $m = 5 \times 10^{12}/9.81 \times 220 = 23.17 \times 10^8$ kg

Volume of water $= 23.17 \times 10^8/10^3 =$ $\mathbf{23.17 \times 10^5\ m^3}$

($\because$ 1m^3 of water weighs 10^3 kg)

Example 3.8. *Calculate the current required by a 1,500 volts d.c. locomotive when drawing 100 tonne load at 45 km.p.h. with a tractive resistance of 5 kg/tonne along* **(a)** *level track* **(b)** *a gradient of 1 in 50. Assume a motor efficiency of 90 percent.*

Solution. As shown in Fig. 3.1 (*a*), in this case, force required is equal to the tractive resistance only.

(*a*) Force required at the rate of 5 kg-wt/tonne = 100 × 5 kg-wt. = 500 × 9.81 = 4905 N

Distance travelled/second = 45 × 1000/3600 = 12.5 m/s

Power output of the locomotive = 4905 × 12.5 J/s or watt = 61,312 W

η = 0.9 ∴ Power input = 61,312/0.9 = 68,125 W

∴ Currnet drawn = 68,125/1500 = **45.41 A**

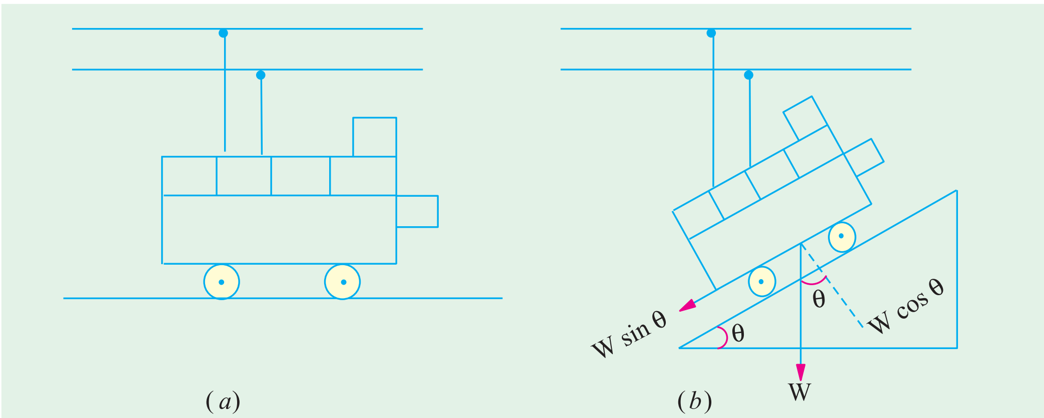


Fig. 3.1

(*b*) When the load is drawn along the gradient [Fig. 3.1 (*b*)], component of the weight acting downwards = 100 × 1/50 = 2 tonne-wt = 2000 kg-wt = 2000 × 9.81 = 19,620 N

Total force required = 19,620 + 4,905 = 24,525 N

Power output = force × velocity = 24,525 × 12.5 watt

Power input = 24,525 × 12.5/0.9 W ; Current drawn = $\dfrac{24,525 \times 12.5}{0.9 \times 1500}$ = **227 A**

Example 3.9. *A room measures 4 m × 7 m × 5 m and the air in it has to be always kept 15°C higher than that of the incoming air. The air inside has to be renewed every 35 minutes. Neglecting radiation loss, calculate the rating of the heater suitable for this purpose. Take specific heat of air as 0.24 and density as 1.27 kg/m^3.*

Solution. Volume of air to be changed per second = 4 × 7 × 5/35 = 60 = 1/15 m^3

Mass of air to be changed/second = (1/15) × 1.27 kg

Heat required/second = mass/second × sp. heat × rise in temp.

= (1.27/15) × 0.24 × 15 kcal/s = 0.305 kcal/s

= 0.305 × 4186 J/s = **1277 watt.**

Example 3.10. *A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. Angular speed ω = 2π *N*/60 rad/s = 2π × 75/60 = 7.85 rad/s

Power required for rotating the engine at this angular speed is

P = torque × angular speed = ωT watt = 60 × 7.85 = **471 W**

Energy required per start is = power $\times$ time per start = 471 $\times$ 8 = 3,768 watt-s = 3,768 J
= 3,768/3600 = 1.047 Wh

Energy drawn from the battery taking into consideration the efficiency of the motor and gearing
= 1.047/0.25 = 4.188 Wh

No. of start possible with a fully-charged battery = 100/4.188 = **24 (approx.)**

Example 3.11. *Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75ºC. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy ? Assume efficiencies of the heating equipment and lifting equipment to be 90% and 70% respectively.* **(Elect. Engg. A.M. Ae. S.I.)**

Solution. Mass of water heated = 45 kg. Heat required = 45 $\times$ 75 = 3,375 kcal

Heat produced electrically = 3,375/0.9 = 3,750 kcal. Now, 1 kcal = 4,186 J

$\therefore$ electrical energy expended = 3,750 $\times$ 4,186 J

Energy available for lifting the load is = 0.7 $\times$ 3,750 $\times$ 4,186 J

If h metre is the height through which the load of 5 tonnes can be lifted, then potential energy of the load = mgh joules = 5 $\times$ 1000 $\times$ 9.81 h joules

$\therefore$ $5000 \times 9.81 \times h = 0.7 \times 3{,}750 \times 4{,}186$ $\therefore$ h = **224 metres**

Example 3.12. *An hydro-electric station has a turbine of efficiency 86% and a generator of efficiency 92%. The effective head of water is 150 m. Calculate the volume of water used when delivering a load of 40 MW for 6 hours. Water weighs 1000 kg/m^3.*

Solution. Energy output = 40 $\times$ 6 = 240 MWh
= $240 \times 10^3 \times 36 \times 10^5 = 864 \times 10^9$ J

Overall η = 0.86 $\times$ 0.92 $\therefore$ Energy input = $\dfrac{864 \times 10^9}{0.86 \times 0.92} = 10.92 \times 10^{11}$ J

Since the head is 150 m and 1 m^3 of water weighs 1000 kg, energy contributed by each m^3 of water = 150 $\times$ 1000 m-kg (wt) = 150 $\times$ 1000 $\times$ 9.81 J = 147.2×10^4 J

$\therefore$ Volume of water for the required energy = $\dfrac{10.92 \times 10^{11}}{147.2 \times 10^4}$ = **74.18×10^4 m^3**

Example 3.13. *An hydroelectric generating station is supplied form a reservoir of capacity 6 million m^3 at a head of 170 m.*

(i) *What is the available energy in kWh if the hydraulic efficiency be 0.8 and the electrical efficiency 0.9 ?*

(ii) *Find the fall in reservoir level after a load of 12,000 kW has been supplied for 3 hours, the area of the reservoir is 2.5 km^2.*

(iii) *If the reservoir is supplied by a river at the rate of 1.2 m^3/s, what does this flow represent in kW and kWh/day ? Assume constant head and efficiency.*

Water weighs 1 tonne/m^3. **(Elect. Engineering-I, Osmania Univ.)**

Solution. *(i)* Wt. of water W = $6 \times 10^6 \times 1000$ kg wt = $6 \times 10^9 \times 9.81$ N

Water head = 170 m

Potential energy stored in this much water
= $Wh = 6 \times 10^9 \times 9.81 \times 170 \text{ J} = 10^{12}$ J

Overall efficiency of the station = 0.8 $\times$ 0.9 = 0.71

$\therefore$ energy available = 0.72×10^{13} J = $72 \times 10^{11}/36 \times 10^5$
= **2×10^6 kWh**

(ii) Energy supplied = 12,000 $\times$ 3 = 36,000 kWh

Energy drawn from the reservoir after taking into consideration the overall efficiency of the station = 36,000/0.72 = 5×10^4 kWh
= $5 \times 10^4 \times 36 \times 10^5 = 18 \times 10^{10}$ J

If m kg is the mass of water used in two hours, then, since water head is 170 m

$$mgh = 18 \times 10^{10}$$

or $$m \times 9.81 \times 170 = 18 \times 10^{10} \quad \therefore \; m = 1.08 \times 10^{8} \text{ kg}$$

If h metre is the fall in water level, then

$$h \times \text{area} \times \text{density} = \text{mass of water}$$

$$\therefore \quad h \times (2.5 \times 10^{6}) \times 1000 = 1.08 \times 10^{8} \quad \therefore \; h = 0.0432 \text{ m} = \mathbf{4.32 \text{ cm}}$$

(iii) Mass of water stored per second = 1.2 × 1000 = 1200 kg

$$\text{Wt. of water stored per second} = 1200 \times 9.81 \text{ N}$$

$$\text{Power stored} = 1200 \times 9.81 \times 170 \text{ J/s} = \mathbf{2{,}000 \text{ kW}}$$

$$\text{Power actually available} = 2{,}000 \times 0.72 = 1440 \text{ kW}$$

$$\text{Energy delivered /day} = 1440 \times 24 = \mathbf{34{,}560 \text{ kWh}}$$

Example 3.14. *The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is 45 × 10^{10} kg. What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85%. Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.*

(Power System, Allahabad Univ.)

Hydroelectric generators

Solution. Actual head available = 230 –30 = 200 m

Energy available at the turbine house = mgh

$$= 45 \times 10^{10} \times 9.81 \times 200 = 88.29 \times 10^{13} \text{ J}$$

$$= \frac{88.29 \times 10^{13}}{36 \times 10^{5}} = \mathbf{24.52 \times 10^{7} \text{ kWh}}$$

Overall $$\eta = 0.85$$

$$\therefore \quad \text{Energy output} = 24.52 \times 10^{7} \times 0.85 = \mathbf{20.84 \times 10^{7} \text{ kWh}}$$

The kinetic energy of water is just equal to its loss of potential energy.

$$\frac{1}{2} mv^2 = mgh \quad \therefore \; v = \sqrt{2\,gh} = \sqrt{2 \times 9.81 \times 200} = 62.65 \text{ m/s}$$

Power available from a mass of m kg when it flows with a velocity of v m/s is

$$P = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times 62.65^2 \text{ J/s} \quad \text{or} \quad \text{W}$$

Equating this to the maximum demand on the station, we get

$$\frac{1}{2} m\, 62.65^2 = 45 \times 10^{6} \quad \therefore \; m = 22{,}930 \text{ kg/s}$$

If A is the total area of the pipes in m^2, then the flow of water is Av m^3/s. Mass of water flowing/second $= Av \times 10^3$ kg (∴ 1 m^3 of water = 1000 kg)

$$\therefore \quad A \times v \times 10^{3} = 22{,}930 \quad \text{or } A = \frac{22{,}930}{62.65 \times 10^{3}} = 0.366 \text{ m}^2$$

If 'd' is the diameter of each pipe, then $\pi d^2/4 = 0.183$ ∴ d = **0.4826 m**

Example 3.15. *A large hydel power station has a head of 324 m and an average flow of 1370 cubic metres/sec. The reservoir is a lake covering an area of 6400 sq. km, Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate*

(i) *the available electric power ;*

(ii) *the number of days this power could be supplied for a drop in water level by 1 metre.*

(AMIE Sec. B Power System I (E-6) Winter)

Solution. ***(i)*** Available power = 9.81 ηQH kW = (0.9 × 0.95) × 1370 × 324 = 379, 524 kW = **379.52 MW.**

(ii) If A is the lake area in m^2 and h metre is the fall in water level, the volume of water used is $= A \times h = m^3$. The time required to discharge this water is Ah / Q second.

Now, $A = 6400 \times 10^6$ m^2 ; h = 1 m; Q = 1370 m^3 /s.

$\therefore \quad t = 6400 \times 10^6 \times 1/1370 = 4.67 \times 10^6$ second = **540686 days**

In a hydel plant, potential energy of water is converted into kinetic energy and then into electricity.

Example 3.16. *The reservoir area of a hydro-electric generating plant is spread over an area of 4 sq km with a storage capacity of 8 million cubic-metres. The net head of water available to the turbine is 70 metres. Assuming an efficiency of 0.87 and 0.93 for water turbine and generator respectively, calculate the electrical energy generated by the plant.*

Estimate the difference in water level if a load of 30 MW is continuously supplied by the generator for 6 hours.

(Power System I-AMIE Sec. B)

Solution. Since 1 cubic metre of water weighs 1000 kg., the reservoir capacity = 8×10^6 m^3 = $8 \times 10^6 \times 1000$ kg. = 8×10^9 kg.

Wt. of water, $W = 8 \times 10^9$ kg. Wt. $8 \times 10^9 \times 9.81 = 78.48 \times 10^9$ N. Net water head = 70 m.

Potential energy stored in this much water = Wh = $78.48 \times 10^9 \times 70 = 549.36 \times 10^{10}$ J

Overall efficiency of the generating plant = 0.87 × 0.93 = 0.809

Energy available = $0.809 \times 549.36 \times 10^{10}$ J = 444.4×10^{10} J

= $444.4 \times 10^{10}/36 \times 10^5$ = **12.34×10^5 kWh**

Energy supplied in 6 hours = 30 MW × 6 h = 180 MWh

= 180,000 kWh

Energy drawn from the reservoir after taking into consideration, the overall efficiency of the station = 180,000/0.809 = 224,500 kWh = $224,500 \times 36 \times 10^5$

= 80.8×10^{10} J

If m kg. is the mass of water used in 6 hours, then since water head is 70 m,

$mgh = 80.8 \times 10^{10}$ or $m \times 9.81 \times 70 = 80.8 \times 10^{10}$ $\therefore$ $m = 1.176 \times 10^9$ kg.

If h is the fall in water level, then $h \times$ area × density = mass of water

$\therefore \quad h \times (4 \times 10^6) \times 1000 = 1.176 \times 10^9 \quad \therefore \quad h$ = 0.294 m = **29.4 cm.**

Example 3.17. *A proposed hydro-electric station has an available head of 30 m, catchment area of 50×10^6 sq.m, the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies : Penstock 95%, Turbine 80% and Generator 85.* **(Elect. Engg. AMIETE Sec. A Part II)**

Solution. Volume of water available = $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7$m^3

Mass of water available = $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10}$ kg

This quantity of water is available for a period of one year. Hence, quantity available per second $= 4.2 \times 10^{10}/365 \times 24 \times 3600 = 1.33 \times 10^3$.

Available head = 30 m

Potential energy available = $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3$ J

Since this energy is available per second, hence power available is $= 391 \times 10^3$ J/s $= 391 \times 10^3$ W = 391 kW

Overall efficiency = 0.95 × 0.80 × 0.85 = 0.646

The power that could be generated = 391 × 0.646 = **253 kW.**

Example 3.18. *In a hydro-electric generating station, the mean head (i.e. the difference of height between the mean level of the water in the lake and the generating station) is 400 metres. If the overall efficiency of the generating stations is 70%, how many litres of water are required to generate 1 kWh of electrical energy ? Take one litre of water to have a mass of 1 kg.*

(F.Y. Engg. Pune Univ.)

Solution. Output energy = 1 kWh = 36×10^5 J

Input energy = $36 \times 10^5/0.7 = 5.14 \times 10^6$ J

If *m* kg. water is required, then

$mgh = 5.14 \times 10^6$ or $m \times 9.81 \times 400 = 5.14 \times 10^6$, $\therefore$ = **1310 kg.**

Example 3.19. *A 3-tonne electric-motor-operated vehicle is being driven at a speed of 24 km/hr upon an incline of 1 in 20. The tractive resistance may be taken as 20 kg per tonne. Assuming a motor efficiency of 85% and the mechanical efficiency between the motor and road wheels of 80%, calculate*

(a) the output of the motor

(b) the current taken by motor if it gets power from a 220-V source.

Calculate also the cost of energy for a run of 48 km, taking energy charge as 40 paise/kWh.

Solution. Different forces acting on the vehicle are shown in Fig. 3.2.

Wt. of the vehicle = 3×10^3 = 3000 kg-wt

Component of the weight of the vehicle acting downwards along the slope = $3000 \times 1/20$ = 150 kg-wt

Tractive resistance = 3×20 = 60 kg-wt

Total downward force = 150 + 60 = 210 kg-wt

= 210×9.81 = 2,060 N

Distance travelled/second = 24,000/3600 = 20/3 m/s

Output at road wheels = $2{,}060 \times 20/3$ watt

Mechanical efficiency = 80% or 0.8

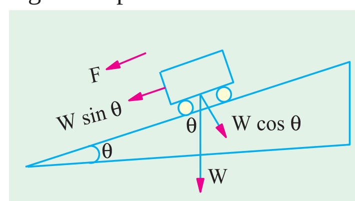


Fig. 3.2

(*a*) Motor output = $\dfrac{2{,}060 \times 20}{3 \times 0.8}$ = **17,167 W**

(*b*) Motor input = 17,167/0.85 = 20,200 W

Current drawn = 20,200/220 = **91.7 A**

Motor power input = 20,200 W = 20.2 kW

Time for 48 km run = 2 hr.

$\therefore$ Motor energy input = 20.2×2 = 40.4 kW

Cost = Rs. 40.4×0.4 = **Rs. 16 paise 16**

Example 3.20. *Estimate the rating of an induction furnace to melt two tonnes of zinc in one hour if it operates at an efficiency of 70%. Specific heat of zinc is 0.1. Latent heat of fusion of zinc is 26.67 kcal per kg. Melting point is 455ºC. Assume the initial temperature to be 25ºC.*

(Electric Drives and Utilization Punjab Univ.)

Solution. Heat required to bring 2000 kg of zinc from 25°C to the melting temperature of 455° C = $2000 \times 0.1 \times (455 - 25)$ = 86,000 kcal.

Heat of fusion or melting = mL = 2000×26.67 = 53,340 kcal

Total heat reqd. = 86,000 + 53,340 = 139,340 kcal

Furnace input = 139,340/0.7 = 199,057 kcal

Now, 860 kcal = 1 kWh $\therefore$ furnace input = 199.057/860 = 231.5 kWh.

Power rating of furnace = energy input/time = 231.5 kWh/1 h = **231.5 kW.**

Example 3.21. *A pump driven by an electric motor lifts 1.5 m^3 of water per minute to a height of 40 m. The pump has an efficiency of 90% and motor has an efficiency of 85%. Determine :* ***(a)*** *the power input to the motor.* ***(b)*** *The current taken from 480 V supply.* ***(c)*** *The electric energy consumed when motor runs at this load for 4 hours. Assume mass of 1 m^3 of water to be 1000 kg.*

(Elect. Engg. Pune Univ.)

Solution. (*a*) Weight of the water lifted = 1.5 m^3 = 1.5×1000 = 1500 kg. Wt = 1500×9.8 = 14700 N.

Height = 40 m; time taken = 1 min. = 60 s

$\therefore$ Motor output power = $14700 \times 40/60$ = 9800 W

Combined pump and motor efficiency = 0.9×0.85

$\therefore$ Motor power input = $9800/0.9 \times 0.85$ = 12810 W = **12.81 kW.**

(*b*) Current drawn by the motor = 12810/480 = **26.7 A**

Electrical energy consumed by the motor = 12.81 kW × 4 h = **51.2 kWh.**

Example 3.22. *An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWhr supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230 V d.c. Assume the efficiency of the motor at 90%.* **(Elect. Engg. A.M. Ae. S.I. June)**

Solution. Work done by the lift = $Wh = mgh = (5 \times 1000) \times 9.8 \times 30 = 1.47 \times 10^6$ J

Since 25% of the electric current input is wasted, the energy supplied to the lift is 75% of the input.

$\therefore$ input energy to the lift $= 1.47 \times 10^6/0.75 = 1.96 \times 10^6$ J

Now, 1 kWh $= 26 \times 10^5$ J

$\therefore$ energy input to the lift $= 1.96 \times 10^6/36 \times 10^5 = 0.544$ kWh

Motor energy output $= 1.96 \times 10^6$ J; $\eta = 0.9$

Motor energy input $= 1.96 \times 10^6/0.9 = 2.18 \times 10^6$ J : time taken $= 27 \times 60 = 1620$ second

Power rating of the electric motor = work done/time taken

$= 2.18 \times 10^6/1620 = 1.345 \times 10^3$ J/s = 1345 W

Current taken by the motor = 1345/230 = **5.85 A**

Example 3.23. *An electrical lift make 12 double journey per hour. A load of 5 tonnes is raised by it through a height 50 m and it returns empty. The lift takes 65 seconds to go up and 48 seconds to return. The weight of the cage is 1/2 tonne and that of the counterweight is 2.5 tonne. The efficiency of the hoist is 80 per cent that of the motor is 85 %. Calculate the hourly consumption in kWh.* **(Elect. Engg. Pune Univ.)**

Solution. The lift is shown in Fig. 3.3.

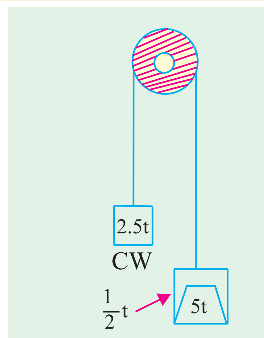


Fig. 3.3

Weight raised during upward journey

$= 5 + 1/2 - 2.5 = 3$ tonne $= 3000$ kg-wt

Distance travelled = 50 m

Work done during upward journey

$= 3000 \times 50 = 15 \times 10^4$ m-kg

Weight raised during downward journey

$= 2.5 - 0.5 = 2$ tonne $= 2000$ kg

Similarly, work done during downward journey

$= 2000 \times 50 = 10 \times 10^{-4}$ m-kg.

Total work done per double journey

$= 15 \times 10^4 + 10 \times 10^4 = 25 \times 10^4$ m-kg

Now, 1, m-kg = 9.8 joules

$\therefore$ Work done per double journey $= 9.8 \times 25 \times 10^4$ J $= 245 \times 10^4$ J

No. of double journey made per hour = 12

$\therefore$ work done per hour $= 12 \times 245 \times 10^4 = 294 \times 10^5$ J

Energy drawn from supply $= 294 \times 10^5/0.8 \times 0.85 = 432.3 \times 10^5$ J

Now, 1 kWh $= 36 \times 10^5$ J

$\therefore$ Energy consumption per hour $= 432.3 \times 10^5/36 \times 10^5 =$ **12 kWh**

Example 3.24. *An electric hoist makes 10 double journey per hour. In each journey, a load of 6 tonnes is raised to a height of 60 meters in 90 seconds. The hoist cage weighs 1/2 tonne and has a balance load of 3 tonnes. The efficiency of the hoist is 80 % and of the driving motor 88 %. Calculate* **(a)** *electric energy absorbed per double journey* **(b)** *hourly energy consumption in kWh* **(c)** *hp (British) rating of the motor required* **(d)** *cost of electric energy if hoist works for 4 hours/day for 30 days. Cost per kWh is 50 paise.* **(Elect. Power - 1, Bangalore Univ.)**

Solution. Wt. of cage when fully loaded $= 6\frac{1}{2}$ tonne-wt.

Force exerted on upward journey $= 6\frac{1}{2} - 3 = 3\frac{1}{2}$ tonne-wt.

$= 3\frac{1}{2} \times 1000 = 3{,}500$ kg-wt.

Force exerted on downward journey $= 3 - \frac{1}{2} = 2\frac{1}{2}$ tonnes-wt. = 2500 kg-wt

Distance moved = 60 m

Work done during upward journey $= 3{,}500 \times 60$ m-kg

Work done during downward journey $= 2{,}500 \times 60$ m-kg

Work done during each double journey $= (3{,}500 + 2{,}500) \times 60 = 36 \times 10^4$ m-kg

$= 36 \times 10^4 \times 9.81 = 534 \times 10^4$ J

Overall $\eta = 0.80 \times 0.88$

$\therefore$ Energy input per double journey $= 534 \times 10^4/0.8 \times 0.88 = 505 \times 104$ J

(*a*) Electric energy absorbed per double journey $= 505 \times 10^4/36 \times 10^5 =$ **1.402 kWh**

(*b*) Hourly consumption $= 1.402 \times 10 =$ **14.02 kWh**

(*c*) Before calculating the rating of the motor, maximum rate of working should be found. It is seen that maximum rate of working is required in the upward journey.

Work done $= 3{,}500 \times 60 \times 9.81 = 206 \times 10^4$ J

Time taken = 90 second

B.H.P of motor $= \dfrac{206 \times 10^4}{90 \times 0.8 \times 746} =$ **38.6(British h.p.)**

(*d*) Cost $= 14.02 \times (30 \times 4) \times 50/100 =$ **Rs. 841.2**

Example 3.25. *A current of 80 A flows for 1 hr, in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 tonne must move in order that its kinetic energy shall be equal to the energy dissipated in the resistance.*

(Elect. Engg. A.M.A.e. S.I.)

Solution. Energy dissipated in the resistance $= V\,It = 2 \times 80 \times 3600 = 576{,}000$ J

A weight of one tonne represents a mass of one tonne *i.e.*, 1000 kg. Its kinetic energy is = (1/2) $\times 1000 \times v^2 = 500\,v^2$

$\therefore \quad 500\,v^2 = 576{,}000 \quad \therefore\ v = 1152$ m/s.

Tutorial Problems No. 3.1

1. A heater is required to give 900 cal/min on a 100 V. d.c. circuit. What length of wire is required for this heater if its resistance is 3 Ω per metre ? **[53 metres]**
2. A coil of resistance 100 Ω is immersed in a vessel containing 500 gram of water of 16º C and is connected to a 220-V electric supply. Calculate the time required to boil away all the water (1kcal = 4200 joules, latent heat of steam = 536 keal/kg). **[44 min 50 second]**
3. A resistor, immersed in oil, has 62.5 Ω resistance and is connected to a 500-V d.c. supply. Calculate
   (*a*) the current taken
   (*b*) the power in watts which expresses the rate of transfer of energy to the oil.
   (*c*) the kilowatt-hours of energy taken into the oil in 48 minutes. **[8A ; 4000 W ; 3.2 kWh]**
4. An electric kettle is marked 500-W, 230 V and is found to take 15 minutes to raise 1 kg of water from 15º C to boiling point. Calculate the percentage of energy which is employed in heating the water. **[79 per cent]**
5. An aluminium kettle weighing 2 kg holds 2 litres of water and its heater element consumes a power of 2 kW. If 40 percent of the heat supplied is wasted, find the time taken to bring the kettle of water to boiling point from an initial temperature of 20ºC. (Specific heat of aluminium = 0.2 and Joule's equivalent = 4200 J/kcal.) **[11.2 min]**

6. A small electrically heated drying oven has two independent heating elements each of 1000 Ω in its heating unit. Switching is provided so that the oven temperature can be altered by rearranging the resistor connections. How many different heating positions can be obtained and what is the electrical power drawn in each arrangement from a 200 V battery of negligible resistance ?
[Three, 40, 20 and 80 W]
7. Ten electric heaters, each taking 200 W were used to dry out on site an electric machine which had been exposed to a water spray. They were used for 60 hours on a 240 V supply at a cost of twenty paise/kWh. Calculate the values of following quantities involved :
(*a*) current (*b*) power in kW (*c*) energy in kWh (*d*) cost of energy.
[(*a*) 8.33 A (*b*) 2 kW (*c*) 120 kWh (*d*) Rs. 24]
8. An electric furnace smelts 1000 kg of tin per hour. If the furnace takes 50 kW of power from the electric supply, calculate its efficiency, given : the smelting tempt. of tin = 235°C ; latent heat of fusion = 13.31 kcal/kg; initial temperature = 15ºC ; specific heat = 0.056. Take J = 4200 J/kcal.
[59.8%] ***(Electrical Engg.-I, Delhi Univ.)***
9. Find the useful rating of a tin-smelting furnace in order to smelt 50 kg of tin per hour. Given : Smelting temperature of tin = 235ºC, Specific heat of tin = 0.055 kcal/kg-K. Latent heat of liquefaction = 13.31 kcal per kg. Take initial temperature of metal as 15ºC. **[1.5 kW]**
(F.Y. Engg. Pune Univ.)
10. State the relation between
(*i*) Kcal and kWh (*ii*) Horse power and watts (*iii*) kWh and joule (watt sec) (*iv*) K.E and joules.
(Gujrat University, Summer 2003)
11. The electrical load in a small workshop consists of 14 lamps, each rated at 240 V, 60 W and 3 fans each rated at 240 V, 1 kW. What is the effective resistance of the total load, total current and energy utilised if run for 8 hrs.
(Pune University 2003) (Gujrat University, Summer 2003)

OBJECTIVE TESTS – 3

1. If a 220 V heater is used on 110 V supply, heat produced by it will be —— as much.
(*a*) one-half (*b*) twice
(*c*) one-fourth (*d*) four times
2. For a given line voltage, four heating coils will produce maximum heat when connected
(*a*) all in parallel (*b*) all in series
(*c*) with two parallel pairs in series
(*d*) one pair in parallel with the other two in series
3. The electric energy required to raise the temperature of a given amount of water is 1000 kWh. If heat losses are 25%, the total heating energy required is — kWh.
(*a*) 1500 (*b*) 1250
(*c*) 1333 (*d*) 1000
4. One kWh of energy equals nearly
(*a*) 1000 W (*b*) 860 kcal
(*c*) 4186 J (*d*) 735.5 W
5. One kWh of electric energy equals
(*a*) 3600 J (*b*) 860 kcal
(*c*) 3600 W (*d*) 4186 J
6. A force of 10,000 N accelerates a body to a velocity 0.1 km/s. This power developed is —— kW
(*a*) 1,00,000 (*b*) 36,000
(*c*) 3600 (*d*) 1000
7. A 100 W light bulb burns on an average of 10 hours a day for one week. The weekly consumption of energy will be —— unit/s
(*a*) 7 (*b*) 70
(*c*) 0.7 (*d*) 0.07
(Principles of Elect. Engg. Delhi Univ.)
8. Two heaters, rated at 1000 W, 250 volts each, are connected in series across a 250 Volts 50 Hz A.C. mains. The total power drawn from the supply would be ——— watt.,
(*a*) 1000 (*b*) 500
(*c*) 250 (*d*) 2000
(Principles of Elect. Engg. Delhi Univ.)

ANSWERS

1. *c* **2.** *a* **3.** *c* **4.** *b* **5.** *b* **6.** *d* **7.** *a* **8.** *b*

Top