C H A P T E R

## 51

## Learning Objectives

$>$ The Atom
$>$ Bohr's Atomic Model
$>$ Electron Energy Levels in Hydrogen Atom
$>$ Orbital (or Azimuthal) Quantum Number
> Orbital Magnetic Quantum Number ( $m_{l}$ )
$>$ Magnetic Spin Quantum Number ( $m_{s}$ )
> Pauli's Exclusion Principle
> Energy Bands in Lithium and Their Occupancy
> Valence and Conduction Bands
> Insulators, Conductors and Semiconductors
> Crystal Structure
> Types of Semiconductors
> Mobile Charge Carriers and Immobile Ions
> Electron Conductivity of a Metal
$>$ Combined Drift and Diffusion Currents
$>$ Relation Between D and $\mu$
> Carrier Life Time
> P-N Junction
$>$ Formation of Depletion Layer
$>$ Junction or Barrier Voltage
$>$ Forward Biased P-N Junction
$>$ Reverse Biased P-N Junction

## SEMI-

CONDUCTOR PHYSICS


[^0]
### 51.1. The Atom

According to the model of atom proposed by Bohr in 1913, an atom is composed of a number of electrons moving in circular or elliptical orbits around a relatively heavy nucleus of protons and neutrons as shown in Fig. 51.1. Although, this simple model of an atom has been replaced by later models, yet it affords a convenient method of understanding the working of semi-conductor devices. Electron has a mass of nearly $9.1 \times 10^{-31} \mathrm{~kg}$ and a charge of $1.6 \times 10^{-19} \mathrm{C}$. The diameter of an atom is approximately $10^{-10} \mathrm{~m}$ and that of the nucleus about $10^{-15} \mathrm{~m}$. The number of protons in the atom of an element gives its atomic number while the atomic mass number is determined by the number of protons and neutrons present in the nucleus.

### 51.2. Bohr's Atomic Model



Fig. 51.1

2. The electrons revolve round their nucleus in circular orbits, the centrifugal force being balanced by the electrostatic pull between the nucleus and electrons;
3. An electron cannot revolve round the nucleus in any arbitrary orbit but in just certain definite discrete orbits. Only those orbits are possible (or permitted) for which the orbital angular momentum (i.e. moment of momentum) of the electron is equal to an integral multiple of $h / 2 \pi$ i.e. orbital angular momentum $=n h / 2 \pi$ where $n$ is an integer and $h$ is Planck's constant. Such orbits are also known as stationary orbits;
4. While revolving in these permitted stationary (or stable) orbits, the electron does not radiate out any electromagnetic energy. In other words, the permissible orbits are non-radiating paths of the electron;
5. The atom radiates out energy only when an electron jumps from one orbit to another. If $E_{2}$ and $E_{1}$ are the energies corresponding to two orbits before and after the jump, the frequency of the emitted photon is given by the relation

$$
E_{2}-E_{1}=h f \quad \text { or } \quad \Delta E=h f
$$

where $f$ is the frequency of the emitted radiations.
Explanation. If $I$ is the moment of inertia of an electron and $\omega$ its angular velocity, then as per assumption (3) given above
$\omega I=n h / 2 \pi \quad$ or $\quad\left(m r^{2}\right) \omega=n h / 2 \pi \quad$ or $\left(m r^{2}\right) v / r=n h / 2 \pi$ or $m v r=n \cdot h / 2 \pi$

Alternatively, since the momentum of the revolving electron is $m v$, its moment about the nucleus is $m v r$ (Fig. 51.2).


Hence,
$m v r=n h / 2 \pi$
when $n=1,2,3$ etc. for the first, second and third orbits respectively. It is called the principal quantum number and because it can take whole number values only, it fixes the size of the allowed orbits (also called Bohr's circular orbits).

Let the different permitted orbits have energies of $E_{1}, E_{2}, E_{3}$ etc. as shown in Fig. 51.3 (a). The electron can be raised from $n=1$ orbit to any other higher orbit if it is given proper amount of energy.


Fig. 51.2
Fig. 51.3
When it drops back to $n=1$ orbit after a short interval of time, it gives out the energy difference $\Delta E$ in the form of a radiation as shown in Fig. 51.3 (b). The relation between the energy released and frequency of the emitted radiation is

$$
E_{2}-E_{1}=h f \quad \text { or } \quad \Delta E=h f
$$

### 51.3. Calculations Conceming Bohr's Atomic Model

The above postulates concerning Bohr's atomic model can be utilized to calculate not only the radii of different electron orbits but also the velocity and orbital frequency possessed by different electrons.

Now, the stability of the atom requires that the centrifugal force acting on the revolving electron be balanced by the electrostatic pull exerted by the positively charged nucleus on the electron.

$$
\therefore \quad \frac{m v^{2}}{r}=\frac{Z e \cdot e}{4 \pi \epsilon_{0} r^{2}} \quad \text { or } \quad m v^{2} r=\frac{Z e^{2}}{4 \pi \epsilon_{0}}
$$

Also, according to Bohr's postulates, $m v r=n h / 2 \pi$.
The above two equations may be used to find the radii of different Bohr's circular orbits.
(a) Radii of Orbits. Eliminating $v$ from the above two equations, we get

$$
r=\frac{\epsilon_{0} n^{2} h^{2}}{\pi m Z e^{2}} \quad \text { or } \quad r_{n}=\frac{\epsilon_{0} n^{2} h^{2}}{\pi m Z e^{2}}
$$

It is seen that the radii of the permitted orbits vary as the square of the principal quantum number $n$. Also, $r_{n}=n^{2} r_{1}$.

For hydrogen atom, $r_{1}=0.53 \times 10^{-10} \mathrm{~m}, r_{2}=2^{2} r_{1}=2.12 \times 10^{-10} \mathrm{~m}$. All values between $r_{1}$ and $r_{2}$ are forbidden.
(b) Velocity of Revolving Electrons. The velocity of a revolving electron as found from the above equations is

$$
v=Z e^{2} / 2 \epsilon_{0} n h=9 \times 10^{9} .2 \pi Z e^{2} / n h .
$$

It shows that velocity is inversely proportional to $n$ i.e. $v \propto 1 / n$. Hence, the electron in the innermost orbit has the highest velocity (nearly $1 / 37$ of the velocity of light).
(c) Orbital Frequency. The orbital rotational frequency* of an electron is

$$
f=\frac{v}{2 \pi r}=\frac{m Z^{2} e^{4}}{4 \epsilon_{0}^{2} n^{3} h^{3}} . \quad \text { It is seen that } f \propto 1 / n^{3}
$$

(d) Electron Energy. The orbital energy of a revolving electron is of two types:
(i) Kinetic energy due to the motion of the electron. Its value is $=(1 / 2) m v^{2}$.

As seen from above, $m v^{2}=\mathrm{Ze}^{2} / 4 \pi \epsilon_{0} r$.
Hence, K.E. $=(1 / 2) m v^{2}=Z e^{2} / 8 \pi \in_{0} r$. -as represented by curve 2 in Fig. 51.4.
Eliminating $r$ from above,

$$
\text { K.E. }=m Z^{2} e^{4} / 8 \epsilon_{0}^{2} n^{2} h^{2} .
$$

(ii) Potential energy-because the electron lies in the electric field of the positive nucleus.

Now, the potential at a point distant $r$ from the nucleus is

$$
V=Q / 4 \pi \epsilon_{0} r=Z e / 4 \pi \epsilon_{0} r
$$

The potential energy of an electron (of charge- $e$ ) is

$$
\begin{aligned}
\text { P.E. } & =V \times(-e)=-\frac{Z e^{2}}{4 \pi \epsilon_{0} r} \\
\text { or } \quad & =-\frac{m Z^{2} e^{4}}{4 \epsilon_{0}^{2} n^{2} h^{2}}-\text { curve } 1 \text { in Fig. } 51.4
\end{aligned}
$$

$\therefore \quad$ Orbital energy $=$ K.E. + P.E.

$$
\begin{aligned}
& =\frac{Z e^{2}}{8 \pi \epsilon_{0} r}-\frac{Z e^{2}}{4 \pi \epsilon_{0} r} \\
& =-\frac{Z e^{2}}{8 \pi \epsilon_{0} r} \quad-\text { curve } 3 \text { in Fig. } 51.4
\end{aligned}
$$



Fig. 51.4

Variations of potential energy and total energy are shown in Fig. 51.4.
If $r$ is eliminated from the above equation, the orbital energy** is
$E_{n}=-m Z^{2} e^{4} / 8 \epsilon_{0}{ }^{2} n^{2} h^{2}$
where $n=1,2,3 \ldots \ldots$. etc. for the energy states that it is possible for the electron to have. It will be seen that $E_{n}=E_{1} / n^{2}$.

### 51.4. Normal, Excited and Ionized Atom

Consider the case of the simplest atom i.e. hydrogen atom. When its only electron is in its innermost orbit $(n=1)$, then the atom is said to be in its normal (or unexcited) state. Generally, it is this condition in which most of the free hydrogen atoms in a gas are found to exist at normal room temperature and pressure. However, if spark is passed through hydrogen gas contained in a vessel, then high-speed electrons produced by the spark collide with hydrogen atoms and may either completely remove the $n=1$ electron from them or raise it to higher permitted orbits having $n=2,3,4$ etc.

When the electron is completely removed from the atom, the atom is said to be ionized. If, however, the electron is forced into an outer or higher $n$-value orbit, then the atom, is said to be $\bar{*}$ It is the mechanical frequency of $\overline{\text { rotation }} \overline{\text { and }} \overline{\text { should not be confused with the frequency of emitted }}$ radiations.
** The negative sign only indicates that this much energy is required to remove the electron from the atom.
excited (or in an excited state). The atom does not remain in the excited state longer than $10^{-8}$ second because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. In doing so, the electron loses the energy it had earlier gained during collision. However, the electron may return by several jumps, thereby emitting many different radiations of different frequencies.

### 51.5. Electron Energy Levels in Hydrogen Atom

As seen from Art. 51.3, orbital energy of an electron revolving in $n_{\text {th }}$ orbit or shell is

$$
E_{n}=-m Z^{2} e^{4} / 8 \epsilon_{0}^{2} n^{2} h^{2}
$$

In the case of hydrogen atom, $Z=1$, hence

$$
\begin{aligned}
E_{n} & =-\frac{m e^{4}}{8 \epsilon_{0}^{2} n^{2} h^{2}}=-\frac{m e^{4}}{8 \epsilon_{0}^{2} h^{2}} \cdot \frac{1}{n^{2}}=-\frac{21.7 \times 10^{-19}}{n^{2}} \text { joules } \\
& =\frac{-21.7 \times 10^{-19}}{1.6 \times 10^{-19}} \cdot \frac{1}{n^{2}}=-\frac{13.6}{n^{2}} \mathrm{eV} \quad\left(\because 1 \mathrm{eV}=1.6 \times 10^{-19} \text { joules }\right)
\end{aligned}
$$

This expression gives the total energy of an electron when it occupies any one of the different orbits (or shells) of the hydrogen atom.


Fig. 51.5
For $K$-shell, $\quad n=1$,
$E_{1}=-13.6 \mathrm{eV}$
For $L$-shell, $n=2$,
$E_{2}=-13.6 / 2^{2}=-3.4 \mathrm{eV}$
For $M$-shell, $n=3$,
$E_{3}=-13.6 / 3^{2}=-1.51 \mathrm{eV}$ etc.
Instead of drawing various electron orbits to the scale of their radii as in Fig. 51.5 (a), it is customary to draw horizontal lines to an energy scale as shown in Fig. 51.5 (b) and such a diagram is called energy level diagram ( $E L D$ ) of an atom. In this array of energies, the higher (i.e. less negative) energies are at the top while the lower (i.e. more negative) energies are at the bottom. The various electron jumps between allowed orbits now become vertical arrows between energy levels. Greater the length of the arrow, greater is the energy ' $h f$ ' of the radiated photon.

Example 51.1. Calculate the value of the kinetic, potential and total energy of an electron revolving in Bohr's first orbit in a hydrogen atom.

Solution. (i)

$$
\begin{aligned}
K . E . & =\frac{m e^{4}}{8 \epsilon_{0}^{2} n^{2} h^{2}}=\frac{9.1 \times 10^{-31} \times\left(1.6 \times 10^{-19}\right)^{4}}{8 \times\left(8.854 \times 10^{-12}\right)^{2} \times 1^{2} \times\left(6.625 \times 10^{-34}\right)^{2}} \\
& =21.7 \times 10^{-19} \text { joules }=\mathbf{1 3 . 6} \mathbf{e V}
\end{aligned}
$$

$$
\begin{equation*}
\text { P.E. }=\frac{m e^{4}}{4 \epsilon_{0}^{2} n^{2} h^{2}}=-43.4 \times 10^{-19} \text { joules }=-27.2 \mathrm{eV} \tag{ii}
\end{equation*}
$$

(iii) Total energy $=$ K.E. + P.E. $=13.6+(-27.2)=-\mathbf{1 3 . 6} \mathbf{e V}$

### 51.6. Orbital (or Azimuthal) Quantum Number

According to Bohr's postulates (Art. 51.2), there is only one orbit (and hence one energy level) corresponding to each value of the principal quantum number $n$. However, subsequent experimental evidence revealed that all orbits (except $n=1$ orbit) consist of more than one orbit called sub-orbits. This group of sub-orbits is collectively known as shell. For example, $n=1$ shell or $K$-shell consists of only one orbit which may also be called its own sub-orbit or sub-shell. The $n=2$ shell or $L$-shell consists of two sub-shells. Similarly, $n=3$ shell or $M$-shell consists of three sub-shells. In other words, the number of sub-shells is equal to the $n$-value of the shell. The $2 n^{2}$ electrons of the shell now get divided between these sub-shells.

In order to distinguish between different sub-shells belonging to a given shell, a new quantum number called orbital (or azimuthal) quantum number $l$ has been introduced. This quantum number can have integral values lying between zero and $(n-1)$ i.e. $0 \leq l \leq(n-1)$. It is helpful in the following ways:
(a) It gives the number of sub-shells which are contained in one shell. The number of sub-shells is equal to the number of values which $l$ can have subject to the restriction $0 \leq l \leq(n-1)$ as shown in Fig. 51.6 (a).


Fig. 51.6
(b) It helps to distinguish between different sub-shells of a shell by its different values for each sub-shell. Consider the following cases :

1. $n=1$ shell. Here, $l$ can have only one value i.e. zero. Hence, $K$-shell has only one subshell with two quantum numbers of $n=1$ and $l=0$.
2. $n=\mathbf{2}$ shell. Here, $l=0,1$. Hence, this shell has two sub-shells* which are distinguishable from each other by their different quantum numbers of $n=2, l=0$ and $n=2, l=1$. (Fig. 51.6).
3. $n=3$ shell. Here $l=0,1,2$, showing that $M$-shell has three sub-shells which differ in their $l$-values. The two quantum numbers for the three sub-shells are :

$$
n=3, l=0 ; n=3, l=1 ; n=3, l=2 .
$$

(c) It helps to determine the shape of different sub-shells of a given shell. Some sub-shells are circular whereas others are elliptical in shape. If $a$ and $b$ are semi-major and semi-minor axes respectively of an ellipse, $n$ and $l$ are its principal quantum number and orbital quantum number respectively, then the relation between them is

$$
\frac{b}{a}=\frac{l+1}{n}
$$

Consider the following cases :

1. $K$-shell or $\mathbf{n}=\mathbf{1}$ shell

Here, $l=0$, hence $b / a=(0+1) / 1$ or $b / a=1$ or $b=a$. In other words, the first shell (also called sub-shell or $K$-shell) around the nucleus of an atom is circular in shape as shown in Fig. 51.7 (a).
2. L-shell or $\mathbf{n}=\mathbf{2}$ shell

As stated earlier and shown in Fig. 51.6, it has two more sub-shells.
First sub-shell with $l=0$. Here, $\frac{b}{a}=\frac{0+1}{2}=\frac{1}{2}$ or $b=\frac{a}{2}$. It is elliptical in shape as shown in Fig. 51.7 (b).
Second sub-shell with $l=1$. Here $\frac{b}{a}=\frac{1+1}{2}=1$ or $b=a$.


Fig. 51.7
This sub-shell has circular shape as shown in Fig. 51.7 (b).
3. M-shell or $\mathbf{n}=3$ shell

It has three sub-shells with $l=0,1,2$.
First sub-shell with $l=0$. Here, $b / a=(0+1) / 3=1 / 3$. It is elliptical in shape.
Second sub-shell with $l=1$. Here, $b / a=(1+1) / 3=2 / 3$. It is also elliptical in shape.
Third sub-shell with $l=2$. Here, $b / a=(2+1) / 3=1$ or $b=a$. Obviously, this sub-shell is circular in shape.
Fig. 51.7 shows to shapes of different sub-shells per values of $n$ from 1 to 4 .
(d) It determines the distribution of electrons in various sub-shells of a shell. The maximum

* Different sub-shells have been shown circular for the sake of simplicity although, in practice, most of them are elliptical.
number of electrons which a sub-shell of any $n$-value can accommodate is $=2(2 l+1)$. Consider the following :

1. K-shell. Here $l=0$, hence it can have $2(2 \times 0+1)=2$ electrons. They are designated as $1 s^{2}$ electrons.
2. L-shell. First sub-shell with $l=0$ can have $2(2 \times 0+1)=2$ electrons which are designated as $2 s^{2}$ electrons.
Second sub-shell with $l=1$ can have a maximum of $2(2 \times 1+1)=6$ electrons which are designated as $2 p^{6}$ electrons.
Total number of electrons in the two sub-shells is $2+6=8$ which tallies with $2 n^{2}=$ $2 \times 2^{2}=8$.
3. M-shell. First sub-shell with $l=0$ has, as explained above, 2 electrons which are designated as $3 s^{2}$ electrons.
Second sub-shell $l=1$ can have 6 electrons written as $3 p^{6}$ electrons.
Third sub-shell with $l=2$ can accommodate a maximum of $2(2 \times 2+1)=10$ electrons which are known as $3 d^{10}$ electrons.
Again, it will be seen that total number of electrons in the three sub-shells of $n=3$ shell is $=2+6+10=18$ which equals $2 n^{2}=2 \times 3^{2}=18$.
4. $\mathbf{N}$-shell. First sub-shell with $l=0$ can have maximum of two electrons designated as $4 s^{2}$ electrons. Second sub-shell can contain 6 electrons known as $4 p^{6}$ electrons.
Similarly, third sub-shell can have 10 electrons designated as $4 d^{10}$ electrons.
The fourth sub-shell can have a maximum of $2(2 \times 3+1)=14$ electrons whose designation is $4 f^{14}$ electrons.
The total number of electrons is $=2+6+10+14=32$ which is in accordance with the number $2 n^{2}=2 \times 4^{2}=32$.

Incidentally, it should be noted that different $l$ values of $0,1,2,3,4 \ldots \ldots$. etc. are identified as $s, p$, d, f, g.... etc.
The different sub-shells and maximum number of electrons they can accommodate are tabulated below.

| Shell <br> $n$ | $\begin{gathered} \mathrm{K} \\ 1 \end{gathered}$ | $\begin{aligned} & \mathrm{L} \\ & 2 \end{aligned}$ |  | $\begin{gathered} M \\ 3 \end{gathered}$ |  |  | $\begin{gathered} \mathrm{N} \\ 4 \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} l \\ \text { sub-shell } \end{gathered}$ | 0 $s$ | $\begin{aligned} & \hline 0 \\ & s \end{aligned}$ | $\begin{gathered} 1 \\ p \end{gathered}$ | $0$ | $\begin{aligned} & 1 \\ & p \end{aligned}$ | $\begin{aligned} & 2 \\ & d \end{aligned}$ | 0 | 1 | 2 $d$ | 3 $f$ |
| Maximum No. of electrons $=2(2 l+1)$ | 2 | 2 | 6 | 2 | 6 | 10 | 2 | 6 | 10 | 14 |
| Total No. of electrons $=2 n^{2}$ | 2 |  | 8 | 18 |  |  | 32 |  |  |  |

(e) The orbital quantum number $l$ also quantizes the orbital angular momentum $p_{l}$ associated with each sub-shell

$$
p_{l}=l . h / 2 \pi \quad \text { where } l=0,1,2,3, \ldots . \text { etc. }{ }^{*}
$$

However, quantum mechanical considerations indicate that $p_{l} \neq l . h / 2 \pi$ as stated above but instead $p_{l}=l^{*} . h / 2 \pi$ where $l^{*}=[l(l+1)]^{1 / 2}$.

[^1]
### 51.7. Electron Configuration of Atoms

By electron configuration of an atom is meant the distribution of its electrons in its various subshells around the nucleus. Following three rules govern the electron distribution :

1. Maximum number of electrons a shell can have is $=2 n^{2}$.
2. In the $n$th shell, there are $n$ sub-shells having different values of $l$ such as $0,1,2 \ldots .(n-1)$.
3. Each sub-shell can accommodate a maximum of $2(2 l+1)$ electrons.

Consider the following atoms :
(i) Sodium atom, $\mathbb{Z}=11$. It has 11 electrons. Hence, its electronic configuration is $1 s^{2}$, $2 s^{2}, 2 p^{6}, 3 s^{1}$. Obviously, $N a$ has a single electron in its outermost sub-shell and hence is said to be monovalent. Same property is possessed by other alkali metals like $\mathrm{Li}, \mathrm{K}, \mathrm{Rb}$ and Cs. They have similar chemical properties and are, therefore, included in the same group in the periodic table.
The electron in the inner sub-shells are very tightly bound to the nucleus and cannot be easily removed. In other words, they have high binding energy.
(ii) Copper atom, $\mathbb{Z}=29$. It has 29 electrons. Its electronic configuration is $1 s^{2}, 2 s^{2}, 2 p^{6}$, $3 s^{2}, 3 p^{6}, 3 d^{10}, 4 s^{1}$.

### 51.8. Orbital Magnetic Quantum Number (ml.)

It determines the spatial orientation of elliptical electron orbits with respect to an applied magnetic field. There are restrictions on the orientations of the electron orbits because of which they are said to be space quantized. Whereas $l$ determines the orbital angular momentum, $m_{l}$ represents the magnitude of the component of angular momentum along the direction of the magnetic field.

Looked from a different angle, $m_{l}$ determines the number of sub-subshells in a given shell.
This quantum number can have any one of the $(2 l+1)$ values ranging from $+l$ to $-l$ including zero i.e. $l,(l-1),(l-2) \ldots \ldots . . .2,1,0,-1,-2, \ldots \ldots \ldots . .-(l-2),-(l-1),-l$.

Each sub-subshell can accommodate a maximum of 2 electrons, so that maximum number of electrons in a shell becomes $2(2 l+1)$ as stated earlier in Art. $51.6(d)$.

### 51.9. Magnetic Spin Quantum Number ( $m_{s}$ )

It has been found that an electron spins around its own mechanical axis as it rotates in orbit around the nucleus. When the electron is subjected to a magnetic field, its spin axis orientates itself either parallel to or antiparallel to the direction of the field.

This quantum number arises out of quantization of the electron spin angular momentum. It determines the spin orientation up or down and has correspondingly two values of $+1 / 2$ and $-1 / 2$.

### 51.10. Pauli's Exclusion Principle

This principle which was enunciated by Pauli in 1925 states that in an atom, no two electrons can have the same set of values for its four quantum number $n, l, m_{l}$ and $m_{s}$. In other words, no two electrons can be described by an identical set of four quantum numbers. They may have at the most three numbers alike but at least one must be different. Consequently, it restricts the number of electrons an atom can have.

Consider the case of He atom which has two electrons. These electrons occupy $K$-shell ( $n=1$ ) and are designated as $1 s^{2}$ electrons. Their four quantum numbers are as follows :

|  | $n$ | $l$ | $m_{l}$ | $m_{s}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1st electron: | 1 | 0 | 0 | $+1 / 2$ |
| 2nd electron: | 1 | 0 | 0 | $-1 / 2$ |

As seen, the two electrons have different sets of four quantum numbers $(1,0,0,+1 / 2)$ and ( 1,0 , $0,-1 / 2$ ) as required by Pauli's Exclusion Principle.

This principle affords a very rational theoretical justification for the electronic shell structure of the atoms deduced experimently.

Pauli's Exclusion Principle may be used to find the total number of electrons an atom can have in its various shells. Consider the following examples :
$\mathrm{n}=1$, K -shell
$l=0,1 s$ sub-shell, $m_{l}=0 ; m_{s}= \pm 1 / 2$
Total $\frac{2 \text { electrons }}{2 \text { electrons }}$
$\mathrm{n}=2$, L-shell
(i) $l=0,2 s$-shell, $m_{l}=0 ; m_{s}= \pm 1 / 2$
(ii) $l=1,2 p$ sub-shell, $m_{l}=-1,0,+1 ; m_{s}= \pm 1 / 2$

Total | 2 electrons |
| ---: |
| 6 electrons |

$\mathrm{n}=3, \mathrm{M}$-shell
(i) $l=0,3 s$ sub-shell, $m_{l}=0 ; m_{s}= \pm 1 / 2$

2 electrons
(ii) $l=1,3 p$ sub-shell, $m_{l}=-1,0,+1 ; m_{s}= \pm 1 / 2$ 6 electrons
(iii) $l=2,3 d$ sub-shell, $m_{l}=-2,-1,0,+1,+2 ; m_{s}= \pm 1 / 2$

Total $\frac{10 \text { electrons }}{18 \text { electrons }}$
$\mathrm{n}=4, \mathrm{~N}$-shell
(i) $l=0,4 s$ sub-shell, $m_{l}=0, m_{s}= \pm 1 / 2$

2 electrons
(ii) $l=1,4 p$ sub-shell, $m_{l}=-1,0,1 ; m_{s}= \pm 1 / 2$
(iii) $l=2,4 d$ sub-shell, $m_{l}=-2,-1,0,+1,+2, m_{s}= \pm 1 / 2$
(iv) $l=3,4 f$ sub-shell, $m_{l}=-3,-2,-1,0,+1,+2,+3, m_{s}= \pm 1 / 2$

| 2 electrons |
| ---: |
| 6 electrons |
| 10 electrons |
| 14 electrons |
| Total |
| 22 electrons |

### 51.11. Energy Bands in Solids

In the case of an isolated single atom, there are single energy levels as shown for a hydrogen atom in Fig. 51.5. But there are significant changes in the energy levels when atoms are brought close together as in solids.

It is found that each of the energy levels of an atom splits into $N$ levels of energy where $N$ is the number of atoms in the crystal. Each original energy level becomes a band of very closely-spaced levels of slightly different energy. The individual energies within the band are so close together that, for many purposes, the energy band may be considered to be a continuous one.

Fig. 51.8 (a) shows the splitting of $K, L$ and $M$ levels as the distance between different atoms is reduced. At first, only valence level or $M$-level is affected as shown by dotted vertical line marked $A$, then as separation is reduced, inner shells also become affected as indicated by dotted vertical line $B$.

Consider the case of Na crystal which consists of an ordered array of many closely-packed sodium atoms usually referred to as crystal lattice. Each $N a$ atom has 11 electrons arranged in different shells and sub-sheels as shown in Fig. 51.9 (a). As seen, the $1 s, 2 s$ and $2 p$ sub-shells are filled but $3 s$ sub-shell is incomplete and could hold one electron more. The electrons in each sub-shell occurs specific energy levels as shown in Fig. 51.9 (b). For a small sodium crystal containing $10^{20}$ atoms, the band formed by splitting of $s$-subshell has 2 electronic levels (one with spin up and the others with spin down). Similarly in a $p$-band there will be $6 \times 10^{20}$ closely packed levels because there are 6 electrons in a filled $p$ subshell.

In general, in an assembly of $N$ atoms, the number of possible energy states is $N$. Since only two electrons of opposite spin can occupy the same state (as per Pauli's Exclusion Principle discussed earlier), the maximum number of electrons which these $N$ states can occupy is $2 N$.


Fig. 51.8

### 51.12. Spacing Between Energy Levels of a Band

It will be quite interesting as well as instructive to calculate the spacing between different energy levels in an energy band. A crystal weighing one milligram contains about $10^{19}$ atoms. If we assume the valency band to be an s-band, it will contain $2 \times 10^{19}$ levels. Suppose the width of the energy band is 2 eV . Then, it is obvious that $2 \times 10^{19}$ levels per milligram are spread over an energy band width of 2 eV . Hence, spacing between different levels $=2 /\left(2 \times 10^{19}\right)=10^{-19} \mathrm{eV}$. It will be appreciated that even though energy levels are discrete, the picture of a band as a continuum of energy levels is a very good approximation. This splitting of the single energy level of an isolated atom into a band of


Fig. 51.9
energy in the case of a solid is responsible for most of the electrical, magnetic and optical properties of that solid. It is worth pointing out here that in gases under normal conditions of temperature and pressure, the atomic spacing is so great that there is no splitting of energy levels and hence no band formation.

### 51.13. Energy Bands in Lithium and their Occupancy

Consider the case of lithium metal-the simplest atom which forms a solid at ordinary temperature. Its atom has three electrons, two of which have the same energy and the third one has higher value of energy. In an isolated single atom, two electrons move round the electron orbit with $n=1$ whereas the third occupies the orbit with $n=2$ as shown in Fig. 51.10 (a). Now, consider a piece of lithium metal containing 100 atoms. It will be found that the lower level (with $n=1$ ) forms a band of 200 electrons occupying 100 different energy states. The higher level (with $n=2$ ) forms a wider band of 100 energy states which could, as before, accommodate 200 electrons. But as there are only 100 electrons available (one from each atom), this energy band remains half-filled [Fig. 51.10 (b)].


(b)

Fig. 51.10

### 51.14. Valence and Conduction Bands

The outermost electrons of an atom i.e. those in the shell furthermost from the nucleus are called valence electrons and have the highest energy* or least binding energy. It is these electrons which are most affected when a number of atoms are brought very close together as during the formation of a solid. The states of lower-energy electrons orbiting in shells nearer to the nucleus are little, if at all, affected by this atomic proximity.

The band of energy occupied by the valence electrons is called the valence band and is, obviously, the highest occupied band. It may be completely filled or partially filled with electrons but never empty.

The next higher permitted energy band is called the conduction band and may either be empty or partially filled with electrons. In fact, it may be defined as the lowest unfilled energy band.

In conduction band, electrons can move freely and hence are known as conduction electrons. The gap between these two bands is known as the forbidden energy gap.

It may be noted that the covalent force of the crystal lattice have their source in the valence band. If a valence electron happens to absorb enough energy, it jumps across the forbidden energy gap and enters the conduction band (Fig. 51.12). An electron in the conduction band can jump to an adjacent conduction band more readily than it can jump back to the valence band from where it had come earlier. However, if a conduction electron happens to radiate too much energy, it will suddenly reappear in the valence band once again.

When an electron is ejected from the valence band, a covalent bond is broken and a positivelycharged hole is left behind. This hole can travel to an adjacent atom by acquiring an electron from

* In the algebraic sense only. Their binding energy is the least.

that atom which involves breaking an existing covalent bond and then re-establishing a covalent bond by filling up the hole. It is to be noted carefully that holes are filled by electrons which move fromadjacent atoms without passing through the forbidden energy gap.


Fig. 51.13
It is simply another way of saying that conditions in the conduction band have nothing to do with the hole flow. It points to a very important distinction between the hole current and electron currentalthough holes flow with ease, they experience more opposition than electron flow in the conduction band.

To summarize the above, it may be repeated that :

1. conduction electrons are found in and freely flow in the conduction band ;
2. holes exist in and flow in the valence band ;
3. conduction electrons move almost twice as fast as the holes.

Fig. 51.13 (a) shows the energy band diagram of an unexcited silicon atom $(Z=14)$ with its electronic distribution. When silicon crystal is given thermal or light energy from outside [Fig. 51.13 (b)], some electrons gain sufficient energy to jump the gap from the valence band into the conduction band thereby becoming free electrons [Fig. 51.13 (c)]. For every electron which jumps to conduction band, a hole is created in the valence band. In this way, an electron-hole pair is created.

### 51.15. Insulators, Conductors and Semiconductors

The electrical conduction properties of different elements and compounds can be explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.
(i) Insulators. Stated simply, insulators are those materials in which valence electrons are
bound very tightly to their parents atoms, thus requiring very large electric field to remove them from the attraction of their nuclei. In other words, insulators have no free charge carriers available with them under normal conditions.

In terms of energy bands, it means that insulators (a) have a full valence band,

1. have an empty conduction band,
2. have a large energy gap (of several eV ) between them and
3. at ordinary temperatures, the probability of electrons from full valence band gaining sufficient energy so as to surmount energy gap and thus become available for conduction in the conduction band, is slight.


Fig. 51.14
This is shown in Fig. 51.14 (a). For conduction to take place, electrons must be given sufficient energy to jump from the valence band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which fact accounts for the negative resistance-temperature coefficient of insulators.
(ii) Conductors. Put in a simple way, conducting materials are those in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Fig. 51.14 (b).

In fact, there is no physical distinction between the two bands. Hence, the availablity of a large number of conduction electrons.

Another point worth noting is that in the absence of forbidden energy gap in good conductors, there is no structure to establish holes. The total current in such conductors is simply a flow of electrons. It is exactly for this reason that the existence of holes was not discovered until semi-conductors were studied thoughly.
(iii) Semiconductors. A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. Examples are : germanium and silicon.


In terms of energy bands, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV ) separating the two.

At $0^{\circ} \mathrm{K}$, there are no electrons in the conduction band and the valence band is completely filled. However, with increase in temperature, width of the forbidden energy bands is decreased so that
some of the electrons are liberated into the conduction band. In other words, conductivity of semiconductors increases with temperature. Moreover, such departing electrons leave behind positive holes in the valence band (Fig. 51.12). Hence, semiconductor current is the sum of electron and hole currents flowing in opposite directions.

### 51.16. Crystal Structure

Semiconductors like germanium and silicon, have crystalline structure. That is the atoms are arranged in three-dimensional periodic fashion. The periodic arrangement of atoms in a crystal is called lattice. In a crystal, an atom strays far from a single, fixed position. The thermal vibrations associated with the atom are centred about this position. For a given semiconduc-
 tor (silicon or germanium), there is a unit cell that is representative of the entire lattice. By repeating the unit cell throughout the crystal, we can generate the entire lattice.

There are several different types of crystal lattice depending upon the symmetry and internal structure. One of them is the cubic crystal lattice. There are three basic types of unit cells in a cubic crystal lattice. These are simple cubic (SC), base centred cubic (BCC) and face centred cubic (FCC). Fig. 51.15 (a) shows a SC crystal. In this unit cell, each corner of the cubic lattice is occupied by an atom (indicated by a small sphere) that has six equidistant nearest neighbouring atoms. The dimension " $a$ " is called the lattice constant. Only podium is crystallised into the simple cubic lattice.

Fig. 51.15 (b) shows a BCC unit cell. In this unit cell, each atom has eight nearest neighbouring atoms. Crystals exhibiting the BCC lattices include those of sodium and tungsten. Fig. 51.15 (c) shows a FCC unit cell. This unit cell has one atom at each of the six cubic faces in addition to the eight corner atoms. In an FCC lattice, each atom has 12 nearest neighbouring atoms. A large number of elements exhibit the FCC lattice form. These include aluminium, copper, gold and platinum.


Fig. 51.15
Fig. 51.16 shows the crystal structure of silicon and germanium - the element semiconductors. This type of structure is called diamond crystal structure and it belongs to cubic crystal family. All atoms are identical in a diamond lattice. Note that each atom in diamond lattice is surrounded by four equidistant neighbours that lie at the corners of a tetrahedron.


Fig. 51.16

### 51.17. Representation of Crystal Planes and Directions

Referring to Fig. 51.15 (b) again, we note that the there are four atoms in the ABCD plane and give atoms in ACEF plane (four atoms from the corners and one from the centre). Moreover, the atomic spacing are different for the two planes. Therefore the crystal properties along different planes are different. The electrical and other device characteristics are dependent on the crystal orientation. A convenient method of defining or representing the various planes in a crystal is to use Miller indices. These indices are obtained using the following steps :

1. Find the intercepts of the plane on the three coordinate axes in terms of lattice constant.
2. Take the reciprocals of these numbers and reduce them to the smallest three integers having the same ratio.
3. Enclose the result in parentheses $(h k 1)$ as the Miller indices for a single plane.

For example, consider the plane ABC having the intercept at $a$, $2 a$ and $2 a$ along the three rectangular coordinate axes as shown in Fig. 51.17. Taking the reciprocal of these intercepts, we get $1,1 / 2$, $1 / 2$. Multiplying each fraction by the least common multiplier 2, we find that the smallest three integers having the same ratio are 2 , 1 and 1 . Thus the Miller indices from the plane are (211). The plane ABC can also be referred to as (211) plane.

Fig. 51.18 shows the Miller indices of important planes in cubic crystal. It may be noted that for a plane that intercepts the $X$-axis on the negative side of the origin, the Miller indices are represented by (hk1). Similarly for a plane that intercepts the $Y$-axis on the negative


Fig. 51.17 side of the origin, the Miller indices are represented by (hk1) and so on.

Sometimes it convenient to represent the Miller indices of the planes of equivalent symmetry by (hk1). For example, in cubic crystal, the planes (100), (010), (001), ( $\overline{1} 00$ ), ( $0 \overline{1} 0$ ) and ( $00 \overline{1}$ ) have the same symmetry. Therefore, these planes can be represented by Miller indices as (100). Note that we have used the curly brackets to represent a set of planes with equivalent symmetry.

The line that originates from the origin and passes through the plane at right angles to it is called crystal direction. The crystal direction is indicated by enclosing the Miller indices in a square brackets


Fig. 51.18
i.e., [hk1]. For example, the direction idicated by [100] is a direction for the $X$-axis and it is perpendicualr to (100) plane. Similarly [111] direction is perpendicular to (111) plane. It is possible to represent a set of equivalent directions by < hk1 >. Notice the use of carat signs. Thus < 100 > represents a set of directions for [100], [010], [100], [0 $\overline{1} 0]$ and $[00 \overline{1}]$.

### 51.18. Atomic Binding in Semiconductors

Semiconductors like germanium* and silicon, have crystalline structure. Their atoms are arranged in an ordered array known as crystal lattice. Both these materials are tetravalent i.e. each has four valence electrons in its outermost shell. The neighbouring atoms form covalent bonds by sharing four electrons with each other so as to achieve inert gas structure (i.e. 8 electrons in the outermost orbit). A twodimensional view of the germanium crystal lattice is shown in Fig. 51.19 (b) in which circles represent atom cores consisting of the nuclei and inner 28 electrons. Each pair of lines represents a covalent bond. The dots represent the valence electrons. It is seen that each atom has 8 electrons under its influence.


Fig. 51.19

[^2]A 3-dimensional view of germanium crystal lattice is shown in Fig. 51.19 (a) where each atom is surrounded symmetrically by four other atoms forming a tetrahedral crystal. Each atom shares a valence electron with each of its four neighbours, thereby forming a stable structure.

In the case of pure (i.e. intrinsic) germanium, the covalent bonds have to be broken to provide electrons for conduction. There are many ways of rupturning the covalent bond and thereby setting the electrons free. One way is to increase the crystal temperature above $0^{\circ} \mathrm{K}$.

It may be noted that covalent crystals are characterised by their hardness and brittleness. Their brittleness is due to the fact that in such crystals, adjacent atoms must remain in accurate alignment since the bond is strongly directional and formed along a line joining the atoms. The hardness is due to the great strength of the covalent bond itself.

### 51.19. Types of Semiconductors



Semiconductors can be classified as shown below :

### 51.20. Intrinsic Semiconductors

An intrinsic semiconductor is one which is made of the semiconductor material in its extremely pure form.

Common examples of such semiconductors are : pure germanium and silicon which have forbidden energy gaps of 0.72 eV and 1.1 eV respectively. The energy gap is so small that even at ordinary room temperature, there are many electrons which possess sufficient energy to jump across the small energy gap between the valence and the conduction bands. However, it is worth noting that for each electron liberated into the conduction band, a positively charged hole is created in the valence band (Fig. 1.20). When an electric field is applied to an intrinsic semiconductor at a temperature greater than $0^{\circ} \mathrm{K}$, conduction electrons move to the anode and the holes in the valence band move to the cathode. Hence semiconductor current consists of movement of electrons and holes in opposite directions. Electron current is due to movement of electrons in the conduction band whereas hole current is within the valence band as a result of the holes 'jumping' from one atom to another.

As stated above, in pure semiconductors, electric conduction is due to the thermally-generated electron hole pairs. Hence in pure semiconductors kept in the dark, thermally-generated charge carriers are the only means of conduction. The number of such charge carriers per unit volume (i.e. intrinsic carrier density) is given by

$$
n_{i}=N \exp \left(-E_{g} / 2 k T\right)
$$

where $N$ is constant for a given semiconductor, $E_{g}$ is the band gap energy in joules, $k$ is Boltzmann's constant and $T$ is the temperature in ${ }^{\circ} \mathrm{K}$.

Example 51.2. Find the intrinsic carrier concentration in silicon at $300^{\circ} \mathrm{K}$ for which $N=$ $3 \times 10^{25} \mathrm{~m}^{-3}, E_{g}=1.1 \mathrm{eV}, \mu_{e}=0.14 \mathrm{~m}^{2} / V-s$ and $\mu_{h}=0.05 \mathrm{~m}^{2} / V-s$. Also, find the conductivity of silicon.
(Electronics-II, Madras Univ. 1993)
Solution. The intrinsic carrier concentration in pure silicon is given by

$$
n_{i}=N \exp \left(-E_{g} / 2 k T\right)
$$

Now,

$$
N=3 \times 10^{25} \mathrm{~m}^{-3}, E_{g}=1.1 \mathrm{eV}=1.1 \times 1.6 \times 10^{-19}=1.76 \times 10^{-19} \mathrm{~J}
$$

$$
k=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}, T=300^{\circ} \mathrm{K}
$$

$$
\left.\therefore \quad n_{i}=3 \times 10^{25}\left(-1.76 \times 10^{-19}\right) / 2 \times 1.38 \times 10^{-23} \times 300\right)=2 \times 10^{16} \mathrm{~m}^{-3}
$$

$$
\sigma=n_{i} e\left(\mu_{e}+\mu_{h}\right)=2 \times 10^{16} \times 1.6 \times 10^{-19}(0.14+0.05)=0.61 \times 10^{-3} \mathrm{~S} / \mathrm{m}
$$

### 51.21. Hole Formation in Semiconductors

The formation of a hole which is a positive charge carrier is explained below :
As shown in Fig. 51.21, suppose the covalent bond is broken at $A$ and the electron has moved through the crystal lattice leaving behind a hole in the covalent bond. An electron at $B$ may jump into the vacant hole at $A$ and later, an electron at $C$ may jump into the hole at $B$ and so on. In this way, by a succession of electron movements, a hole will appear at $G$ and a negative charge would have moved from $G$ to $A$. It would, however, be more convenient to regard positive charge to have moved from $A$ to $G$ and this conception gives rise to a hole as a positive charge carrier as if it were an electron with a positive charge. It should be clearly understood that these holes are due to the movement of electrons in the valence band and that each electron movement corresponds to a collision. The drift velocity of holes is, obviously, much less than the drift velocity of electrons.

Alternatively, an intrinsic semiconductor may be defined as one in which the number of conduction electrons is equal to the number of holes.


Fig. 51.20
Fig. 51.21
Schematic energy band diagram of an intrinsic semiconductor at room temperature is shown in Fig. 51.20. Only two bands i.e. valence and conduction bands have been shown since lower filled bands are not of any consequence. Here, Fermi level* lies exactly in the middle of the forbidden energy gap.

### 51.22. Fermi Level in an Intrinsic Semiconductor

It can be proved that in an intrinsic semiconductor, Fermi energy level ${ }^{*} E_{F}$ lies in the middle of the energy gap i.e. midway between the conduction and valence bands.

Let, at any temperature $T^{\circ} \mathrm{K}$

$$
\begin{aligned}
& n_{c}=\text { No. of electrons in the conduction band } \\
& n_{v}=\text { No. of electrons in the valence band }
\end{aligned}
$$

[^3]\[

$$
\begin{aligned}
N & =n_{c}+n_{v} \\
& =\text { No. of electrons in both bands. }
\end{aligned}
$$
\]

Further, let us make the following simplifying assumptions :

1. width of energy bands are small as compared to forbidden energy gap between them;
2. since band widths are small, all levels in a band have the same energy;
3. energies of all levels in valence band are zero as shown in Fig. 51.22;


Fig. 51.22
4. energies of all levels in the conduction band are equal to $E_{g}$.

In Fig. 51.22, the zero-energy reference level has been arbitrarily taken at the top of valence band.

Now, number of electrons in the conduction band is

$$
n_{c}=N . P\left(E_{g}\right)
$$

where $P\left(E_{g}\right)$ represents the probability of an electron having energy $E_{g}$. Its value can be found from Fermi-Dirac probability distribution function given by

$$
P(E)=\frac{1}{1+e^{\left(E-E_{g}\right) / k T}}
$$

where $P(E)$ is the probability of finding an electron having any particular value of energy $E$ and $E_{F}$ is Fermi level.

$$
\therefore \quad P\left(E_{g}\right)=\frac{1}{1+e^{\left(E_{g}-E_{F}\right) / k T}} \quad \therefore \quad n_{c}=\frac{N}{1+e^{\left(E_{g}-E_{F}\right) / k T}}
$$

Now, number of electrons in the valence band is $n_{v}=N . P(0)$
The probability $P(0)$ of an electron being found in the valence band with zero energy can again be calculated by putting $E=0$ in the Femi-Dirac probability distribution function.

$$
\begin{aligned}
P(0) & =\frac{1}{1+e^{\left(0-E_{F}\right) / k T}}=\frac{1}{1+e^{-E_{F} / k T}} . \quad \text { Hence, } n_{c}=\frac{N}{1+e^{-E_{F} / k T}} \\
\text { Now } N & =n_{c}+n_{v} \text { or } N=\frac{N}{1+e^{\left(E_{g}-E_{F}\right) / k T}}+\frac{N}{1+e^{-E_{F} / k T}} \\
\therefore \quad 1-\frac{1}{1+e^{-E_{F} / k T}} & =\frac{1}{1+e^{\left(E_{g}-E_{F}\right) / k T}}
\end{aligned}
$$

which on simplification gives $E_{F}=E_{g} / 2$.

Hence, it shows that in an intrinsic semiconductor, the Fermi level lies midway between the conduction and valence bands. That this conclusion is physically possible can be best seen with the help of Fig. 51.23.


Fig. 51.23

### 51.23. Extrinsic Semiconductors

Those intrinsic semiconductors to which some suitable impurity or doping agent or dopant has been added in extremely small amounts (about 1 part in $10^{8}$ ) are called extrinsic or impurity semiconductors.

The usual doping agents are :

1. pentavalent atoms having five valence electrons (arsenic, antimony, phosphorus) or
2. trivalent atoms having three valence electrons (gallium, indium, aluminium, boron).

Pentavalent doping atom is known as donor atom because it donates or contributes one electron to the conduction band of pure germanium. The trivalent atom, on the other hand, is called acceptor atom because it accepts one electron from the germanium atom.

Depending on the type of doping material used, extrinsic semiconductors can be sub-divided into two classes :
(i) $N$-type semiconductors and (ii) $P$-type semiconductors.
(a) $N$-type ExtrinsicSemiconductor. This type of semiconductor is obtained when a pentavalent material like antimonty $(\mathrm{Sb})$ is added to pure germanium crystal. As shown in Fig. 51.24 (a), each antimony atom forms covalent bonds with the surrounding four germanium atoms with the help of four of its five electrons. The fifth electron is superfluous and is loosely bound to the antimony atom. Hence, it can be easily excited from the valence band to the conduction band by the application of electric field or increase in thermal energy. Thus, practically every antimony atom introduced into the germanium lattice, contributes one conduction electron into the germanium
 lattice without creating a positive hole. Antimony is called donor impurity and makes the pure germanium an $N$-type ( $N$ for negative) extrinsic semi-conductor. As an aid to memory, the student should associate the $\mathbf{N}$ in doNor with $\mathbf{N}$ in the $N$-type material and $N$ in Negative charge carrier.

It may be noted that by giving away its one valence electron, the donor atom becomes a positivelycharged ion. But it cannot take part in conduction because it is firmly fixed or tied into the crystal lattice. It will be seen that apart from electrons and holes intrinsically available in germanium, the addition of antimony greatly increases the number of conduction electrons. Hence, concentration of electrons in the conduction band is increased and exceeds the concentration of holes in the valence band. Because of this, Fermi level shifts upwards towards the bottom of the conduction band as shown in Fig. 51.24 (b),* because the number of charge carriers has become more in conduction band than in valence band.

In terms of energy levels, the fifth antimony electron has an energy level (called donor level) just below the conduction band. Usually, the donor level is 0.01 eV below conduction band for germanium and 0.054 eV for silicon.

It is seen from the above description that in $N$-type semiconductors, electrons are the majority carriers while holes constitute the minority carriers. Hence, $N$-type semiconductor conducts principally by electrons in the nearly empty conduction band and the process is called 'excess' conduction.

Another point worth noting is that even though $N$-type semiconductor has excess of electrons,

[^4]still it is electrically neutral. It is so because by the addition of donor impurity, number of electrons available for conduction purposes becomes more than the number of holes available intrinsically. But the total charge of the semiconductor does not change because the donor impurity brings in as much negative charge (by way of electrons) as positive charge (by way of protons in its nucleus).


Fig. 51.24
(b) P-type Extrinsic Semiconductor. This type of semiconductor is obtained when traces of a trivalent like boron $(B)$ are added to a pure germanium crystal.

In this case, the three valence electrons of boron atom form covalent bonds with four surrounding germanium atoms but one bond is left incomplete and gives rise to a hole as shown in Fig. 51.25 (a).

Thus, boron which is called an acceptor impurity causes as many positive holes in a germanium crystal as there are boron atoms thereby producing a $P$-type ( $P$ for positive) extrinsic semiconductor. As an aid to memory, the student should associate the $\mathbf{P}$ in acce $\mathbf{P t o r}$ with $\mathbf{P}$ in $P$-type material and $\mathbf{P}$ with Positive charge carrier.

In this type of semiconductor, conduction is by the movement of holes in the valence band. Accordingly, holes form the majority carriers whereas electrons constitute minority carriers. The process of conduction is called 'deficit' conduction.


Fig. 51.25

Since concentration of holes in the valence band is more than the concentration of electrons in the conduction band, Fermi level shifts nearer to the valence band [Fig. 51.25 (b)]. The acceptor level lies immediately above the Fermi level. Conduction is by means of hole movement at the top of valence band, acceptor level readily accepting electrons from the valence band.

Again, it may be noted that even though $P$-type semiconductor has excess of holes for conduction purposes, on the whole it is electrically neutral for the same reasons as given above.

## 5 1.24. Majority and Minority Carriers

In a piece of pure germanium or silicon, no free charge carriers are available at $0^{\circ} \mathrm{K}$. However, as its temperature is raised to room temperature, some of the covalent bonds are broken by heat energy and as a result, electron-hole pairs are produced. These are called thermally-generated charge carriers. They are also known as intrinsicallyavailable charge carriers. Ordinarily, their number is quite small.

An intrinsic of pure germanium can be converted into a $P$-type semiconductor by the


Fig. 51.26 addition of an acceptor impurity which adds a large number of holes to it. Hence, a $P$-type material contains following charge carriers :
(a) large number of positive holes-most of them being the added impurity holes with only a very small number of thermally generated ones;
(b) a very small number of thermally-generated electrons (the companions of the thermally generated holes mentioned above).
Obviously, in a $P$-type material, the number of holes (both added and thermally-generated) is much more than that of electrons. Hence, in such a material, holes constitute majority carriers and electrons form minority carriers as shown in Fig. 51.26 (a).

Similarly, in an $N$-type material, the number of electrons (both added and thermally-generated) is much larger than the number of thermally-generated holes. Hence, in such a material, electrons are majority carriers whereas holes are minority carriers as shown in Fig. 51.26 (b).

### 51.25. Mobile Charge Carriers and Immobile Ions

As discussed in Art. 51.23, $P$-type material is formed by the addition of acceptor impurity atoms like boron to the pure Ge or Si crystals. The number of holes added is equal to the number of boron atoms because each such atom contributes one hole. Now, when a hole moves away from its parent atom, the remaining atom becomes a negative ion. Unlike the mobile and free-moving hole, this ion


Fig. 51.27
Fig. 51.28
cannot take part in conduction because it is fixed in the crystal lattice. In Fig. 51.27 (a), these immobile ions are shown by circled minus signs whereas free and mobile holes are shown by uncircled plus signs. Thermally-generated electrons (which form minority carriers) are shown by uncircled minus signs.

Similarly, addition of penta-valent atoms like antimony to pure Ge or Si crystal produces an N type material. The number of free and mobile electrons which are added equals the number of donor $S b$ atoms. Again, when an electron moves away from its parent atom, it leaves behind a positive ion. This ion, being fixed in crystal structure, cannot take part in conduction. As shown in Fig. 51.27 (b); these immobile ions are represented by circled plus signs whereas free and mobile electrons are represented by uncircled minus signs. The thermally-generated holes (which form minority carriers in this case) are shown by uncircled plus signs. In Fig. 51.28, minority carriers of both types have been neglected. Hence, the figure does not show the small number of free electrons in the $P$-type material or the small number of holes in the $N$-type material.

### 51.26. Electron Conductivity of a Metal

According to free electron model of an atom, the valence electrons are not attached to individual atoms but are free to move about in all directions among the atoms. These electrons are called conduction electrons and are said to form 'free electron cloud' or free electron 'gas' or the Fermi gas. For example in copper there is one such free electron per atom, the
 other 28 electrons remaining bound to the copper nuclei to form positive ion cores.

When no external field is applied to the metal, the free electrons move randomly in all directions as shown in Fig. 51.29 (a). However, when an external electric field is applied to the metal, the free electron motion becomes directed a shown in Fig. 51.29 (b). This directed flow of electrons results in a net charge displacement in a definite direction. This type of motion is known as drift and the phenomenon is referred to as process of conduction by drift charge. The drift velocity $(v)$ of the electrons is dependent upon the electron mobility $\left(\mu_{e}\right)$ and the


Fig. 51.29 applied electric field $E$. The actual relation is $v=\mu_{e} E$

Let, $\quad e=$ electron charge (coloumb)

$$
v=\text { electron drift velocity }(\mathrm{m} / \mathrm{s})
$$

$$
A=\text { conductor cross-section }\left(\mathrm{m}^{2}\right)
$$

$$
n=\text { number of free electrons per unit volume of the conductor }
$$

$$
\text { i.e. electron density }\left(\text { per } \mathrm{m}^{3}\right)
$$

$l=$ length of the conductor (m)
$E=V / l$ —applied electric field (V/m)
Now, electric current flowing in any conductor is given by the amount of charge which flows in one second across any plane of the conductor (Fig. 51.30). The total number of electrons which cross the plane $P$ of cross-section $A$ in one second $=n \times(\nu \times A)$.

Charge carried by them per second is $=e n v A$. Hence, $I=v e n A$.
Substituting the value of $v$, we get

$$
\begin{aligned}
& I & =n e A \mu_{e} E=n A e \mu_{e} V / l \\
\therefore & R & =\frac{V}{I}=\frac{l}{A}\left(\frac{1}{n e \mu_{e}}\right)=\rho \frac{l}{A}
\end{aligned}
$$

$\therefore \quad$ resistivity $\rho=1 / n e \mu_{e}$ ohm-m and conductivity $\sigma=n e \mu_{e}$ Siemens/m


Fig. 51.30

Incidentally, it may be noted that conductivity of a semiconductor differs from that of a metal in one important respect i.e. in a semiconductor, charge carriers are both holes as well as electrons whereas in metals, electrons are the only charge carriers (Art. 51.27).

Example 51.3. A copper wire of 2 mm diameter with conductivity of $5.8 \times 10^{7}$ Siemens $/ \mathrm{m}$ and electron mobility of $0.0032 \mathrm{~m}^{2} / \mathrm{V}$-s is subjected to an electric field of $20 \mathrm{~V} / \mathrm{m}$. Find (a) the charge density of free electrons, (b) the current density, (c) current flowing in the wire, (d) the electron drift velocity.
(U.P.S.C. Engg. Services 2002)

Solution.

$$
\begin{aligned}
d & =2 \mathrm{~mm}=2 \times 10^{-3} \mathrm{~m}, \quad \sigma=5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}, \\
\mu_{e} & =0.0032 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, \quad E=20 \mathrm{~m} \mathrm{~V} / \mathrm{m} \\
\sigma & =q n \mu
\end{aligned}
$$

(a)

$$
n=\frac{\sigma}{q \mu_{e}}=\frac{5.8 \times 10^{7}}{1.6 \times 10^{-19} \times 0.0032}=1.132 \times 10^{29} / \mathrm{m}^{3}
$$

(b) $J=\sigma \mathrm{E}=\left(5.8 \times 10^{7}\right) \times\left(20 \times 10^{-3}\right)=1.16 \times 10^{6} \mathrm{~A} / \mathrm{m}^{2}$
(c) Let $\mathrm{I}=$ current flowing through the wire.

Area of cross-section of a wire,

$$
\begin{aligned}
A & =\frac{\pi d^{2}}{4}=\frac{\pi \times\left(2 \times 10^{-3}\right)^{2}}{4}=3.16 \times 10^{-6} \mathrm{~m}^{2} \\
J & =\frac{I}{A} \text { or } I=J A=\left(1.16 \times 10^{6}\right) \times\left(3.16 \times 10^{-6}\right)=3.67 \mathrm{~A} \\
v & =\mu E=0.0032 \times\left(20 \times 10^{-3}\right)=64 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(d)

### 51.27. Conductivity of Intrinsic Semiconductors

In their case, current flow is due to the movement of electrons and holes in opposite directions. However, since their charges are of opposite sign, the current due to each is in the same direction. Even though the number of electrons equals the number of holes, hole mobility $\mu_{h}$ is practically half of electron mobility $\mu_{e}$.

As shown in Fig. 51.31, the total current flow which is due to the sum of electron flow and hole flow, is given by

$$
\begin{aligned}
I= & I_{e}+I_{h} \\
\mathrm{v}_{e}= & \text { drift velocity of electrons } \\
& (\mathrm{m} / \mathrm{s}) \\
v_{h}= & \text { drift velocity of holes }(\mathrm{m} / \mathrm{s}) \\
n_{i}= & \text { density of free electrons in } \\
& \text { an intrinsic semiconductor } \\
& \left(\text { per } \mathrm{m}^{3}\right)
\end{aligned}
$$

Let


Fig. 51.31

$$
\begin{aligned}
p_{i} & =\text { density of holes in an intrinsic semiconductor }\left(\text { per } \mathrm{m}^{3}\right) \\
e & =\text { electron charge }(\text { coulomb }) \\
A & =\text { cross-section of the semiconductor }\left(\mathrm{m}^{2}\right)
\end{aligned}
$$

Since in an intrinsic semiconductor $n_{i}=p_{i}$

$$
\therefore \quad I=n_{i} e\left(v_{e}+v_{h}\right) A=n_{i} e\left(\mu_{h}+\mu_{e}\right) E A
$$

where

$$
\begin{aligned}
& \mu_{e}=\text { electron mobility }=v_{e} / E \\
& \mu_{h}=\text { hole mobility }=v_{h} / E
\end{aligned}
$$

Since $E=V / l$ where $l$ is the length of the intrinsic semiconductor,

$$
\begin{array}{ll}
\therefore & I \\
\therefore & =n_{i} e\left(\mu_{e}+\mu_{h}\right) A V / l \\
\therefore & \frac{V}{I}
\end{array}=\frac{l}{A} \cdot \frac{1}{n_{i} e\left(\mu_{e}+\mu_{h}\right)}=\rho_{i} \frac{l}{A}
$$

where $\rho$ is the resistivity of the semoconductor. It is given by

$$
\rho_{i}=\frac{1}{n_{i} e\left(\mu_{e}+\mu_{h}\right)} \text { ohm-m }
$$

The electrical conductivity which is the reciprocal of resistivity is given by

$$
\sigma_{i}=n_{i} e\left(\mu_{e}+\mu_{h}\right) \mathrm{S} / \mathrm{m}
$$

Now, current density, $J=I / A \quad \therefore \quad J=n_{i} e\left(\mu_{e}+\mu_{h}\right) E=\sigma_{i} E \quad \therefore \quad \sigma_{i}=J / E$
Obviously, conductivity of semiconductors depends on two factors (i) number of current carriers present per unit volume and (ii) the mobility of the current carriers. It is found that with increase in temperature, $n_{i}$ as well $p_{i}$ increase and correspondingly, the conductivity of intrinsic semiconductors increases i.e. resistivity decreases.

### 51.28. Conductivity of Extrinsic Semiconductors

The general expression for current density (derived above) in the case of an extrinsic semiconductor when an electric field is employed is

$$
\begin{equation*}
J=\left(n e \mu_{e}+p e \mu_{h}\right) E \tag{i}
\end{equation*}
$$

(a) If it is an $N$-type semiconductor, then the above expression becomes

$$
J_{n}=e\left(n_{n} \mu_{e}+p_{n} \mu_{h}\right) E
$$

where $n_{n}$ and $p_{n}$ represent the electron and hole densities in the $N$-type semiconductor after doping.
(b) If it is a $P$-type semiconductor, then

$$
J_{p}=e\left(n_{p} \mu_{e}+p_{p} \mu_{h}\right) E
$$

where $n_{p}$ and $p_{p}$ represent similar quantities in a $P$-type semiconductor after doping.
The conductivity is given by $\sigma=J / E$
$\therefore \quad \sigma=n e \mu_{e}+p e \mu_{h}$
or $\quad \sigma_{n}=e\left(n_{n} \mu_{e}+p_{n} \mu_{h}\right) \quad$ - for $N$-type
and $\quad \sigma_{p}=e\left(n_{p} \mu_{e}+p_{p} \mu_{h}\right) \quad$ - for $P$-type
(i) In $N$-type semiconductors, electrons form the majority carriers although holes are also available as minority carriers.

The current density in such a semiconductor is given by Eq. (i) above. However, since electron density in such extrinsic semiconductors is much more than hole density i.e. $n_{n}$ » $p_{n}$, the above expressions are simplified to

$$
\begin{equation*}
J_{n}=n_{n} e \mu_{e} E \quad \text { and } \quad \sigma_{n}=n_{n} e \mu_{e} \tag{ii}
\end{equation*}
$$

(ii) In $P$-type semiconductors, conduction is by means of holes in the valence band which form majority carriers in this case although electrons are available as minority carriers.

Since in such extrinsic semiconductors, $n_{p}<p_{p}$, the above expressions become

$$
\begin{equation*}
J_{p}=p_{p} e \mu_{h} E \quad \text { and } \quad \sigma_{p}=p_{p} e \mu_{h} \tag{iii}
\end{equation*}
$$

### 51.29. Conductivity when Intrinsic Charge Carrier Densities are Neglected

In case density of charge carriers available intrinsically is negligible as compared to the added impurity atoms (whether of donor or a acceptor type), then the formulae for conductivity given by Eq. (ii) and (iii) above will be changed as follows :
(a) For $N$-type semiconductor

As seen from Eq. (ii) above, the conductivity is given by $\sigma_{n}=n_{n} e \mu_{e}$ where $n_{n}$ is electron density after doping.

In this relation, intrinsic hole density has already been neglected. The remaining electron density is also made up of the following two components :

1. intrinsic hole density due to holes available in a pure semiconductor ;
2. electron density $N_{d}$ contributed by added donor impurity.

However, if we further neglect the intrinsic electron density, then $\sigma_{n}=N_{d} e \mu_{e}$
(b) For $\boldsymbol{P}$-type semiconductor

As seen from Eq. (iii) above, the conductivity is given by $\sigma_{p}=p_{p} e \mu_{h}$ where $p_{p}$ represents hole density. Again, in this relation, intrinsic electron density has been already neglected. This hole density further consists of the following two components :

1. intrinsic hole density due to holes available in a pure semiconductor ;
2. hole density $\left(N_{a}\right)$ contributed by added acceptor impurity.

However, if we further neglect the intrinsic hole density, then $\sigma_{p}=N_{a} e \mu_{h}$

### 51.30. Conductivity of Pure and P-type Germanium

As shown in Art. 51.25, the conductivity of pure germanium is given by

$$
\sigma=n_{i} e\left(\mu_{e}+\mu_{h}\right)=p_{i} e\left(\mu_{e}+\mu_{h}\right)
$$

When germanium is doped by a trivalent impurity like indium, it becomes a $P$-type semiconductor. After doping, its conductivity depends on the number of charge carriers available in it. The law of Mass Action can be used for finding this number. For acceptor impurity, the law may be stated as follows :

$$
n_{p} p_{p}=n_{i} p_{i}=n_{i}^{2}=p_{i}^{2}
$$

where $n_{p}$ and $p_{p}$ represent the 'free' electron and hole densities respectively in the semi-conductor after doping and $n_{i}$ and $p_{i}$, the electron and hole densities before doping i.e. in an intrinsic semiconductor. Put in another way, it simply means that, at constant temperature, the product of the number of electron carriers and the number of hole carriers is independent of the density of acceptor atoms. In physical terms, it means that the introduction of $P$-type impurity fills some of the electron levels produced by thermal action.

By calculating $n_{p}$ and $p_{p}$ from above and knowing $\mu_{e}$ and $\mu_{h}$, conductivity after doping can be found out as illustrated by the following example.
(a) Pure Germanium

$$
\sigma=n_{i} e\left(\mu_{e}+\mu_{h}\right)=p_{i} e\left(\mu_{e}+\mu_{h}\right)
$$

Let,

$$
n_{i}=p_{i}=2 \times 10^{19} \text { per } \mathrm{m}^{3}, \quad \mu_{e}=0.36 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}
$$

$$
\mu_{h}=0.17 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s} \text { and } e=1.6 \times 10^{-19} \mathrm{C}
$$

$$
\sigma=2 \times 10^{19} \times 1.6 \times 10^{-19}(0.36+0.17)=1.69 \mathrm{~S} / \mathrm{m}
$$

(b) P-type Germanium
here,

$$
\sigma_{p}=e\left(n_{p} \mu_{e}+p_{p} \mu_{h}\right)
$$

Suppose, we add $10^{22}$ atoms $/ \mathrm{m}^{3}$ of indium and that $n_{i}=2 \times 10^{19}$ charge carriers (either electrons or holes) per $\mathrm{m}^{3}$.

Then,

$$
\begin{array}{lr}
\text { Then, } & n_{p} p_{p}=n_{i}^{2}=\left(2 \times 10^{19}\right)^{2}=4 \times 10^{38} \text { and } p_{p}-n_{p}=10^{20} \\
\therefore & p_{p}-4 \times 10^{38} / p_{p}=10^{20} \quad \text { or } \quad p_{p}{ }^{2}-10^{20} p_{p}-\left(4 \times 10^{38}\right)=0
\end{array}
$$

Solving the above quadratic equation and taking positive value only.

$$
\begin{aligned}
p_{p} & =1.04 \times 10^{20} \quad \text { and } \quad n_{p}=0.04 \times 10^{20} \\
\sigma_{p} & =1.6 \times 10^{-19}\left(0.04 \times 10^{20} \times 0.36+1.04 \times 10^{20} \times 0.17\right)=3.1 \mathrm{~S} / \mathrm{m}^{*}
\end{aligned}
$$

It is seen that conductivity is almost doubled.
Example 51.4. What length of a round copper wire of diameter 1 mm will have a resistance of $1 \mathrm{k} \Omega$ if copper conductivity is $60 \mathrm{MS} / \mathrm{m}$. A cylindrical piece of silicon having a diameter of 1 mm is doped with $10^{20} \mathrm{~m}^{-3}$ atoms of phosphorous which are fully ionized. What length of this silicon would be required to give a resistance of $1 \mathrm{k} \Omega$ if electronic mobility in silicon is $0.1 \mathrm{~m}^{2} / V-s$ ?
(Electronic Devices \& Circuits, Pune Univ. 1994)
Solution. $\quad R=1 \mathrm{k} \Omega=1000 \Omega, \sigma=60 \times 10^{6} \mathrm{~S} / \mathrm{m}, A=\pi d^{2} / 4=\pi \times\left(1 \times 10^{-3}\right)^{2} / 4 \mathrm{~m}^{2}$

$$
\begin{aligned}
& R=l / \sigma A \\
& l=\sigma A R=60 \times 10^{6} \times\left(\pi \times 10^{-6} / 4\right) \times 1000=47,100 \mathrm{~m}=47.1 \mathrm{~km}
\end{aligned}
$$

For Silicon Wire

$$
\begin{aligned}
\sigma & =n_{i} e \mu_{e}=10^{20} \times 1.6 \times 10^{-19} \times 0.1=1.6 \mathrm{~S} / \mathrm{m} \\
l & =\sigma A R=1.6 \times\left(\pi \times 10^{-6} / 4\right) \times 1000 \\
& =1.26 \times 10^{-3}=1.26 \mathrm{~mm}
\end{aligned}
$$

Example 51.5. Calculate the intrinsic conductivity of silicon at room temperature if $n=1.41$ $\times 10^{16} \mathrm{~m}^{-3}, \mu_{e}=0.145 \mathrm{~m}^{2} / V-s, \mu_{h}=0.05 \mathrm{~m}^{2} / V-s$ and $e=1.6 \times 10^{-19} \mathrm{C}$. What are the individual contributions made by electrons and holes ?
(Electronic Engg., Nagpur Univ. 1991)
Solution. As seen from Art. 1.27, the conductivity of an intrinsic semiconductor is given by

$$
\begin{aligned}
\sigma_{i} & =n_{i} e \mu_{e}+n_{i} e \mu_{h} \\
& =1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.145+1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.05 \\
& =0.325 \times 10^{-3}+0.112 \times 10^{-3} \mathrm{~S} / \mathrm{m}=0.437 \times 10^{-3} \mathrm{~S} / \mathrm{m}
\end{aligned}
$$

Contribution by electrons $=0.325 \times 10^{-3} \mathrm{~S} / \mathrm{m}$
Contribution by holes $\quad=0.112 \times 10^{-3} \mathrm{~S} / \mathrm{m}$
Example 51.6. Calculate the donor concentration in N-type germanium having resistivity of $100 \Omega-m$. Derive the formula you use. Take $e=1.6 \times 10^{-19} \mathrm{C}, \mu_{e}=0.36 \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$.
(Electronics ; Nagpur Univ. 1990)

$$
\begin{array}{ll}
\text { Solution. As seen from Art. 1.29, } & \\
\therefore \quad 100=1 / N_{d} \times 1.6 \times 10^{-19} \times 0.36 ; & N_{d}=1 / N_{d} e \mu_{e} \\
\therefore \quad 1.74 \times \mathbf{1 0}^{17} \text { atoms } / \mathrm{m}^{3}
\end{array}
$$

Example 51.7. An $N$-type silicon has a resistivity of $1500 \Omega-m$ at a certain temperature. Compute the electron-hole concentration given that $n_{i}=1.5 \times 10^{16} \mathrm{~m}^{-3}, \mu_{e}=0.14 \mathrm{~m} / \mathrm{V}-\mathrm{s}, \mu_{h}=$ $0.05 \mathrm{~m}^{2} / V$-s and $e=1.6 \times 10^{-19} \mathrm{C}$.

Solution. Being $N$-type silicon, it is assumed that $n » p$
$\therefore \quad \sigma=e\left(n \mu_{e}+p \mu_{h}\right)=n_{e} \mu_{e}$
$\therefore \quad \rho=1 / n_{2} \mu_{e} \quad$ or $15=1 / n \times 1.6 \times 10^{-19} \times 0.14$ or $n=3.1 \times 10^{20} \mathrm{~m}^{-3}$
$\quad$ Now, $\quad n p=n_{i}^{2{ }^{2}}$ or $p=n_{i}^{2} / n=\left(1.5 \times 10^{16}\right)^{2} /\left(3.1 \times 10^{20}\right)=2 \times 10^{12} \mathrm{~m}^{-3}$
Example 51.8. A specimen of pure germanium at $300^{\circ} \mathrm{K}$ has a density of charge carriers of $2.5 \times 10^{19} / \mathrm{m}^{3}$. It is doped with donor impurity atoms at the rate of one impurity atom for every $10^{6}$ atoms of germanium. All impurity atoms may be supposed to be ionized. The density of germanium atom is $4.2 \times 10^{28}$ atoms $/ \mathrm{m}^{3}$.

Find the resistivity of the doped germanium if electron mobility is $0.36 \mathrm{~m}^{2} / V-s$.
Solution. Density of added impurity atoms is

$$
N_{d}=4.2 \times 10^{28} / 10^{6}=4.2 \times 10^{22} \text { atoms } / \mathrm{m}^{3}
$$

*     - It is the new name for the old unit of mho/m.

As seen, it is very large as compared to the intrinsic charge carrier density of $2.5 \times 10^{19} / \mathrm{m}^{3}$ which will, therefore, be neglected. Now, as seen from Art. 1.29.

$$
\begin{aligned}
& \sigma_{n}=N_{d} \text { e } \mu_{e}=4.2 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.36=2.42 \times 10^{3} \mathrm{~S} / \mathrm{m} \\
& \rho_{n}=1 / \sigma_{n}=1 / 2.42 \times 10^{3}=0.413 \times 10^{-3} \Omega-\mathrm{m} .
\end{aligned}
$$

Example 51.9. Compute the relative concentration of silicon atoms and electron-hole pairs at $300^{\circ} \mathrm{K}$. Also, calculate intrinsic resistivity of silicon. Given Avogadro's number $=6.02 \times 10^{23}$ atoms/ $g$-atom, density $=2.33 \times 10^{6} \mathrm{~g} / \mathrm{m}^{3}$, atomic Wt. $=28.1$, intrinsic carrier density $=1.5 \times 10^{16} \mathrm{~m}^{-3}$, $\mu_{e}=0.14 \mathrm{~m}^{2} / V-s, \mu_{h}=0.05 \mathrm{~m}^{2} / V-\mathrm{s}$.

Solution. $\quad n_{A}=$ Avogadro's No. $\times$ density/atomic Wt.

$$
=6.02 \times 10^{23} \times 2.33 \times 10^{16} / 28.1 \cong 5 \times 10^{28} \text { atoms } / \mathrm{m}^{3}
$$

Since intrinsic concentration $n_{i}$ i.e., electron-hole pairs $/ \mathrm{m}^{3}$ is $1.5 \times 10^{16}$

$$
\therefore \quad \frac{n_{A}}{n_{i}}=\frac{5 \times 10^{28}}{1.5 \times 10^{16}} \cong 3.3 \times 10^{12}
$$

It means that there are $3.3 \times 10^{12} \mathrm{Si}$ atoms for each electron-hole pair.
Since in a pure semiconductor, $n=p=n_{i}$, intrinsic conductivity is given by

$$
\sigma_{i}=n_{i} e\left(\mu_{e}+\mu_{h}\right)=1.5 \times 10^{16} \times 1.6 \times 10^{-19}(0.14+0.05)=0.456 \times 10^{-3} \mathrm{~S} / \mathrm{m}
$$

Hence, intrinsic resistivity is given by

$$
\rho_{i}=1 / \sigma_{i}=1 / 0.456 \times 10^{-3}=2193 \Omega-\mathrm{m}
$$

Example 51.10. Silicon is doped with acceptor atoms to a density of $10^{22} \mathrm{~m}^{-3}$. If it is assumed that all acceptor centres are ionized, calculate the conductivity of the extrinsic silicon. Given that intrinsic density is $1.4 \times 10^{16} \mathrm{~m}^{-3}, \mu_{e}=0.145 \mathrm{~m}^{-3}$ and $\mu_{h}=0.05 \mathrm{~m}^{-3}$.
(Electronic Devices \& Circuits, Pune Univ. 1992)
Solution. The minority carrier density can be found from the equation given in Art. 1.30, i.e $n p=n_{i}^{2}$.

Now, $\quad p=10^{22} \quad \therefore \quad n \times 10^{22}=\left(1.4 \times 10^{16}\right)^{2} ; \quad n=1.96 \times 10^{10} / \mathrm{m}^{3}$
Now, $\quad \sigma=n e \mu_{e}+p e \mu_{h}$

$$
=1.96 \times 10^{10} \times 0.145 \times 1.6 \times 10^{-19}+10^{22} \times 0.05 \times 1.6 \times 10^{-19}=80 \mathrm{~S} / \mathrm{m}
$$

### 51.31. Drift

Directed motion of charge carriers in semiconductors occurs through two mechanisms :
(i) charge drift under the influence of applied electric field and
(ii) diffusion of charge from a region of high charge density to one of low charge density.

Consider the drift phenomenon first. When no electric field is applied to the semiconductor which is above $0^{\circ} \mathrm{K}$, the conduction electrons (as well as holes) move within the crystal with random motion and repeatedly collide with each other and the fixed ions. Due to randomness of their motion, the net average velocity of these charge carriers in any given direction is zero. Hence, no current exists in the crystal under this condition of no field.

Now, consider the case when an electric field is applied to the crystal. Under the influence of this field, the charge carriers attain a directed motion which is superimposed on their random thermal motion. This results in a net average velocity called drift velocity in the direction of the applied electric field. Of course, electrons and holes move in opposite directions but because of their opposite charges, both produce current in the same direction. In extrinsic semiconductors, this current is essentially a majority carrier flow.

The drift velocity is proportional to electric field strength $E$, the constant of proportionality being called mobility $\mu$. The exact relation between the two is $v=\mu E$.

Let us find the value of drift current in a semiconductor.
(i) current density due to electron drift is $J_{e}=e \mu_{e} n E$ where $\mu_{e}$ is electron mobility, $n$ is electron density and $E$ is the electric field strength.
(ii) current density due to hole drift is $J_{h}=e \mu_{h} p E$ where $p$ is hole density.

Total current density due to electron and hole drift is

$$
\begin{aligned}
J & =J_{e}+J_{h}=e \mu_{e} n E+e \mu_{h} p E=e\left(n \mu_{e}+p \mu_{h}\right) E \\
& =e \mu_{h}(p+b n) E \quad \text { where } b=\mu_{e} / \mu_{h}
\end{aligned}
$$

### 51.32. Diffusion

It is gradual flow of charge from a region of high density to a region of low density. It is a forcefree process based on non-uniform distribution of charge carriers in a semiconductor crystal. It leads to an electric current without the benefit of an applied field. This flow or diffusion of carriers is proportional to the carrier density gradient, the constant of proportionality being called diffusion constant or diffusion coefficient $D$ which has a unit of $\mathrm{m}^{2} / \mathrm{s}$.

Current density due to hole diffusion is $J_{h}=-e D_{h} d p / d x$. Similarly, current density due to electron diffusion is $J_{e}=e D_{e} d n / d x$.
where $\quad D_{e}, D_{h}=$ electron and hole diffusion constants respectively
$d n / d x=$ density gradient of electrons
$d p / d x=$ density gradient of holes.
It is obvious that diffusion depends on charge in homogeneity or on the presence of a space gradient of charge density. It can occur in regions free of electric field. On the other hand, drift current is a function of both electric field and charge density.

Incidentally, it may be noted that, generally, diffusion leads to redistribution of charges which further results in the development of potential difference between different parts of the semiconductor. The electric field due to this potential difference sets up drift current in opposition to diffusion current. Final equilibrium is achieved when the potential difference developed becomes sufficiently large so as to create a drift current equal and opposite to the diffusion current thus resulting in zero net flow of current.

### 51.33. Combined Drift and Diffusion Currents

In semiconductors, drift and diffusion processes may be present simultaneously. The expressions for total electron and hole densities become.

$$
J_{e}=e \mu_{e} n E+e D_{e} d n / d x \mathrm{~A} / \mathrm{m}^{2} \quad \text { and } \quad J_{h}=e \mu_{h} p E-e D_{h} d p / d x \mathrm{~A} / \mathrm{m}^{2}
$$

### 51.34. Relation Between $D$ and $\mu$

Both diffusion constant and mobility are statistical thermodynamic phenomena and are related to each other by the following equation.

$$
\mu_{e}=\frac{e}{k T} D_{e} \text { and } \mu_{h}=\frac{e}{k T} D_{h} \quad \text { or } \quad \frac{D_{e}}{\mu_{e}}=\frac{D_{h}}{\mu_{h}}=\frac{k T}{e}=\frac{T}{11,600}
$$

The relationship is known as Einstein's equation
At $t=23^{\circ} \mathrm{C}, T=300^{\circ} \mathrm{K}$, hence $D / \mu=300 / 11,600=1 / 39$ or $\quad \mu=39 \mathrm{D}$
Example 51.11. Calculate diffusion constants for electrons and holes at $300^{\circ} \mathrm{K}$ in silicon if $\mu_{e}=0.15 \mathrm{~m}^{2} / V$-s and $\mu_{h}=0.05 \mathrm{~m}^{2} / V$-s.

Solution. According to Einstein's equation,

$$
\begin{aligned}
D & =\mu k T / e \quad \text { or } \quad D=\mu / \mathbf{3 9} \mathbf{m}^{2} / \mathrm{s} \quad \text { at } 300^{\circ} \mathrm{K} \\
D_{e} & =\mu_{e} / 39=0.15 / 39=\mathbf{3 . 8 5} \times \mathbf{1 0}^{-\mathbf{3}} \mathrm{m}^{2} / \mathrm{s} \\
D_{h} & =\mu_{h} / 39=0.05 / 39=\mathbf{6 . 4} \times \mathbf{1 0}^{-5} \mathbf{m}^{\mathbf{2}} / \mathrm{s}
\end{aligned}
$$

Example 51.12. Find the diffusion coefficients of holes and electrons for germanium at 300 K . The carrier mobilities in $\mathrm{cm}^{2} /$ volt-sec at 300 K for electrons and holes are respectively 3600 and 1700. Density of carriers is $2.5 \times 10^{13}$. Boltzmann constant, $k=1.38 \times 10^{23}$.
(U.P.S.C. Engg. Services, 1996)

Solution. According to the Einstein's equation,

$$
\begin{aligned}
D & =\mu k T / e \quad \text { or } \quad D=\mu / 39 \mathrm{~m}^{2} / \mathrm{s}-\text { at } 300^{\circ} \mathrm{K} \\
D_{e} & =\mu_{e} / 39=3600\left(\mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}\right) / 39=0.36\left(\mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}\right) / 39=9.2 \times 10^{-3} \mathrm{~m}^{2} / \mathrm{s} \\
D_{h} & =\mu_{h} / 39=1700\left(\mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}\right) / 39=0.17\left(\mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}\right) / 39=4.36 \times 10^{-3} \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

### 51.35. Recombination

Apart from drift and diffusion, a third phenomenon which occurs in semiconductors is called recombination that results from the collision of an electron with a hole.

The process is essentially the return of a free conduction electron to the valence band and is accompanied by the emission of energy. Obviously, the recombination rate is directly proportional to the carrier concentration for the simple reason that larger the number of carriers, the more likely is the occurrence of electron-hole recombination. This phenomenon is important in describing minority carrier flow.

As is well-known, in a semiconductor, thermal generation of electron-hole pairs also takes place continuously. Hence, there is net recombination rate given by the difference between the recombination and generation rates.

### 51.36. Carrier Life Time

It is defined as the time for which, on an average, a charge carrier will exist before recombination with a carrier of opposite charge. Its value varies from nanoseconds $\left(10^{-9}\right)$ to hundreds of microseconds $(\mu s)$ and depends on temperature and impurity concentration in the semiconductor material.

### 51.37. Total Carrier Flow

The total carrier flow in a semiconductor is the sum of the three flow phenomena discussed above. Each type of carrier has to be treated separately and the number of electrons or holes leaving the sample being accounted for by drift or diffusion or net recombination. The current in the semiconductor is then the sum of the electron and hole currents.

### 51.38. P-N Junction

It is possible to manufacture a single piece of a semiconductor material half of which is doped by P-type impurity and the other half by $N$-type impurity as shown in Fig. 51.32. The plane dividing the two zones is called junction. Theoretically, junction plane is assumed to lie where the density of donors and acceptors is equal. The $P-N$ junction is fundamental to the operation of diodes, transistors and other solid-state devices.

Let us see if anything unusual happens at the junction. It is found that following three phenomena take place :


Fig. 51.32

1. A thin depletion layer or region (also called space-charge region or transition region) is established on both sides of the junction and is so called because it is depleted of free charge carriers. Its thickness is about $10^{-6} \mathrm{~m}$.
2. A barrier potential or junction potential is developed across the junction.
3. The presence of depletion layer gives rise to junction and diffusion capacitances (Art. 51.5).

### 51.39. Formation of Depletion Layer

Suppose that a junction has just been formed. At that instant, holes are still in the $P$-region and electrons in the $N$-region. However, there is greater concentration of holes in $P$-region than in $N$-region (where they exist as minority carriers). Similarly, concentration of electrons is greater in $N$-region than in $P$-region (where they exist as minority carriers). This concentration differences establishes density gradient across the junction resulting in carrier diffusion. Holes diffuse from
 $P$ to $N$-region and electrons from $N$-to $P$-region and terminate their existence by recombination [Fig. $51.33(a)$ ]. This recombination of free and mobile electrons and holes produces the narrow region at the junction called depletion layer. It is so named because this region is devoid of (or depleted of) free and mobile charge carriers like electrons and holes-there being present only positive ions which are not free to move.


Fig. 51.33
It might seem from above that eventually all the holes from the $P$ side would diffuse to the $N$-side and all the electrons from the $N$-side would diffuse to the $P$-side but this does not occur due to the formation of ions on the two sides of the junction. The impurity atoms which provide these migratory electrons and holes are left behind in an ionized state bearing a charge which is opposite to that of the departed carrier. Also, these impurity ions, just like germanium atoms, are fixed in their positions in the crystal lattice in the $P$ - and $N$ - regions of the diode. Hence, as shown in Fig. 51.33 (b), they form parallel rows or 'plates' of opposite charges facing each other across the depletion layer. Obviously, row of fixed positive ions in the $N$-region is produced by the migration of electrons from the $N$ - to $P$ - region. Similarly, the row of fixed negative ions in the $P$-region is produced by the migration of holes from the $P$ - to $N$-region.

If a majority carrier (either an electron or a hole) tries to cross into depletion layer, it can meet either of the following two facts :
(i) either it can be trapped or captured by the row of fixed impurity ions of opposite sign which guard its own region. For example, a hole trying to approach the depletion layer may be neutralized by the row of fixed negative ions situated in the $P$-region itself

at the edge of the depletion layer. So will be the case with the electron trying to approach the depletion layer from $N$-region [Fig. 51.33 (c)]; or
(ii) it may succeed in entering the depletion layer where it will be repelled by the row of similarlycharged impurity ions guarding the other region. But its life will be cut short by recombination with a majority carrier of opposite sign which has similarly entered the depletion layer from the other half of the diode.
Ultimately, an equilibrium condition is reached when depletion layer has widened to such an extent that no electrons or holes can cross the $P-N$ junction.

### 51.40. Junction or Barrier Voltage

Even though depletion layer is cleared of charge carriers, it has oppositely-charged fixed rows of ions on its two sides. Because of this charge separation, an electric potential difference $V_{B}$ is established across the junction even when the junction is externally isolated (Fig. 51.34). It is known as junction or barrier potential. It stops further flow of carriers across the junction unless supplied by energy from an external source. At room temperature of $300^{\circ} \mathrm{K}, V_{B}$ is about 0.3 V for Ge and 0.7 V for Si.

The value of barrier voltage is given by $V_{B}=V_{T} \log _{e} N_{a} N_{d} / n_{i}^{2}$ where $N_{a}, N_{d}, n_{i}^{2}$ and $V_{T}$ have the meanings explained in Art. 51.29 and 34. The value of $V_{T}$ at room temperature of $300^{\circ} \mathrm{K}$ is given by

$$
\begin{array}{ll} 
& V_{T}=V_{300}=\frac{k T}{e}=\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}=26 \mathrm{mV} \\
\therefore & V_{B}=26 \log _{e}\left(N_{a} N_{d} / n_{i}^{2}\right) \mathrm{mV}
\end{array}
$$

Barrier voltage depends on doping density, electronic charge and temperature. For a given junction, the first two factors are constant, thus making $V_{B}$ dependent on temperature. With increase in temperature, more minority charge carriers are produced, leading to their increased drift across the junction. As a result, equilibrium occurs at a lower barrier potential. It is found that both for Ge and Si, $V_{B}$ decreases by about $2 \mathrm{mV} /{ }^{\circ} \mathrm{C}$.
$\therefore \quad \Delta V_{B}=-0.002 \Delta t \quad$ where $\Delta t$ is the rise in temperature in ${ }^{\circ} \mathrm{C}$.
The strong field set up by $V_{B}$ causes drift of carriers through depletion layer. As seen from |Fig. 51.35 (b), under the influence of this field, holes drift from $N$-to $P$-region and electrons from $P$-to - $N$ region. This drift current must be equal and opposite to the diffusion current [Fig. 51.35 (a)] because under condition of equilibrium and with no external supply, net current through the crystal is zero.


(a)


Drift Current
(b)

Fig. 51.34
Fig. 51.35
To summarize the main points of Art. 51.39 and 51.40, we have :

1. As soon as $P-N$ junction is formed, free electrons and holes start diffusing across the junction and recombining.
2. Their recombination leads to the appearance of a depletion layer across the junction which contains no mobile carriers but only immobile ions.
3. These immobile ions set up a barrier potential and hence an electric field which sets up drift current that is equal and opposite to the diffusion current when final equilibrium is reached.

Example 51.13. Calculate the barrier potential at room temperature for $P-N$ junction in silicon which is doped to a carrier density of $10^{21} \mathrm{~m}^{-3}$ on the $P$-side and $10^{22} \mathrm{~m}^{-3}$ on the $N$-side. The intrinsic carrier density for silicon is $1.4 \times 10^{16} \mathrm{~m}^{-3}$.
(Electronics-I, Bangalore Univ. 1992)
Solution. Using the relation given in Art 1.40, we have

$$
\begin{aligned}
V_{B} & =26 \log _{e}\left(N_{a} N_{d} / n_{i}^{2}\right) \mathrm{mV} \\
& =26 \log _{e} 10^{21} \times 10^{22} /\left(1.4 \times 10^{16}\right)^{2}=641 \mathrm{mV}=0.641 \mathrm{~V}
\end{aligned}
$$

Example 51.14. Calculate the change in barrier potential of a $P-N$ junction at $300^{\circ} \mathrm{K}$ if doping on the $N$-side is increased 1000 times while keeping doping on $P$-side unchanged.

Solution. As seen from Art. 51.40, at $300^{\circ} \mathrm{K}$

$$
\begin{array}{lrl}
\therefore & V_{B} & =26 \log _{e}\left(N_{a} N_{d} / n^{2}\right) \mathrm{mV} \\
\therefore & V_{B 1} & =26 \ln \left(N_{a 1} N_{d 1} / n_{i}^{2}\right) ; \quad V_{B 2}=26 \ln \left(N_{a 2} N_{d 2} / n_{i}^{2}\right) \\
\therefore \quad V_{B 2}-V_{B 1} & =26 \log _{e}\left(N_{a 2} N_{d 2}\right) /\left(N_{a 1} N_{d 1}\right) \\
& =26 \log _{e}\left(N_{d 2} / N_{d 1}\right)=26 \ln 1000=\mathbf{1 7 9} \mathbf{~ m V}
\end{array}
$$

### 51.41. Energy Band Diagram of a P-N Junction

Now, let us consider the operation of a $P-N$ junction in terms of its energy bands. At the instant of junction formation, energy bands of the trivalent impurity atoms in the $P$-region are at a slightly higher level than those of the pentavalent impurity atoms in the $N$-region as shown in Fig. 51.36. It is so because core attraction for valence electrons ( +3 ) in a trivalent atom is less than the core attraction for valence electrons ( +5 ) in a pentavalent atom. Consequently, trivalent valence electrons are in slightly higher orbit and, hence, at a higher energy level. However, there is some over-lap between respective bands of the two regions. Due to this reason, some high-energy electrons near the top of $N$-region conduction band diffuse into the lower part of the $P$-region conduction band. Soon after, they recombine with the holes in the valence band as shown in Fig. 51.36 (a). As diffusion continues, depletion layer begins to form. Another side-effect of this electron diffusion is that energy bands in

(a)

(b)

Fig. 51.36
the $N$-region shift downward due to loss of high-energy electrons. When the top of the conduction band in $N$-region reaches the same level as the bottom of the conduction band in $P$-region, further diffusion ceases and equilibrium condition is reached as shown in Fig. 51.36 (b). To an electron which might still try to diffuse across the junction, the path looks like a steep energy hill. It cannot climb this energy hill unless it receives energy from an external source.

### 51.42. Forward Biased P-N Junction

Suppose, positive battery terminal is connected to $P$-region of a semiconductor and the negative battery terminal to the $N$-region as shown in Fig. 51.37 (a). In that case the junction is said to be biased in the forward direction because it permits easy flow of current across the junction. This current flow may be explained in the following two ways :


Fig. 51.37
(i) As soon as battery connection is made, holes are repelled by the positive battery terminal and electrons by the negative battery terminal with the result that both the electrons and the holes are driven towards the junction where they recombine. This en masse movement of electrons to the left and that of holes to the right of the junction constitutes a large current flow through the semiconductor. Obviously, the junction offers low resistance in the forward direction.
A more detailed picture of carrier flow is as under :
As free electrons move to the left, new free electrons are injected by the negative battery terminal into the $N$-region of the semiconductor. Thus, a flow of electrons is set up in the wire connected to the negative battery terminal. As holes are driven towards the junction, more holes are created in the $P$-region by the breakage of covalent bonds (Art. 51.21). These newly-created holes are driven towards the junction to keep up a continuous supply. But the electrons so produced are attracted to the left by the positive battery terminal from where they go to the negative terminal and finally to the N -region of the crystal.

Incidentally, it may be noted that though there is movement of both electrons and holes inside the crystal, only free electrons move in the external circuit i.e. in the battery-connecting wires.

Note. There is also present an extremely small amount of current $I_{0}$ due to minority carriers on either side of the junction. But it is negligible as compared to forward current which is due to majority carriers. This current $I_{0}$ depends on temperature but is almost independent of applied voltage.
(ii) Another way to explain current flow in forward direction is to say that forward bias of $V$ volts lowers the barrier potential to $\left(V-V_{B}\right)$ which now allows more current to flow across the junction [Fig. 51.37 (b)].
Incidentally, it may be noted that forward bias reduces the thickness of the depletion layer as shown in Fig. 51.38.

Energy band diagram for forward bias is shown in Fig. 51.39. By comparing this figure with Fig.
51.36 it is seen that energy hill has been reduced. Because of this reduction, conduction electrons in $N$-region are able to cross over to $P$-region. After reaching there, each electron falls into a hole (path $A$ ) and becomes a valence electron. In this way, it is able to continue its journey towards the left end of the crystal.


Fig. 51.38
Fig. 51.39

### 51.43. Forward V/I Characteristic

A typical $V / I$ characteristic for a forwardbiased $P-N$ junction is shown in Fig. 51.40. It is seen that forward current rises exponentially with the applied forward voltage. However, at ordinary room temperature, a p.d. of about 0.3 V is required before a reasonable amount of forward current starts flowing in a germanium junction. This voltage is known as threshold voltage ( $V_{t h}$ ) or cut-in voltage or knee voltage $V_{K}$. It is practically the same as barrier voltage $V_{B}$. Its value for silicon junction is about 0.7 volt. For $V<V_{t h}$, current flow is negligible. But as applied voltage increases beyond the threshold value, the forward current increases sharply. If forward voltage is increased beyond a certain safe value, it will produce an extremely large current which


Fig. 51.40 may destroy the junction due to overheating. Ge devices can stand junction temperatures around $100^{\circ} \mathrm{C}$ whereas $S i$ units can function upto $175^{\circ} \mathrm{C}$.

Obviously, the forward-biased junction has a low resistance. For point $B$ in Fig. 51.40, the forward resistance for Si is

$$
R_{F}=0.8 \mathrm{~V} / 20 \mathrm{~mA}=40 \Omega
$$

Similarly, for point $A$ on the $G e$ curve, $R_{F}=0.36 \mathrm{~V} / 20 \mathrm{~mA}=18 \Omega$
In practice, this static forward resistance is not used. Instead, the dynamic resistance or incremental resistance or ac resistance of the juncion is used. It is given by the reciprocal of the slope of the forward characteristic.

$$
r_{a c}=\frac{1}{\Delta I_{F} / \Delta V_{F}}=\frac{\Delta V_{F}}{\Delta I_{F}}
$$

Here,

$$
\Delta V_{F}=0.19 \mathrm{~V} \quad \text { and } \quad \Delta I_{F}=37.6 \mathrm{~mA}
$$

$$
\therefore \quad r_{a c}=\frac{0.19}{37.6 \times 10^{-3}} \cong 5 \Omega
$$

### 51.44. Reverse Biased P-N Junction

When battery connections to the semiconductor are made as shown in Fig. 51.41 (a), the junction is said to reverse-biased. In this case, holes are attracted by the negative battery terminal and electrons by the positive terminal so that both holes and electrons move away from the junction and away from each other. Since there is no electron-hole combination, no current flows and the junction offers high resistance.

Another way of looking at the process is that in this case, the applied voltage increases the barrier potential to $\left(V+V_{B}\right)$, thereby blocking the flow of majority carriers.


Fig. 51.41
Incidentally, it may be noted that under reverse bias condition, width of depletion layer is increased because of increased barrier potential as shown in Fig. 51.42.

Although, in this case, there is practically no current due to majority carriers, yet there is a small


Fig. 51.42 amount of current (a few $\mu \mathrm{A}$ only) due to the flow of minority carriers across the junction. As explained earlier in Art. 51.24, due to thermal energy, there are always generated some holes in the $N$-type region and some electrons in the $P$-type region of the semiconductor as shown in Fig. 51.28. The battery drives these minority carriers across the junction thereby producing a small current called reverse current or reverse saturation current $I_{0}$ or $I_{S}$. Since minority carriers are thermally-generated, $I_{0}$ is extremely temperature dependent. For the same reason, forward current is also temperature dependent but to a much less degree because minority current forms a very small percentage of the majority current. The name saturation has been used because we cannot get minority current more than what is produced by thermal energy. In other words, $I_{S}$ does not increase with increase in reverse bias.
$I_{S}$ is found to increase approximately 7 percent per $0^{\circ} \mathrm{C}$ rise in temperature both for Ge and Si . Since, $(1.07)^{10}=2$, it means that reverse current doubles for every $10^{\circ} \mathrm{C}$ rise in temperature. It is worth noting that reverse saturation current is also referred to as leakage current of the $P-N$ junction diode.

With reverse bias, energy hill becomes too steep for majority carriers to go up the hill and cross over.

### 51.45. Reverse V/I Characteristic

As said earlier, the reverse saturation current is also referred to as leakage current of the $P-N$ junction. Fig. 51.43 shows $V / I$ characteristics of a reverse-biased $P-N$ junction. It is seen that as reverse voltage is increased from zero, the reverse current quickly rises to its maximum or saturation value. Keeping temperature constant as the reverse voltage is increased, $I_{o}$ is found to increase only slightly. This slight increase is due to the impurities on the surface of the semiconductor which behaves as a resistor and hence obeys Ohm's law. This gives rise to a very small current called surface leakage


Fig. 51.43 current. Unlike the main leakage (or saturation) current, this surface leakage current is independent of temperature but depends on the magnitude of the reverse voltage.

A reverse-biased junction can be represented by a very large resistance. As seen from Fig. 51.43, in the case of Si , for a reverse voltage of about $15 \mathrm{~V}, I_{0}=10 \mu \mathrm{~A}$. Hence, reverse resistance is $R_{R}=15$ $\mathrm{V} / 10 \mu \mathrm{~A}=1.5 \mathrm{M} \Omega$

### 51.46. Hall Effect

If a specimen (whether of a metal or a semiconductor) carrying a current $I$ is placed in a transverse magnetic field of flux density $B$, an electric field is developed along a direction perpendicular to both $B$ and $I$. This phenomenon is known as Hall effect and is used for the following purposes :

1. to determine whether a semiconductor is of $N$-type or $P$-type;
2. to find carrier concentration;
3. to measure the conductivity of the material;
4. to find carrier mobility;
5. to detect and measure magnetic fields one million times smaller than that of earth with the help of Hall-effect magnetometers.
As shown in Fig. 51.44 (a), a current $I$ is flowing through the semiconductor in the direction $M N$ under the influence of an external applied electric field $E_{L}$. Obviously, electrons comprising this current move along $N M$ with a velocity of $v_{e}$. Consider one such electron shown in the figure. The direction of the force exerted on it by the magnetic field $B$ can be found by using Fleming's left-hand rule and is as shown*. The magnitude of force is $B e v_{e}$.

Under the influence of this force,


Fig. 51.44 electrons will tend to crowd towards the left side of semiconductor. This collection of electrons to one side gives rise to electric potential difference $V_{H}$ (called Hall voltage) and hence to an electric field $E_{H}$. This field begins to prevent additional electrons from arriving there. Force exerted by this field on the electrons is $e E_{H}$. Equilibrium is established when the two oppositely-directed magnetic and electric forces acting on the electron become equal in magnitude.

* While using the rule, the middle finger must point in the direction of conventional current $\overline{i . e} \overline{-} \overline{-} \overline{-} \overline{-} \overline{-} \bar{\sim}$ and not along the direction of electronic current i.e. along $N M$.
i.e. $\quad B e v_{e}=e E_{H}$
or

$$
\begin{equation*}
B v_{e}=E_{H} \tag{i}
\end{equation*}
$$

Also
$\therefore \quad B \mu_{e} E_{L}^{e}=E_{H}$ or $\mu_{e} B=E_{H} / E_{L}$

$$
v_{e}=\mu_{e} E_{L}
$$

Hence, knowing $E_{L}$ and $B$ and measuring $E_{H}$, we can find electron mobility.
Now, current density $\quad J=I / A=n e v_{e} A / A=n e v_{e}$
Substituting the value of $v_{\mathrm{e}}$ from $(i)$ above, we get

$$
J=n e E_{H} / B \quad \text { or } \quad n=J B / e E_{H}
$$

This equation may be used to find electron density.
Now, Hall coefficient, $\quad R_{H}=1 /$ charge density $=1 /$ ne or $R_{H}=E_{H} / J B$
As shown in Art. 51.26. $\quad \mu_{e}=1 /$ ne $\rho=R_{H} / \rho$
where $\rho$ is the resistivity of the semiconductor.
It may be noted that the above treatment is equally applicable to $P$-type semiconductors where current flow is made up of hole movement. If holes were to move in the same direction as that of the electrons in Fig. 51.44 (a), then polarity of Hall voltage $E_{H}$ would be reversed as shown in Fig. 51.44 (b). In fact, it was the observation of the polarity of Hall voltage associated with $P$-type material which led to the concept of a positive hole.

Example 51.15. The Hall-coefficient of a specimen of doped semiconductor is $3.66 \times 10^{-4} \mathrm{~m}^{3}$ $C^{-1}$ and the resistivity of the specimen is $8.93 \times 10^{-3} \Omega-m$. Determine the carrier mobility in $m^{2} v^{-1} s^{-1}$.

Solution. Using equation $\mu_{e}=R_{H} / \rho$

$$
\mu_{e}=\frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}}=0.041 \mathrm{~m}^{2} \mathrm{v}^{-1} \mathrm{~s}^{-1}
$$

Example 51.16. A sample of $N$-type semiconductor has a hall coefficient of $160 \mathrm{~cm}^{3} /$ Coulomb. If is resistivity is $0.16 \Omega$-cm, estimate the electron mobility in the sample,

Solution. Using the relation, $\mu_{e}=R_{H} / \rho$, we have

$$
\mu_{e}=\frac{160}{0.16}=1000 \mathrm{~cm}^{2} / \text { Volt-Sec. }
$$

Example 51.17. A current of $50 A$ is passed through a metal strip, which is subjected to a magnetic flux of density $1.2 \mathrm{~Wb} / \mathrm{m}^{2}$. The magnetic field is directed at right angles to the current direction and the thickness of the strip in the direction of magnetic field is 0.5 mm . The Hall voltage is found to be 100 V. Calculate the number of conduction electrons per cubic metre in the metal.

Solution. We know that $n=\frac{J B}{e E_{H}}$
Since $J=I / A=I / d . t$ where $d$ is the distance between the two surfaces between the ends across which the elecric field is measured. The above equation can be rewritten as,

$$
n=\frac{(I / A) B}{e E_{H}}=\frac{(I / d . t) B}{e V_{H} / d}=\frac{B I}{e V_{H} t} \quad\left(\therefore E_{H}=V_{H} / d\right)
$$

Substituting the values of $V_{H}, t, B, I$ and $e$, we get

$$
n=\frac{1.2 \times 50}{\left(1.6 \times 10^{-19}\right) \times 100 \times\left(0.5 \times 10^{-3}\right)}=7.51 \times 10^{21} / \mathrm{m}^{3}
$$

Example 51.18. An N-type semiconductor has a resistivity of $20 \times 10^{-2}$ ohm-m. The mobility of electrons through a separate experiment was found to be $100 \times 10^{-4} \mathrm{~m}^{2} v^{-1} \mathrm{~s}^{-1}$. Find the number of electron carriers per $m^{3}$.

Solution. Using the equation, $\mu_{e}=1 /$ ne $\rho$, we have

$$
\begin{aligned}
n & =\frac{1}{e \mu_{e} \rho}=\frac{1}{1.6 \times 10^{-19} \times\left(100 \times 10^{-4}\right) \times\left(20 \times 10^{-2}\right)} \\
& =3.1 \times 10^{21} / \mathrm{m}^{3}
\end{aligned}
$$

Example 51.19. The Hall-coefficient of a specimen of doped silicon is found to be $3.66 \times 10^{-4}$ $m^{3} C$; the resistivity of the specimen is $8.93 \times 10^{-3} \Omega-m$. Find the mobility and density of charge carriers assuming single carrier conduction.
(U.P.S.C. Engg. Services 2002)

Solution. $\quad R_{H}=3.66 \times 10^{-4} \mathrm{~m}^{3} \mathrm{C} ; \rho=8.93 \times 10^{-3} \Omega-\mathrm{m}$.

$$
\begin{aligned}
\mu & =\sigma R_{H}=\frac{1}{\rho} R_{H}=\frac{1}{8.93 \times 10^{-3}} \times 3.66 \times 10^{-4}=0.041 \mathrm{~m}^{2} / \mathrm{V} . \mathrm{s} \\
R_{H} & =\frac{1}{n e} \text { or } n=\frac{1}{R_{H} e}=\frac{1}{\left(3.66 \times 10^{-4}\right) \times\left(1.6 \times 10^{-19}\right)}=1.7 \times 10^{22} / \mathrm{m}^{3}
\end{aligned}
$$

Example 51.20. Resistivity of a sample semiconductor is $9 m \Omega-m$. Its holes have mobility of $0.03 \mathrm{~m}^{2} /$ Vs. Calculate the value of Hall-coefficient of the sample. (U.P.S.C. Engg. Services 2002)

Solution.

$$
\begin{aligned}
\rho & =9 \mathrm{~m} \Omega-\mathrm{m}=9 \times 10^{-3} \Omega-\mathrm{m}, \mu_{p}=0.03 \mathrm{~m}^{2} / \mathrm{V} . \mathrm{s} \\
\sigma & =\frac{1}{\rho}=\frac{1}{9 \times 10^{-3}}=111.1 \mathrm{~S} / \mathrm{m} \\
\mu_{p} & =\sigma R_{H} \\
R_{H} & =\frac{\mu_{p}}{\sigma}=\frac{0.03}{111.1}=2.7 \times 10^{-4} \mathrm{~m}^{3} \mathrm{C}
\end{aligned}
$$

## Tutorial Problem No. 51.1

1. Compute the intrinsic conductivity of a specimen of pure silicon at room temperature given that $n_{i}=1.4 \times 10^{6} \mathrm{~m}^{-3}, \mu_{e}=0.145 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, \mu_{\mu}=0.05 \mathrm{~m}^{2} / \mathrm{V}-s$ and $e=1.6 \times 10^{-19} \mathrm{C}$. Also, calculate the individual contributions from electrons and holes.

$$
\left[0.437 \times 10^{-3} \mathrm{~S} / \mathrm{m} ; 0.325 \times 10^{-3} \mathrm{~S} / \mathrm{m} ; 0.112 \times 10^{-3} \mathrm{~S} / \mathrm{m}\right]
$$

2. Find (i) conductivity and (ii) resistance of a bar of pure silicon of length 1 cm and cross-sectional area $1 \mathrm{~mm}^{2}$ at $300^{\circ} \mathrm{K}$. Given : $n_{i}=1.5 \times 10^{16}$ per m${ }^{3}, \mu_{e}=0.13 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, \mu_{h}=0.05 \mathrm{~m}^{2} / \mathrm{V}$-s and $e=1.6 \times 10^{-19} \mathrm{C}$.
[(i) $4.32 \times 10^{-4} \mathrm{~S} / \mathrm{m}$ (ii) $23.15 \mathrm{M} \Omega$ ]
3. A specimen of silicon is doped with acceptor impurity to a density of $10^{22}$ atoms per $\mathrm{m}^{3}$. Given that $n_{i}=1.4 \times 10^{16}$ per m${ }^{3}, \mu_{e}=0.145 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, \mu_{h}=0.05 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, e=1.6 \times 10^{-19} \mathrm{C}$. All impurity atoms may be assumed to be ionized.
[nearly $80 \mathrm{~S} / \mathrm{m}$ ]
4. Calculate the conductivity of a specimen of pure $S i$ at room temperature of $300^{\circ} \mathrm{K}$ for which $n_{i}=1.5 \times 10^{16} \mathrm{~m}^{-3}, \mu_{e}=0.13 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, \mu_{h}=0.05 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}, e=1.6 \times 10^{-19} \mathrm{C}$.
The Si specimen is now doped 2 parts per $10^{8}$ of a donor impurity. If there are $5 \times 10^{28} \mathrm{Si}$ atoms $/ \mathrm{m}^{3}$, calculate its conductivity. By what factor has the conductivity increased?
$\left[4.32 \times 10^{-4} \mathrm{~S} / \mathrm{m} ; \mathbf{2 0 . 8 ~ S} / \mathrm{m} ; \simeq \mathbf{4 8 , 0 0 0}\right]$
5. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.36 \mathrm{~m}^{2} / \mathrm{V}-\mathrm{s}$ and $0.17 \mathrm{~m}^{2} / \mathrm{V}$-s respectively. If the electron and hole densities are each equal to $2.5 \times 10^{19}$ per $\mathrm{m}^{3}$, calculate the conductivity.
[2.12 S/m](Electronic-I, Bangalore Univ.)
6. A p.d. of 10 V is applied longitudinally to a rectangular specimen of intrinsic germanium of length 2.5 cm , width 0.4 cm and thickness 0.15 cm . Calculate at room temperature
(i) electron and hole drift velocities;
(ii) the conductivity of intrinsic Ge if intrinsic carrier density is $=2.5 \times 10^{19} / \mathrm{m}^{3}$;
(iii) the total current.

Given, $\mu_{e}=0.38 \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}, \mu_{h}=0.18 \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}, k=1.38 \times 10^{-23} \mathrm{~J}^{-\mathrm{deg}^{-1}}, e=1.6 \times 10^{-19} \mathrm{C}$
[(i) $152 \mathrm{~m} / \mathrm{s}, 72 \mathrm{~m} / \mathrm{s}$ (ii) $2.24 \mathrm{~S} / \mathrm{m}$ (iii) 5.38 mA$]$
(Applied Electronics \& Circuits, Grad. I.E.T.E. 1987)
5. The resistivity of a dopoed silicon material is $9 \times 10^{-3}$. The Hall coefficient is $3.6 \times 10^{-4} \mathrm{Coulomb}^{-1}$. Assuring single carrier conduction, find the mobility and density of charge carriers. $e=1.6 \times 10^{-19}$ Coulomb.
(U.P.S.C. Engg. Services 1994)
6. Consider an intrinsic Ge bar with material time constant of $100 \mu \mathrm{sec}$ across a cross-section of $1 \mathrm{~mm}^{2}$ and length 1 cm . One side of the bar is illuminated with $10^{15}$ photons $/ \mathrm{sec}$. Assume that each incident photon generate one electron hole pair and these are uniformly distributed throughout the bar. Find the bar resistance under constant excitation at room temperature.
(Electronic Devices and Circuits, Nagpur Univ. Summer, 2004)
7. Find the concentration of holes and electrons in a p-type germanium at 300 K , if the conductivity is 100 per ohm-cm. Also find these values for $n$-type silicon, if the conductivity is 0.1 per ohm-cms. Given that
for germanium $n_{i}=2.5 \times 10^{13} / \mathrm{cm}^{3}$
$M_{\mathrm{n}}=3800 \mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}, \quad \mathrm{M}_{\mathrm{p}}=1800 \mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}$
for silicon, $n_{i}=1.5 \times 10^{10}$ per cm ${ }^{3}$
$M_{n}=1300 \mathrm{~cm}^{2} / V-\mathrm{s}$ and $\mathrm{M}_{\mathrm{p}}=500 \mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}$
(Electronics Engg; Bangalore Univ. 2004)
8. A germanium P-N junction at 300 K has the following parameters; $\mathrm{N}_{D}=5 \times 10^{18} / \mathrm{cm}^{3}$, $\mathrm{N}_{\mathrm{A}}=6 \times 10^{16} / \mathrm{cm}^{3}, n_{i}=1.5 \times 10^{10} / \mathrm{cm}^{3}$; Calculate the minority electron density in the P-region and the minority hole density in the N -region.
(Electronics Engg. Bangalore Univ. 2003)

## OBJECTIVE TESTS - 51

1. The total energy of a revolving electron in an atom can
(a) have any value above zero
(b) never be positive
(c) never be negative
(d) not be calculated.
2. An atom is said to be ionised when any one of its orbiting electron
(a) Jumps from one orbit to another
(b) is raised to a higher orbit
(c) comes to the ground state
(d) is completely removed.
3. The maximum number of electrons which the M -shell of an atom can contain is
(a) 32
(b) 8
(c) 18
(d) 50 .
4. Electronic distribution of an Si atom is
(a) $2,10,2$
(b) $2,8,4$
(c) $2,7,5$
(d) $2,4,8$.
5. Semiconductor materials have $\qquad$ bonds.
(a) ionic
(b) covalent
(c) mutual
(d) metallic.
6. The maximum number of electrons which the valence shell of an atom can have is
(a) 6
(b) 8
(c) 18
(d) 2
7. Silicon has $Z=14$. Its outermost orbit is
(a) partially filled
(b) half filled
(c) completely occupied
(d) empty
8. Major part of the current in an intrinsic semiconductor is due to
(a) conduction-band electrons
(b) valence-band electrons
(c) holes in the valence band
(d) thermally-generated electron.
9. Conduction electrons have more mobility than holes because they
(a) are lighter
(b) experience collisions less frequently
(c) have negative charge
(d) need less energy to move them.
10. Doping materials are called impurities because they
(a) decrease the number of charge carriers
(b) change the chemical properties of semiconductors
(c) make semiconductors less than 100 percent pure
(d) alter the crystal structures of the pure semiconductors.
11. Current flow in a semiconductor depends on the phenomenon of
(a) drift
(b) diffusion
(c) recombination
(d) all of the above.
12. The process of adding impurities to a pure semiconductor is called
(a) mixing
(b) doping
(c) diffusing
(d) refining.
13. The most widely used semiconucting material in electronic devices is
(a) germanium
(b) sillicon
(c) copper
(d) carbon
14. Electon-hole pairs are produced by
(a) recombination
(b) thermal energy
(c) ionization
(d) doping
15. Recombination takes place when
(a) an electron falls into a hole
(b) a positive and a negative ion bond together
(c) a valence electron becomes a conduction
(d) a crystal is formed
16. When a $P-N$ junction is formed, diffusion current causes
(a) mixing of current carriers
(b) forward bias
(c) reverse bias
(d) barrier potential.
17. The leakage current of a $P-N$ diode is caused by
(a) heat energy
(b) chemical energy
(c) barrier potential
(d) majority carriers.
18. Electronic components which are made of a semiconductor material are often called
$\qquad$ devices.
(a) solid-state
(b) silicon
(c) germanium
(d) intrinsic.
19. Any voltage that is connected across a $P-N$ junction is called $\qquad$ voltage.
(a) breakdown
(b) barrier
(c) bias
(d) reverse.
20. The area within a semiconductor diode where no mobile current carriers exist when it is formed is called $\qquad$ region.
(a) depletion
(b) saturation
(c) potential barrier
(d) space charge.
21. The depletion region of a semiconductor diode is due to
(a) reverse biasing
(b) forward biasing
(c) crystal doping
(d) migration of mobile charge carriers.
22. If an intrinsic semiconductor is doped with a very small amount of boron, then in the extinsic semiconductor so formed, the number of electrons and holes will,
(a) decrease
(b) increase and decrease respectively
(c) increase
(d) decrease and increase respectively.
23. Two initially identical samples $A$ and $B$ of pure germanium are doped with donors to concentrations of $1 \times 10^{20}$ and $3 \times 10^{20}$ repectively. If the hole concentration in A is $9 \times 10^{12}$, then the hole concentration in B at the same temperature will be
(a) $310^{12} \mathrm{~m}^{-3}$
(b) $7 \times 10^{12} \mathrm{~m}^{-3}$
(c) $11 \times 10^{12} \mathrm{~m}^{-3}$
(d) $27 \times 10^{12} \mathrm{~m}^{-3}$
24. Hall effect is observed in a specimen when it (metal or a semiconductor) is carrying current and is placed in a magnetic field. The resultant electric field inside the specimen will be in
(a) a direction normal to both current and magnetic field
(b) the direction of current
(c) a direction antiparallel to the magnetic field
(d) an arbitrary direction depending upon the conductivity of the specimen.

## ANSWERS

| 1. $(b)$ | 2. $(d)$ | 3. $(c)$ | 4. $(b)$ | 5. $(b)$ | 6. $(b)$ | 7. $(b)$ |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 8. $(a)$ | 9. $(d)$ | 10. $(d)$ | 11. $(d)$ | 12. $(b)$ | 13. $(b)$ | 14. $(b)$ |  |
| 15. $(a)$ | 16. $(d)$ | 17. $(a)$ | 18. $(a)$ | 19. $(c)$ | 20. $(a)$ | 21. $(d)$ |  |
| 22. $(d)$ | 23. $(a)$ | 24. $(a)$ |  |  |  |  |  |

GO To FIRST


[^0]:    A worker at a hydro-electric power station is carrying out routine maintenance on a highvoltage insulator made of a ceramic material.

[^1]:    * According to Bohr's theory, $p_{1}=n . h / 2 \pi$.

[^2]:    * A single germanium atom has 32 electrons out of which only four electrons take part in electrical properties of germanium, the remaining 28 electrons being tightly bound to the nucleus. The four electrons revolve in the outermost shell and are called valence electrons.

[^3]:    * For the present discussion, Fermi level may be defined as $\overline{\text { the }} \overline{\text { energy }} \overline{\text { which }} \overline{\text { corresponds to the centre }} \overline{\text { of }}$ gravity of conduction electrons and holes weighted according to their energies.

[^4]:    * Since the number of electrons as compared to the number of holes increases with temperature, the position of Fermi level also changes considereably with temperature.

